## MIMO-DFE COUPLED WITH V-BLAST FOR ADAPTIVE EQUALIZATION OF WIDEBAND MIMO CHANNELS

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## ABSTRACT

A new adaptive equalizer is proposed by combining a MIMO-DFE and a V-BLAST detector for frequency-selective fading MIMO channels. The MIMO-DFE cancels ISI while the V-BLAST detector cancels ICI and detects the transmitted symbols. The wideband channel concatenated with the MIMO-DFE is considered as a virtual flat-fading channel. Estimation and tracking of this virtual channel, which is used by the V-BLAST detector, is realized by means of an adaptive filter. Simulation results show that the new equalizer outperforms a previously-proposed adaptive wideband MIMO channel equalizer while requiring a lower computational complexity.

### 1. INTRODUCTION

In recent decades, there has been a growing interest in multiple-input multiple-output (MIMO) communication systems. Recent information-theoretical findings corroborate that employing multiple transmit/receive antennas can increase capacity of the wireless communication systems dramatically, growing linearly with the minimum of the number of antennas used at the transmitter and the receiver [1]. To exploit this tremendous spectral efficiency, complicated receiver structures are required. Among them, vertical Bell Labs layered space-time (V-BLAST) architecture [2] is the most famous one, which has been designed to deal with flat-fading channels. It efficiently cancels inter-(sub)channel interference (ICI) to increase reliability of symbol detection. However, increased transmission rates require shorter symbol periods and thus intersymbol interference (ISI) arises. Therefore, an equalizer for a wideband wireless communication system operating in a frequency-selective MIMO channel should be able to counter both ICI and ISI. One way to eliminate ISI is to employ MIMO orthogonal frequency-division multiplexing (OFDM) systems [3]. Using MIMO-OFDM, a frequency-selective fading channel is effectively converted to several flat-fading channels. Despite this advantage, OFDM has some drawbacks such as implementation difficulties due to high peak-to-average power ratio, identifiability of spectral nulls, and sensitivity to carrier synchronization [4]. Although, the industry has adopted MIMO-OFDM in many recent standards, the abovementioned complications make single-

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carrier MIMO communication systems still attractive for certain applications like uplink in 3G LTE where singlecarrier frequency-division multiple-access (SC-FDMA) is used. SC-FDMA is used in view of the fact that its peak-toaverage power ratio is small and more constant power enables high RF power amplifier efficiency in the mobile handsets, an important factor for battery-powered equipment [5]. Most MIMO channel equalization methods reported in the literature require perfect knowledge of the channel. They obtain this knowledge through a separate channel estimation process in the receiver. The process is performed using the information acquired during periodic training sessions in which a training sequence known to the receiver is transmitted [6] [7]. It is proven that in order to achieve the best spec-

which a training sequence known to the receiver is transmitted [6], [7]. It is proven that in order to achieve the best spectral efficiency training sequences should optimally take up as much as half of the whole transmitted data [6]. Channelestimation-based equalizers usually assume that the channel is static during transmission of a burst (packet, frame, or block). However, this assumption does not hold when long bursts of data are transmitted through time-varying channels. As an example, it is shown in [10] that for a typical slowfading channel and an interval of 8000 symbols, channel taps may change significantly, i.e. 75% amplitude variation and  $7\pi/8$  phase rotation. Therefore, the use of adaptive receiver structures, which can adapt to channel variations without requiring excessively frequent channel estimations, is imperative. Adaptive equalizers do not need any explicit channel estimation and are inherently capable of tracking channel variations. In addition, they usually require less training and impose less computational complexity compared to other equalization schemes.

There are a limited number of papers in the literature on adaptive equalization of the frequency-selective fading MIMO channels. In [8], an adaptive MIMO-DFE based on the recursive least squares (RLS) algorithm is proposed which equalizes the received signals by canceling ISI of all the layers. In [9], an adaptive equalizer for wideband fading MIMO channels is developed which in fact enhances the equalizer of [8] by canceling interference of the detected symbols of other layers, i.e. ICI, from ISI-canceled received signals. In [9], the ICI cancelation is carried out successively and in an ordered fashion resembling ordered-successive interference cancelation (OSIC) of V-BLAST [2].

In this paper, we propose a new equalizer that couples the MIMO-DFE of [8] with a V-BLAST detector to suppress



Fig. 1, System model and the proposed equalizer.



Fig. 2, Shortened system model.

both ISI and ICI. We show that the new equalizer is superior to the equalizer of [9] in terms of bit-error-rate (BER) performance whilst enjoying an appreciably reduced computational burden. Exploiting the low-complexity implementation of V-BLAST in [11] further contributes to mitigating the complexity.

Section 2 describes the signal and system model and section 3 explains the proposed equalizer. Section 4 analyzes complexity of the new equalizer and compares it with those of other equalizers. Section 5 provides simulation results and section 6 concludes the paper.

