LOCALIZATION IN WIRELESS NETWORKS BASED ON JOINTLY COMPRESSED SENSING

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ABSTRACT

Location sensing is fundamental for supporting wireless communications services. This paper exploits the signal correlation structure observed in an indoor localization environment in order to provide accurate position estimation by means of a limited amount of signal-strength measurements. Because the mobile devices have limited processing power and battery capacity, the proposed received signal-strength localization protocol avoids putting on extra computational overhead on the mobile device by performing the position estimation at the Access Points (APs). Since the APs observe correlated signals from the mobile devices, the introduced method exploits the common structure of the received measurements in order to jointly estimate the positions precisely. The evaluation of the proposed protocol is performed on real laboratory data through experiments that quantify the impact of the system parameters on the location error.

1. INTRODUCTION

Wireless communication systems are widely deployed to provide various types of services. Location management is critical in order to support many fundamental network services in medicine, entertainment and commerce. Localization or location sensing is a process to determine the physical position of a mobile user and can be accomplished through the efficient gathering of locally correlated network data [1].

Some of the existing localization systems are based on the signal strength transmitted from the APs and require the user to compute its own position. On the other hand, some systems consider the periodic broadcasting of the mobile users and compute their positions remotely, in a central unit or at a specific AP. In a typical scenario, a number of APs capture signals transmitted from several mobile users. Then, the received signals are combined to estimate the positions of the nodes.

Current literature shows a growing interest on leveraging existing infrastructure (*e.g.*, WiFi access points), to perform location sensing [2]. The main advantage is the avoidance of the cost of deploying extra specialized infrastructure for localization. The majority of the signal-strength based systems can be classified into two categories, namely map-based and distance-prediction based.

Distance-prediction based systems estimate the position of a mobile user by measuring its distances from multiple reference points (*e.g.*, APs, anchor nodes) according to a known radio propagation model. The main challenge arising in these systems is the difficulty to formulate a reliable radio propagation model due to the shadow fading, low probability of line-of-sight (LOS) path, and specific parameters (floor layout, moving objects) that appear in an indoor environment [3].

On the contrary, map-based systems create signature maps in order to represent the physical space by capturing the variations of the dynamic nature of indoor radio propagation [4]. The localization process typically consists of two phases. During the training phase, location fingerprints are collected from each potential position of the mobile node. Then, during the runtime phase, the position of the user is estimated by comparing its runtime signal strength measurements with training observations. Map-based systems are fairly accurate but they are time consuming due to the training phase [5].

A significant observation is that the localization problem presents an inherent sparsity in the space domain. Particularly, if we use a grid-based representation of the physical space where each cell of the grid indicates a position, we observe that the mobile location is sparse over the ground plane. A vector is called sparse if its elements are mostly zeros in a certain domain. The sparse nature of the problem motivated us to explore the compressed sensing theory (CS) in order to reformulate the location estimation as a sparse-approximation problem [6].

Compressed sensing provides a novel framework that allows the recovery of a signal that is sparse under a certain basis while enabling a significant reduction in the sampling costs compared to traditional methods. CS is based on the observation that a small collection of linear projections of a sparse signal contains enough information for reconstruction.

In [6] we introduced a signal strength based localization scheme that considers the measurements received at the APs and offers accurate position estimation when compared with traditional localization algorithms. In this work, we extend our previous study by exploiting the intra- and inter- signal correlation structures present in the localization application in order to provide accurate signal reconstruction by means of a limited amount of signal-strength measurements. The main goal is to reduce the amount of data required for accurate localization in order to minimize the time that a mobile node needs to stay at one position. The collection time reduction is fundamental in cases where the mobile user is not willing or permitted to remain stationary for a long time. Moreover, by performing the localization task at a central unit, we reduce the energy consumption of the mobile device. This is important since, in spite of improvements in energy consumption, battery capacity grows slowly and power management is still a challenge in mobile computing.

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Considering the above insights, in this paper we apply the distributed compressed sensing (DCS) theory that rests on the joint sparsity of a signal ensemble and provides effective signal recovery by jointly reconstructing all the signals precisely [7].

