BLIND CHANNEL-SHORTENING FOR WIRELESS COMMUNICATION WITH A RECURSIVE DELAYED PREDICTION-ERROR FILTER

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ABSTRACT

Linear Prediction in Prediction-Error Filter implementation is a well-known technique for communication channel equalization. Here, we show how a recursive version may be used with additional prediction delay for blind channel-shortening of wireless channels with Multi-Carrier Modulation signals. We observe the recursive prediction-error filter impulse response adapts to minimum-phase form, required for stability. The recursive mode allows the filter length to be significantly shorter than in non-recursive form. Good equalization of minimum-phase channels is obtained. Additional prediction delay ameliorates the filter response to maximum-phase terms within a channel, so that partial equalization may be achieved other than for channels with severe maximum-phase terms.

Index Terms**—** Multi-Carrier Modulation, Channel-Shortening, Equalization, Linear Predictor, Prediction-Error Filter

1. INTRODUCTION

The widely-used Multi-Carrier Modulation (MCM) scheme mitigates the effects of Inter-Symbol Interference (ISI) by including a Cyclic Prefix (CP) between symbols. Channel equalization for MCM communication remains of interest however, because the CP reduces the available signal bandwidth. The design challenge is to *sufficiently* equalize a channel—that is, "shorten" its impulse response (IR)—to permit a short CP to be used with less impact on bandwidth.

"Blind" equalization exploits expected general properties of a signal, that are "restored", rather than using known particular signal content such as a training-sequence. Two well-known methods for blind shortening of MCM signals are the "Sum-Squared Autocorrelation Minimization" (SAM) algorithm [1] that uses the low autocorrelation property of an MCM signal, and "Multicarrier Equalization by Restoration of RedundancY" (MERRY) [2], that uses redundancy of CP content. Linear prediction, using the prediction error as the output, is a further way of blindly equalizing a channel where the transmitted signal has a low-autocorrelation property, and has been used in partial solutions, for example [3]. A Linear Predictor may also be used for channel-shortening, shortening being implemented by increasing the prediction delay period [4].

Here we are interested in shortening wireless channels' IRs, which are largely characterized by multiple non-dispersive paths with z-plane zeros. A wireless-channel equalizer using a linear finite impulse response (FIR) filter is long, containing several times more taps than the channel IR. Wireless channels' IRs are also commonly mixed-phase (i.e. with zeros outside the z-plane unit-circle). This is a difficulty for SAM, MERRY and the Linear Predictor, because the signal correlation information they use does not effectively distinguish minimum-phase and maximum-phase terms. (In [6], the cyclostationarity of over-sampled signals is used to overcome the maximum/minimum-phase limitation of a linear predictor, a more complex technique.)

In this paper we propose a recursive version of a linear prediction-error filter as a channel shortener for wireless channels. The recursive filter length is much shorter than that of a predictor FIR, when applied to the multipath wireless channel. For stability, the recursive filter IR must be minimum-phase. Now a non-recursive prediction-error filter in forward form has a minimum-phase IR, and in backward form a maximum-phase IR [5]. In this research we observe that, when initialized to zero, the recursive forward prediction-error filter also always adapts to a minimum-phase IR. A problem is that the filter cannot equalize terms in a channel IR that are maximum-phase. In [7] a recursive FLP is used with QAM signals to equalize an underwater acoustic multipath channel, augmented by a bussgang filter and Decision-directed (DD) equalizer to correct the phase. The latter techniques are not readily applicable to MCM signals. Here, we propose to ameliorate the issue by increasing the delay of the predictor (also described for channel shortening of DSL channels in [4]). Short maximum-phase terms of a channel can be ignored, so not causing spoiling minimum-phase terms in the equalizer.

2. THE RECURSIVE PREDICTION-ERROR FILTER

The Linear Predictor algorithm predicts the value of a sample of a sequence from a linear combination of the other samples. A communication channel and a recursive prediction-error filter are shown in Fig 1. An input signal *x(n)* to the channel **h** is received as *d(n)*. The signal *y(n)* is both the prediction error and the system output. The recursive filter **w** output *d´(n)* is the predicted value of *d(n)*; the prediction is "forward", i.e. based on older system samples. The prediction delay *D* determines which previous samples of *y(n)* up to the length of the vector **y** will be used.

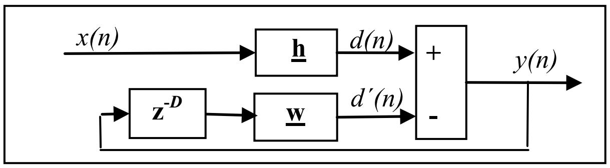

Figure 1 Recursive Prediction-Error Filter

Thus:

$$y(n) = d(n) - d'(n)$$
$$= d(n) - \mathbf{w}^{\mathrm{H}}.\mathbf{y}(n-D)$$
$$y(n).\left[1 + \sum_{k=D}^{L_w} w_k^{*}.z^{-k}\right] = d(n)$$

where w_k is the filter coefficient for the sample $y(n\text{-}k)$, L_w is the length of **w**, and $D \geq 1$. It follows that the IR of the recursive filter has z-transform:

$$\frac{1}{w_r(z)} = \frac{1}{1 + \sum_{k=D}^{L_w} w_k.z^{-k}}$$

The expected value of $|y(n)|^2$ may be minimized in a stochastic manner. The instantaneous gradient of $|y(n)|^2$ w.r.t **w**, assuming a stationary channel **h** and that past values of *y(n)* are not a function of the instantaneous **w,** is:

$$\nabla_w\{|y(n)|^2\} = \nabla_w\{y(n).y^*(n)\}$$
$$= \nabla_w\{(d(n) - \mathbf{w}^{\mathrm{H}}.\mathbf{y}(n-D)).(d^*(n) - \mathbf{y}^{\mathbf{H}}(n-D).\mathbf{w})\}$$
$$= -2.y^*(n).\mathbf{y}(n-D)$$

The prediction filter **w** may then be adaptively updated in the Least Mean Squares (LMS) manner as:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + 2.\mu.y^*(n).\mathbf{y}(n-D),$$

where μ is the adaptation coefficient.

Notice that upon convergence, when **w** has reached a steady state, then: $E[y^*(n).\mathbf{y}(n-D)] = 0$. So, when the cost function is minimized, *y(n)* is uncorrelated with previous samples of itself (of delay *D* and older) within the **y** vector. The recursive prediction-error filter thus delivers an output that is uncorrelated for delays of *D* and greater. Now for MCM signals, the input *x(n)* is uncorrelated. The channel **h** introduces correlated terms to the signal due to the time-spread of its impulse response. The action of the filter is to predict and remove those correlation terms, and therefore recover the original signal.

When delay $D > 1$, correlation delays of less than *D* are ignored by the predictor and thus the taps in a channel impulse response separated in time by less than *D* are not equalized. The channel may then be considered shortened to length *D*, rather than equalized.

Convergence of the prediction filter implicitly uses the autocorrelation of *y(n)*. A channel zero $z_o = (1 - a_o z^{-1})$ may be equalized, and minimize *y(n)* autocorrelation, by an equivalent pole in the recursive filter, i.e. $p_o = 1/(1 - a_o z^{-1})$. However, if the filter pole value is "flipped" (i.e. a_o replaced by a value $1/a_o^{*}$, and * denotes conjugation) the *y(n)* autocorrelation sequence is also minimized. ("Zero flipping" is further discussed in [8].) It is known that a conventional forward prediction-error filter is minimum-phase [5].

It is expected that the recursive filter when initialized to zero also will adapt to a minimum-phase set of zeros, whether or not the channel zeros are minimum-phase. Thus where that channel presents a maximum-phase zero, the filter responds with a "flipped" minimum-phase zero, which lengths rather than shortens the channel. However, by using a prediction delay *D* of up to the CP-length, shorter maximum-phase terms may be ignored.

3. SYSTEM MODEL AND SIMULATION

Simulation tests of the recursive delayed prediction-error filter were conducted with modelled wireless channels. The system model is the same as in foregoing work such as [1] and [4], shown in Fig 2; model code available from [9] was the original basis.