The organization of this paper is as follows. Section 2 presents the CS and DCS background. In Section 3, we discuss the location estimation approach via spatial sparsity and we introduce the proposed localization framework. The performance of the proposed approach is studied in Section 4, while we conclude in Section 5.

2. COMPRESSED SENSING BACKGROUND

Compressed sensing builds on the observation that a signal which has a sparse representation in one basis can be reconstructed from far fewer data or measurements than what is assumed by the Nyquist-Shannon sampling theorem [8, 9]. Denote the discrete-time signal $\mathbf{x}$, an $D \times 1$ column vector in $\mathbb{R}^D$ that can be represented in terms of a basis $\mathbf{\Psi}$ of $D \times 1$ vectors $\{\psi_i\}_{i=1}^{D}$ such that

$$\mathbf{x} = \mathbf{\Psi}\mathbf{b}, \tag{1}$$

where $\mathbf{b}$ is an $D \times 1$ column vector with K non-zero elements ($\|b\|_0 = K$). The signal $\mathbf{x}$ is called K-sparse if it is a linear combination of only K basis vectors, where $K \ll D$.

CS exploits sparsity to acquire a compressive signal representation without collecting D samples. Particularly, the original signal is reconstructed by considering M linear projections $y(m) = \langle \mathbf{x}, \phi_m^T \rangle$ of the signal $\mathbf{x}$ into M measurement basis vectors $\{\phi_m\}_{m=1}^{M}$. The symbol T denotes the transpose of the vector and $\langle \cdot, \cdot \rangle$ denotes the inner product. We can represent the measurement $y(m)$ in a $M \times 1$ vector $\mathbf{y}$ and the measurement basis vectors ϕ_m^T as rows in a $M \times D$ matrix $\mathbf{\Phi}$. Therefore, the measurement process can be written as

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{b}. \tag{2}$$

An essential requirement is that the rows $\{\phi_m\}$ of $\mathbf{\Phi}$ cannot sparsely represent the columns $\{\psi_i\}$ of $\mathbf{\Psi}$ (incoherence property). It has been proved that independent and identically distributed (i.i.d.) Gaussian or Bernoulli vectors provide universal measurement bases that are incoherent with any basis matrix $\mathbf{\Psi}$ with high probability.

When the above conditions hold, the original sparse vector $\mathbf{b}$ can be recovered as the solution of an ℓ_1 optimization problem:

$$\widehat{\mathbf{b}} = \arg\min \|\mathbf{b}\|_1 \;\; s.t.\; \mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{b}. \tag{3}$$

Commonly used approaches to solve (3) are based on convex relaxation and greedy strategies such as the Orthogonal Matching Pursuit (OMP) [10].

The principles of the CS theory have been applied for signal reconstruction, detection and classification by exploiting the intra-signal structures at a single collection point (*e.g.*, a sensor). Multiple collection points usually capture related phenomena and a joint structure is expected for the signals ensemble, in addition to the intra-signal correlation between the individual measurements. Recently, the authors in [7] introduced a theory for *distributed compressed sensing* (DCS) that exploits both intra- and inter-signal correlation structures. DCS considers the joint sparsity of a signal ensemble to obtain accurate signal reconstruction.

Three models for jointly sparse signals have been introduced and algorithms for joint recovery have been proposed [7]. According to the first model, all signals share a common sparse component but they have sparse innovations components that are unique to each signal. This situation arises in applications where global signal processes affect all sensors while local noise affects individual sensors, *e.g.*, a network of multiple microphones placed in a venue. The second model describes signals that are constructed from the same sparse set of basis vectors but with different coefficients. This is the case in our indoor positioning application where the signals are sparse in the spatial domain, yet different propagation path losses cause different attenuations among the received signals at each AP. In this case, accurate recovery is achieved via the DCS-SOMP algorithm [11]. Finally, the third model is an extension of the first model in that the common component need not be sparse in any basis.

3. INDOOR LOCALIZATION VIA SPATIAL SPARSITY

In our setting, we consider a WLAN scenario where a set of APs are connected and one mobile node, equipped with an active wireless adapter card, is located in an indoor environment. An AP that listens to a channel receives the beacons sent by the mobile user (at that channel) periodically and records its *received signal strength indicator* (RSSI) values. We refer the interested reader to [12] for further details concerning the characteristics of RSSI.

The location estimation can be obtained by searching for the sparsest solution of an under-determined linear system of equations that arises in localization. The location of one mobile user can be modeled as an 1-sparse vector which is the case of highest sparsity. Consequently, the CS framework can be applied to efficiently detect the position of a node.

Recently, independently of our work in [6], another sparse approximation approach to mobile localization has been introduced. The authors in [13] propose a two-phase signal strength based system that applies a CS algorithm to improve the final estimation. The localization, unlike in our approach, is performed at the device of the mobile user. More specifically, the main goal in [13] is to minimize the number of APs needed for accurate position. Therefore, a measurement basis $\mathbf{\Phi}$ is defined as an AP selection operator that considers a subset of available APs. The objective of our proposed approach is to reduce the number of measurements needed per AP so as to effectively decrease the time a mobile device is required to remain stationary at a specific position in order to be located.

3.1 Proposed Framework

In the indoor localization scenario, the APs measure signals transmitted from the mobile user that are each individually sparse in a certain basis and also correlated among the APs. The goal is to exploit the spatial correlations among the measurements in order to jointly estimate the sparse coefficients precisely. The joint structure of the signal ensembles at the APs makes the theory of DCS applicable for indoor positioning in WLANs (cf. Figure 1). Specifically, the problem setup follows the same properties associated to the second joint model, as it was described in Section 2.

Our proposed localization framework consists of two phases (cf. Figure 2). During the training phase, the char-

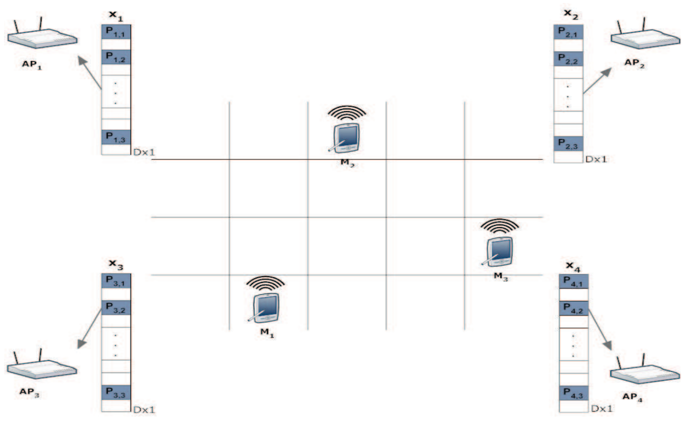

Figure 1: Common sparse support model in a WLAN indoor environment. Each AP gathers signal-strength vectors $\mathbf{x}$, transmitted from the mobile users, on its local grid. Each vector has non-zero coefficients at the positions occupied by the mobile users.

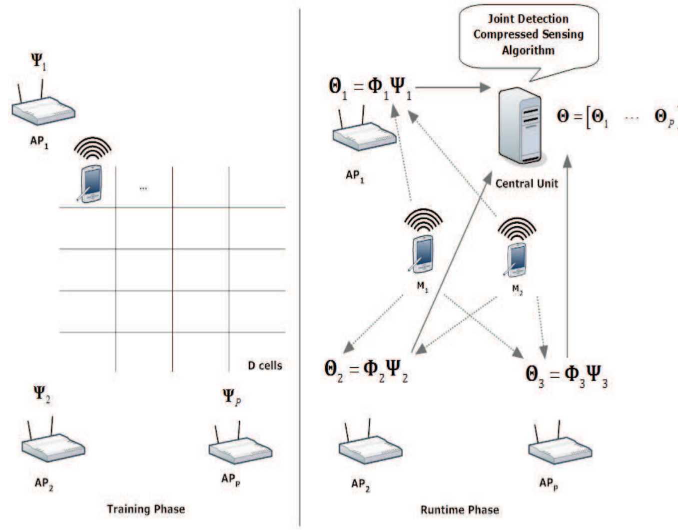

Figure 2: Proposed localization framework.

acteristics of the signal propagation in the indoor environment are captured. More specifically, the APs gather signal-strength data from beacons received from a mobile device at each cell of the grid in order to construct a signature map of the spatial space. In the runtime phase, each AP collects RSSI measurements for a period of time, projects its signal into another incoherent basis and then transmits just a subset of the resulting coefficients to the central unit. The central unit collects the RSSI fingerprints from the APs and applies a *Joint Detection Compressed Sensing* (JDCS) protocol to estimate the positions of the mobile devices.