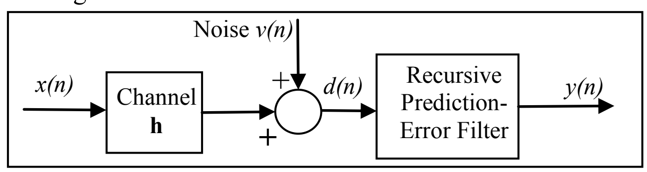

Figure 2 Transmission System Model

The wireless channel **h** model used has Rayleigh fading, Doppler frequency-offset and 4 paths. It is specified such that the channel IR is limited in length to 6.4µs, i.e.64 taps at a signal sample-rate of 10Ms/s. (The shortest and longest path lengths are 2000m and 3830m.) Snapshots of the IR of this channel at a range of times were used to test the shortener performance over a range of channel types. Results for four representative channel snapshot IRs (Ch1...Ch4) are reported here. The non-zero tap values for the channels' IRs are given in Table 1. (Note that the channel IRs used here are real-valued for ease of results display, but that in general a baseband channel IR and the corresponding equalizer IR will be complex-valued. However, the general results and phenomena reported here are also valid for complex channels.)

	h(0)	**h(26)**	**h(37)**	**h(61)**	**Phase**
Ch 1	0.8329	0.1608	-0.2688	-0.1654	Min
Ch 2	0.3771	0.7041	-0.1639	0.5789	Mixed
Ch 3	-0.4809	0.8681	-0.1080	0.0592	Mixed
Ch 4	-0.4615	-0.2440	0.8528	-0.0129	Mixed

Table 1 Wireless Channel Impulse Responses

The input signal *x(n)* is the MCM-modulated signal; the symbol FFT size is 512 samples and the CP-length 32; the signal has 256 real sub-carriers. The sample rate is 10Ms/s, so the symbol of 544 samples has a period of 54.4µs.

The added noise *v(n)* is white, uncorrelated with the channel output, and zero-mean. Results reported here are for a Signal-to-Noise Ratio (SNR) of 20dB. (A correction to the modelled code of [9] was included in the SNR calculation—true modelled SNR is higher than the value indicated in the earlier code.) The length of the recursive equalizing filter **w**, L_w, including the prediction delay *D,* is set to the specified maximum IR length of the channel **h**, i.e. 64 taps. Equalization performance is measured using Achievable Bit-Rate (ABR), which is the bit-rate obtainable by the defined MCM signal and sample-rate, given the effective (equalized) channel **c** and the SNR; it is evaluated as in [1]. The filter **w** is adaptively updated in the LMS manner described earlier. For SNR=20dB, μ is set to 0.0003, selected empirically for speed and stability.

The prediction delay for the main results is set to *D*=28, so that output signal autocorrelation (and thus effective channel IR length) is "don't-care" for up to delays of 28, just less than the CP length. Results for *D*=1 are used for comparison.

4. SIMULATION TESTS AND RESULTS

The tests of channel equalization were of 30-symbol duration scenarios. The filter **w** was initialized to all-zeros, and ABR monitored for the duration of the scenario. The results shown are the scenario ABR, averaged over 16 runs, the channel and effective channel IRs and the filter IR. The channels Ch1...Ch4 are as defined in Table 1.

The filter IR shown is $\mathbf{w_R}(z) = 1 + \sum_{k=D}^{L_w} w_k . z^{-k}$. Thus, $w_R(0)$ = 1, and $w_R(1)...w_R(D\text{-}1)$ are fixed to zero, where *D*>1. When L_w=64 and *D*=28, the number of adaptive taps in both **w** and $\mathbf{w_R}$ is 36. Effective channel **c** is obtained as $\mathbf{c} = \mathbf{h}^*(\mathbf{w_R})^{-1}$, (where * = convolution operator).

Minimum-phase channel Ch1. The convergence results for prediction delays of both *D*=1 and *D*=28 (Fig 3) are shown. Filter non-zero taps evidently equalize the corresponding channel taps. For *D*=1, all channel terms are equalized (*h*(26), *h*(37) and *h*(61)); for *D*=28 only the latter two taps. The ABR for *D*=1 converges to about 90% of the MFB, whereas for *D*=28 it is degraded to about 80% of MFB. Convergence time to 80% of the steady state is about 11 and 9 symbols (0.6ms and 0.5ms) respectively.

Close inspection of the filter IRs for *D*=28 shows two features that help to explain the ABR performance reduction. First, the equalization of channel taps *h*(37) and *h*(61) is less effective than for *D*=1. Secondly the unequalized channel tap *h*(26) convolves with the filter taps *w*(37) and *w*(61). Filter tap *w*(63) responds and compensates for this, but extra non-zero terms in the effective channel are visible at for example *c*(87).