At this point, it is emphasized that in our previous approach [6], during the runtime phase the APs collect measurements for a period of time and then they compute the average value. The average RSSI value is sent to the collection point where the CS algorithm is applied. In the proposed approach, each AP considers the complete time series of the RSSI measurements in a cell, in order to represent the signal's properties for a given environment.

An AP collects signal strength measurements in order to locate the mobile node on its local grid. We assume the physical space of D dimensions and we consider J APs and 1 mobile user ($K = 1 \ll D$). Our objective is to determine the location $\mathbf{n} = [x, y]^T$ of the mobile node by detecting the cell in which he is located.

We discretize the physical space to create a finite set of cells $\mathscr{B} = \{p_1, p_2, \ldots, p_D\}$. The sparse vector $\mathbf{b} \in \mathbb{R}^D$ selects elements from $\mathscr{B}$. All elements of $\mathbf{b}$ are zero except $b(p) = 1$, where p is the index of the grid point that corresponds to the estimated position of node.

During the training phase, each AP j constructs the signature basis matrix $\mathbf{\Psi}_j$ where

$$\mathbf{\Psi}_j = \begin{pmatrix} P_{1,1,j} & P_{1,2,j} & \cdots & P_{1,D,j} \\ P_{2,1,j} & P_{2,2,j} & \cdots & P_{2,D,j} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N,1,j} & P_{N,2,j} & \cdots & P_{N,D,j} \end{pmatrix}.$$

Particularly, each column of $\mathbf{\Psi}_j$ corresponds to the N received RSSI signals the j-th AP perceives from each potential location that a node may occupy. We denote the t-th RSSI sample the j-th AP receives from a node at location k as $P_{t,k,j}$.

In the runtime phase, each AP collects RSSI measurements from the mobile node on its local area. We can express the runtime measurements of the signal $\mathbf{x}_j$ received at the j-th AP as:

$$\mathbf{x}_j = \mathbf{\Psi}_j \mathbf{b}, \tag{4}$$

where $\mathbf{b}$ is supported on the same $\mathscr{B}' \subset \mathscr{B}$ and $|\mathscr{B}'| = 1$. We observe that all $\mathbf{x}_j$ signals are 1-sparse and are constructed from the same elements of signature basis matrix $\mathbf{\Psi}_j$ but with different coefficients. Hence, the joint sparsity model 2 in [7] applies on our case.

Our approach aims to reduce the total number of measurements needed, during the runtime phase, to detect the sparse coefficients in the common support set. Employing a reconstruction algorithm in the central unit that exploits the common structure among the signals will facilitate this task.

During the runtime phase, each AP constructs a random measurement matrix $\mathbf{\Phi}_j \in \mathbb{R}^{M \times N}$, where M is the number of RSSI measurements each AP receives from the mobile users. Therefore, the set of RSSI measurements for the mobile device is expressed as:

$$\mathbf{y}_j = \mathbf{\Phi}_j \mathbf{x}_j = \mathbf{\Theta}_j \mathbf{b} \quad j = 1, \ldots J, \tag{5}$$

where $\mathbf{\Theta}_j = \mathbf{\Phi}_j \mathbf{\Psi}_j$. The set $\{\mathbf{\Phi}_j\}_{j=1}^J$ contains matrices with i.i.d. random variables from a Gaussian propability density function with mean zero and variance $1/D$. The Gaussian measurement matrix $\mathbf{\Phi}_j$ is incoherent with the basis matrix $\mathbf{\Psi}_j$ with high probability to satisfy the conditions imposed by the theory of CS (universality property).

The localization process is performed in the central unit where the location of the mobile node is jointly estimated via an iterative algorithm. Motivated by recent work [11], we adapt the DCS-SOMP algorithm to the requirements of the localization problem (Algorithm 1).

The JDCS algorithm is based on the common sparse support set among the J APs in order to detect the position of the mobile user. Particularly, in Step 1 the sparse coefficient vector $\mathbf{b}_j$ is estimated for each AP. Because of the joint sparsity,

Algorithm 1 JDCS

Inputs: $\mathbf{\Theta}_j$, RSSI measurements $\mathbf{y}_j$, $j = 1, \ldots, J$.
Outputs: Support set $\mathscr{B}'$
Initialize: $\mathscr{B}' \neq \emptyset$

for $j = 1$ to J **do**
  $\mathbf{r}_j = \mathbf{y}_j$
end for
1. $\mathbf{b}_j \leftarrow \mathbf{\Theta}_j^T \mathbf{r}_j, \quad j = 1, \ldots J$
2. $\mathbf{b} = \sum_{j=1}^{J} |\mathbf{b}_j|$
3. $p_l = \arg\max \mathbf{b}$
4. $\mathscr{B}' \leftarrow [\mathscr{B}' \quad p_l]$
return $\mathscr{B}'$

the index of the maximum value of the $\mathbf{b}_j$ vectors should coincide for each AP. In Step 2 the individual absolute $\mathbf{b}_j$ vectors from each AP are added together, in order to estimate the position of the coefficient that has the maximum energy among all the APs. Therefore, in Step 4, the position of the mobile user is detected as the index that corresponds to the largest coefficient of vector $\mathbf{b}$. Finally, the detection algorithm returns the set $\mathscr{B}'$ that contains the estimated position of the mobile user.

In an indoor environment, the central unit applies the JDCS algorithm to select elements from the finite set of cells $\mathscr{B}$. Thanks to the common sparsity of the structure among the signals, JDCS offers accurate estimates of the position of the node.

4. EXPERIMENTAL RESULTS

In this Section, we study the performance of the proposed scheme in terms of location error under different RSSI variations charasteristics. The location error is defined as the Euclidean distance between the estimated position of the mobile node and the true one. The purpose of the experiment is to evaluate the accuracy of the proposed method compared to traditional localization techniques using real data measurements. The evaluation of the joint compressed sensing localization protocol took place in a laboratory area of $7m \times 12m$. For this area, a grid-based structure was considered with cells of size $50cm \times 50cm$. During the training phase, RSSI observations from a mobile device were recorded for a period of 109 seconds. The signature map included measurements from 109 different cells. If no RSSI observations are found for a candidate location in the grid at an AP, the corresponding RSSI entry in the signature map is set to -100 dBm. The experiment involved a total of 13 APs.

Figure 3 indicates the effectiveness of the proposed framework when compared with two well-known localization schemes, the KNN (K = 1) [14] and the Bayesian indoor localization method [15]. To evaluate the performance of the algorithms in terms of location error under different signal-to-noise (SNR) values, we added white Gaussian noise to the runtime measurements. For each possible position in the testbed, we performed 100 Monte Carlo simulations for different SNR values. In this experiment, the JDCS algorithm employs all the available runtime RSSI measurements.

Figure 3 shows that for a certain SNR, the proposed JDCS method achieves a significant reduction in the location error when compared to the KNN and the Bayesian localization techniques. Particularly, for low SNR (SNR = -110 dB) values, we observe that the proposed algorithm cuts the mean location error by approximately 67% and 80% of the value corresponding to the KNN and the Bayesian algorithms, respectively.