Mixed-phase channel Ch2. The ABR performance (Fig 4) is significantly lower than for the minimum-phase channel; the converged ABR is about 30% of MFB. Nonetheless the performance is significantly better than that of the unequalized channel. Further, the settled ABR for *D*=28 is higher than for *D*=1, by ignoring certain maximum-phase channel terms.

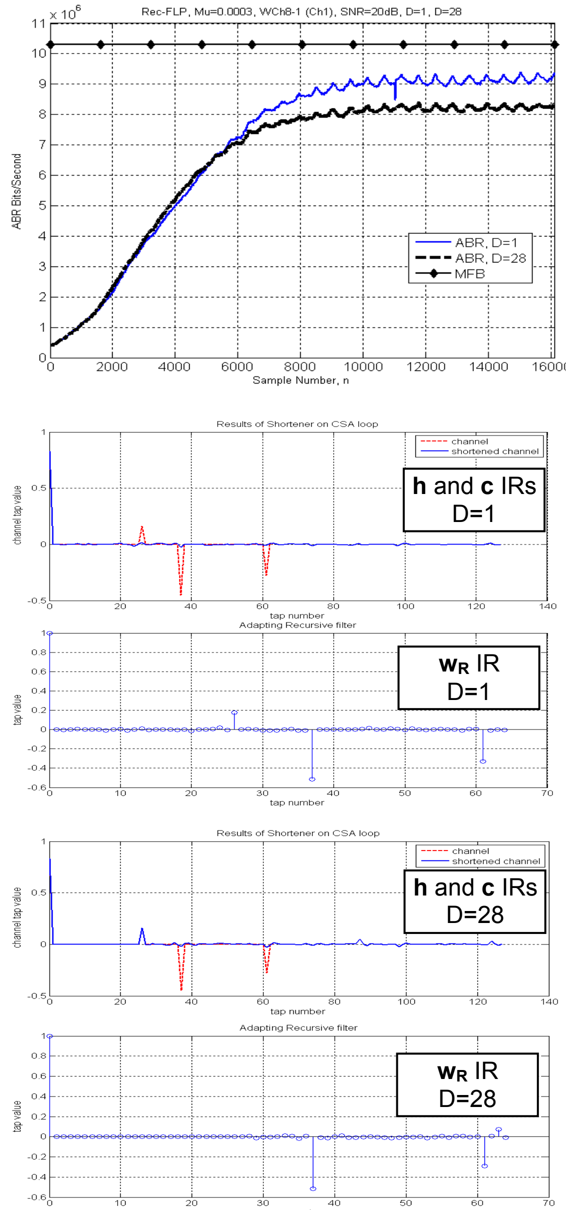

Figure 3 Ch 1 ABR and Ch/Filter IRs

For this channel the filter tap values are less obvious than for Ch 1. Some observations illustrate the operation of the recursive predictor with mixed phase channels. The delay *D*=28 ensures that the correlation of *h*(0) and *h*(26) channel taps is ignored and (correctly) unequalized. The main adapted filter tap is *w*(35), derived from the correlation of channel taps *h*(26) and *h*(61). It provides some equalization of *h*(61), but introduces unhelpful energy at *c*(35) in the effective channel. There is little equalization of *h*(37), as the correlation of *h*(0)/*h*(37) is low.

Short mixed-phase channel Ch3. The main part of the IR of Ch 3, ignoring minor terms, is maximum-phase but is shorter than the CP length. The unequalized channel delivers an adequate ABR performance (Fig 5), but when *D*=1, the recursive predictor creates a minimum-phase filter that lengthens rather than equalizes the channel, and the ABR reduces to 15% of MFB from the unequalized level of 60%. However, by including the prediction delay of *D*=28 the maximum-phase term of this channel is ignored, and the filter action is minor resulting in a modest increase in ABR—hence the value of the prediction delay in avoiding unhelpful responses to short maximum-phase channel terms.

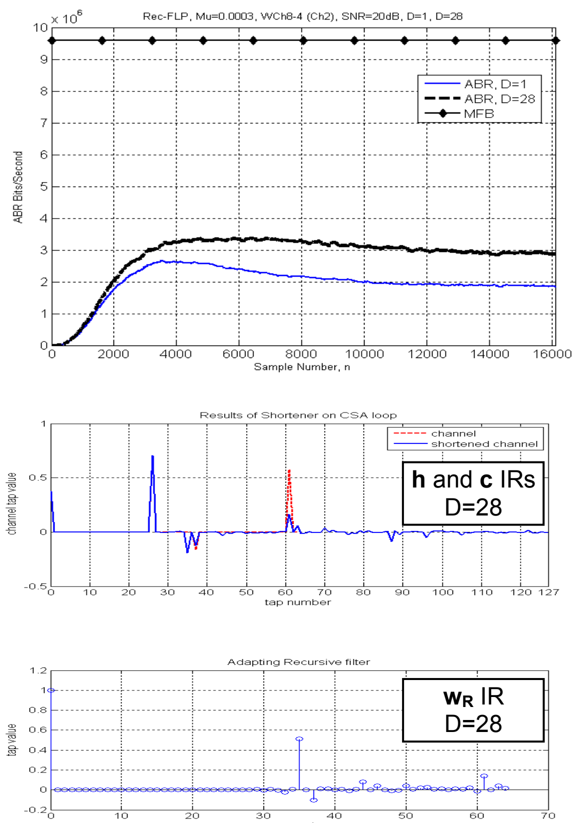

Figure 4 Mix-Ph Ch 2 ABR and Ch/Filter IRs

Mixed-phase channel with long max-phase term. The IR of Ch 4 has a maximum-phase term longer than the CP length. The unequalized channel introduces sufficient ISI to force ABR to zero. The recursive predictor creates a minimum-phase filter that, as for Ch 3 when *D*=1, lengthens rather than equalizes the channel, and does not improve the ABR to a significant level. Unlike Ch 3, a prediction delay of *D*=28 cannot improve equalization, since the channel maximum-phase term is longer than the CP. For this type of channel therefore, the recursive prediction error filter must be considered to be unsuitable for equalization.

General phenomena. We draw attention here to several distinct behaviours of the recursive prediction error filter.

1. Basic Operation. For the minimum-phase channel, the recursive filter adapts to the channel IR and cancels its effects. Addition of prediction delay beyond *D*=1 allows corresponding correlation terms to be ignored, and the effective channel is then shortened rather than equalized.