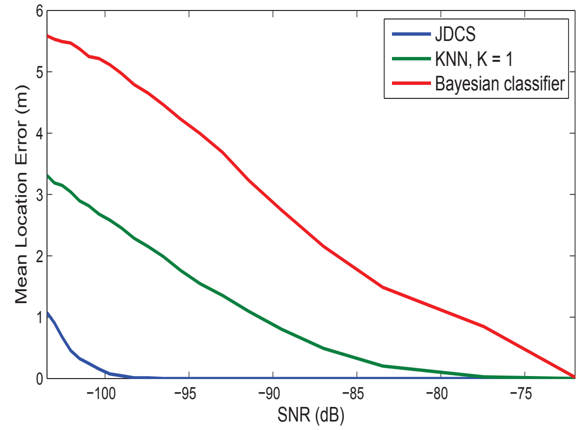

Figure 3: Mean location error vs. SNR for the KNN, Bayesian and the JDCS methods. The JDCS algorithm has better performance in all cases.

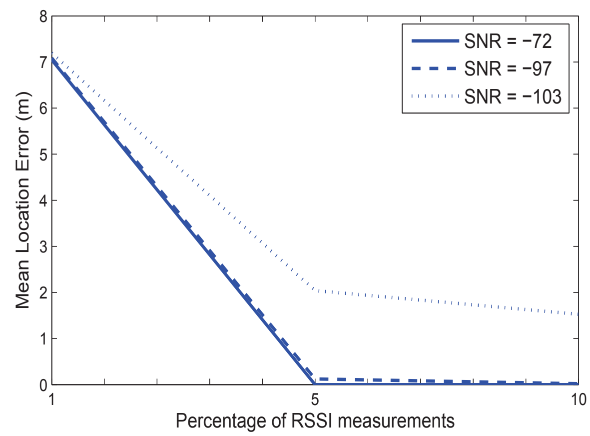

Figure 4: Mean location error vs. percentage of RSSI runtime measurements for the Joint Detection Compressed Sensing localization, for different SNR values.

Figure 4 examines the effectiveness of the JDCS algorithm when it uses a different percentage of the total available runtime RSSI measurements. We observe that as the number of measurements increases, the accuracy of the proposed scheme increases, as expected. But interestingly enough, JDCS achieves its desired accuracy with only 5% to 10% of the runtime RSSI measurements.

Finally, Figure 5 illustrates the empirical cumulative distribution function (CDF) curves $(P(|X| \leq x))$ of the localization error for the three methods and a case of low SNR = -110 dB. JDCS employs 10% of the available runtime RSSI measurements. We observe that 73% of the time the location error of JDCS is less than 0.1 m. On the contrary, the median

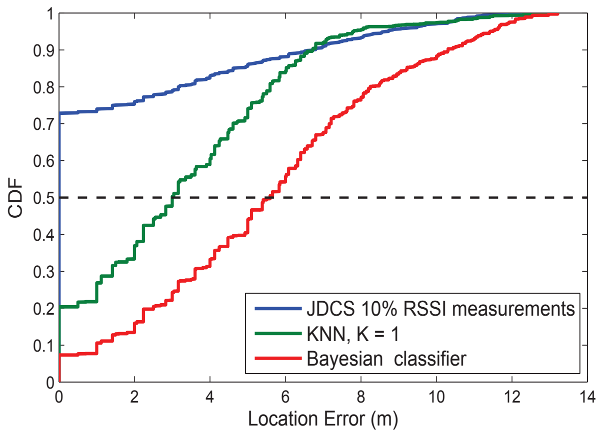

Figure 5: CDF curves ($P|X| \leq x$) for the three methods for low SNR. JDCS considers only 10% of the available runtime RSSI measurements. Observe that the location error of the JDCS method is less than 0.1 m 73% of the time.

location error (*i.e.*, the value bellow which 50% of the location error fall), is 3 m for the KNN and 5.5 m for the Baysian classification method.

5. CONCLUSIONS

In this paper, we proposed a received signal strength localization protocol that exploits the spatial correlations among the received measurements in order to jointly estimate the positions of the mobile users. The proposed method was implemented at the APs in order to reduce the computational overhead and the energy consumption at the mobile device. The experimental results validated the proposed Joint Detection Compressed Sensing approach under different RSSI characteristics.

Future work will investigate the performance of the algorithm in various operational environments. We will focus on the impact of the environmental conditions on the accuracy of the localization protocol. Finally, we are interested in reformulating the localization algorithm in a decentralized manner where all APs will participate in location sensing.

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