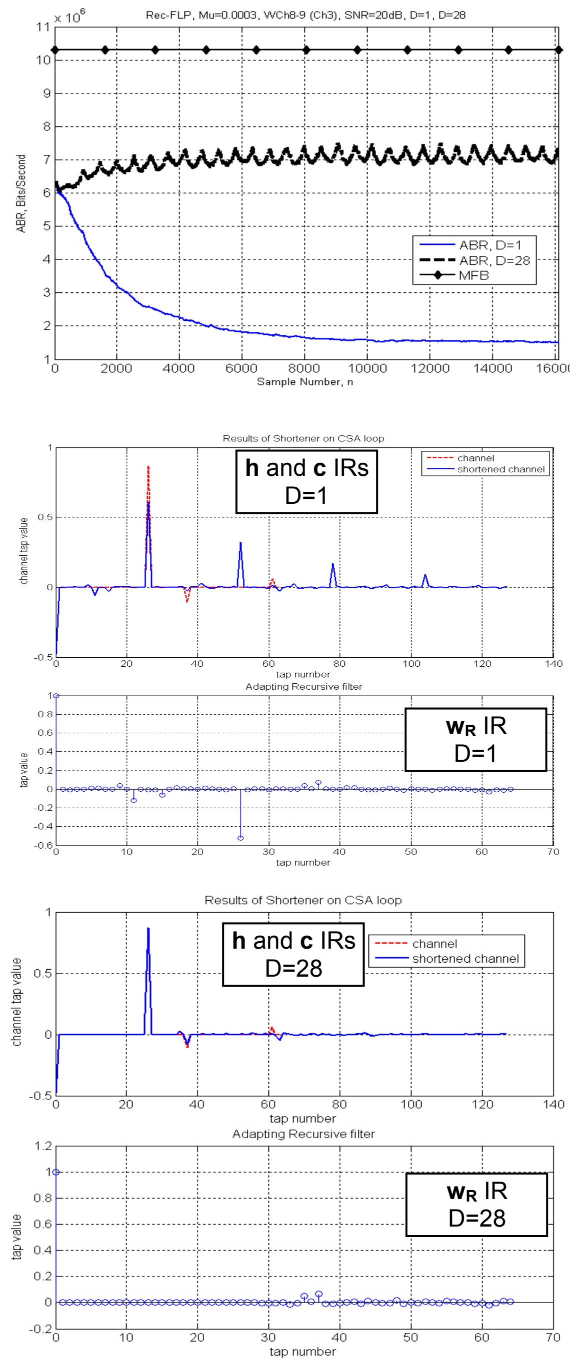

Figure 5 Short Mix-Ph Ch 3 ABR and IRs

2. When some of the channel energy remains unequalized a dilution of the measure of correlation of the remaining signal occurs. Qualitatively, the predictor compares the correlation energy for a given correlation delay to the whole energy of the equalized signal. Correct filter tap coefficients' values are lowered and do not fully cancel channel terms. This dilution effect is a cause of the lowered ABR for the minimum-phase channel when *D*=28.

3. Channel taps that are not fully equalized have a cross-correlation. This is reflected in the filter, and further

degrades the equalization. For example, for the mixed-phase channel the filter has a significant coefficient $w(35)$, due to correlation of the unequalized channel taps $h(26)$ and $h(61)$, and as a result the effective channel has energy at $c(35)$ that degrades the equalization.

4. The residual terms in the effective channel IR (the taps that are unequalized whether by intent when D>1, or because the equalization is incomplete for any reason) will convolve with the coefficients of the recursive filter **w**. This has the effect of propagating further non-zero coefficients into the later part of the effective channel in a recursive manner. This was observed for the minimum-phase channel Ch1. Unequalized channel tap $c(26)$ convolves with the filter tap $w(37)$, causing a compensating response in $w(63)$, and filter tap $w(63)$ causing an extra non-zero terms in the effective channel at $c(87)$. The propagated terms, by making **c** longer, reduce ABR. So use of extra predictor delay to shorten rather than equalize channel IR has this further disadvantage.

5. For the channels reported here, and for all of a wide range of other channel sizes and types, the recursive prediction-error filter IR always converges to a minimum-phase form, ensuring its behaviour remains stable. However, no proof that the recursive version of the FLP must do so is known to the authors.

The results here contain poor as well as good performance, whereas those reported in [4] with the non-recursive delayed FLP and mainly minimum-phase ADSL channels always provided good equalization. Good performance is also reported in [7] with the recursive FLP and bussgang and DD techniques for acoustic multipath channels, but for QAM signals that admit the use of the extra techniques. Consistent good equalization for the more-difficult MCM signals with mixed-phase channels and the recursive FLP cannot be claimed. However, the technique of additional prediction delay with the recursive FLP clearly improves the blind shortening performance obtainable for MCM and mixed-phase channels.

5. CONCLUSION

The results demonstrate that an adaptive recursive prediction-error filter may be used to blindly equalize a zero-based wireless channel of specified maximum length when MCM signals are transmitted, but with limitations for mixed-phase channels. The minimum-phase characteristic of a prediction-error filter is also observed in the recursive version, ensuring it is stable. A significant advantage for wireless channels is the reduced length of the recursive filter. It is only as long as the maximum specified length of the channel IR, reducing computational load in a receiver.

The recursive filter is most suitable as an equalizer for minimum-phase channels. It cannot fully equalize a mixed-phase or maximum-phase channel. However, the results show that the recursive prediction-error filter can nonetheless adequately equalize mixed-phase channels, depending on the channel IR; though for channels with long maximum-phase terms it is likely to be ineffective. The use of additional prediction delay *D*, of up to the length of the MCM signal CP, reduces the degrading of the equalization by short maximum-phase terms. This increases the range of mixed-phase channel IRs that may be adequately or partially equalized. However, it also introduces effects of correlation dilution, and propagation of residual unequalized channel terms into a longer effective channel.

The recursive prediction-error filter is therefore a suitable and efficient blind equaliser for MCM signals on minimum-phase wireless channels. For channels with IR of varying minimum-, mixed- and maximum-phase, it will deliver variable effectiveness. Use of prediction delay ameliorates the effects of mixed-phase channels, so that only severe maximum-phase channels cannot be at least partially equalized.

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