### 2. SIGNAL AND SYSTEM MODEL

Let us consider a MIMO communication system with M transmitters and N receivers. The system operates over a frequency-selective and time-varying wireless channel and can be described via a discrete-time complex baseband model as

$$\mathbf{r}(n) = \sum_{l=0}^{L-1} \mathbf{H}(n,l)\mathbf{s}(n-l) + \mathbf{u}(n)$$
(1)

where  $\mathbf{s}(n) = \frac{1}{\sqrt{M}} [s_1(n), s_2(n), \dots, s_M(n)]^T$  is the  $M \times 1$  vector of symbols simultaneously transmitted by M transmitting antennas,  $\mathbf{r}(n) = [r_1(n), r_2(n), \dots, r_N(n)]^T$  is the  $N \times 1$  vector of received signals,  $\mathbf{H}(n, l)$  is the  $N \times M$  channel impulse response coefficient matrix with lag l at time index n,  $\mathbf{u}(n)$  represents  $N \times 1$  vector of additive white Gaussian noise and  $(\cdot)^T$  denotes matrix transposition. The channel is independent of the noise and its time dispersion, L, is assumed the same for all subchannels associated with all transmitter-receiver antenna pairs.

### 3. ALGORITHM DESCRIPTION

The proposed MIMO channel equalizer is depicted in Fig. 1. It combines an adaptive MIMO-DFE with a V-BLAST detector. The MIMO-DFE suppresses intersymbol interference (ISI) and the V-BLAST detector counters inter-subchannel (inter-substream) interference (ICI).



Fig. 3, Identifying and tracking  $\mathcal{H}(n-\delta)$  using an adaptive filter,  $\hat{\mathcal{H}}(n-\delta)$ , with a delay equal to decision delay  $\delta$ .

The basic idea of the proposed equalizer was inspired by the work of [12] where it is shown that globally optimum joint ISI/ICI suppression can be achieved by carrying out ISI and ICI cancelation in separate concatenated stages. Nonetheless, the nature of the proposed equalizer is totally different from the equalizer of [12], which assumes perfect knowledge of the frequency-selective channel,  $\mathbf{H}(n, l)_{l=0,\dots,L-1}$ , from the channel estimator in a prior stage. In contrast, the adaptive MIMO-DFE of the proposed equalizer does not require any explicit information about the channel impulse response. On the other hand, a V-BLAST detector normally requires an estimated channel matrix to cancel ICI from the received signals and to detect the transmitted symbols in an orderedsuccessive manner. Therefore, assuming acceptable ISI suppression by the MIMO-DFE, we consider the combination of the original frequency-selective channel and the MIMO-DFE as a virtual flat-fading channel,  $\mathcal{H}(n)$ . Hence, the system model of Fig. 1 can be simplified to the model of Fig. 2 with shortened channel. This virtual channel can be easily identified and tracked by an adaptive filter as shown in Fig. 3. The covariance matrix of the equivalent noise for the shortened system, u(n), can also be estimated recursively as we will show later. This noise is colored since the white noise of the original system,  $\mathbf{u}(n)$ , passes through the MIMO-DFE.

The algorithm of the proposed equalizer is described in the following subsections assuming the use of the RLS algorithm for updating the coefficients of the adaptive MIMO-DFE and identifying and tracking the virtual channel. Applying any other adaptive filtering algorithm in the proposed equalizer is straightforward.

#### 3.1 Equalization and detection

Equalization of the received signals and detecting the transmitted symbols are performed in two steps:

- ISI cancelation by the adaptive MIMO-DFE

Intersymbol interference cancelation is carried out via

Table 1, Required number of arithmetic operations for different equalizers at each iteration.

	Multiplication	Addition	Division	Square-root
Proposed equalizer using RLS	$3K^2 + 2KM + K + \frac{7}{6}M^3 + \frac{23}{2}M^2$	$2K^2 + 2KM + \frac{7}{6}M^3 + \frac{13}{2}M^2$	K + M	_
Proposed equalizer using NLMS	$3KM + \frac{7}{6}M^3 + \frac{21}{2}M^2$	$2KM + \frac{7}{6}M^3 + \frac{11}{2}M^2$	2	_
Equalizer of [9]	$3K^{2} + \frac{1}{2}KM^{2} + \frac{13}{2}KM + 5K + \frac{1}{6}M^{3} + 3M^{2}$	$K^{2} + \frac{1}{2}KM^{2} + \frac{7}{2}KM + \frac{1}{6}M^{3} + 2M^{2}$	K + M	K + M
Equalizer of [8]	$3K^2 + 2KM + 2K + M^2$	$2K^2 + 2KM + M^2$	K	_

$$\hat{\mathbf{d}}(n-\delta) = \mathbf{W}^*(n)\mathbf{x}(n) \tag{2}$$

where  $\hat{\mathbf{d}}(n-\delta) = [\hat{d}_1(n-\delta), \hat{d}_2(n-\delta), ..., \hat{d}_M(n-\delta)]^T$ is an  $M \times 1$  vector of ISI-suppressed outputs of the MIMO-DFE with a decision delay of  $\delta$ ,  $\mathbf{W}(n)$  is the  $K \times M$  matrix of MIMO-DFE filter coefficients and  $(\cdot)^*$  stands for complex-conjugate transposition. In addition,  $\mathbf{x}(n)$  is the input regressor vector of size *K* defined as

$$\mathbf{x}(n) = [\mathbf{r}_1^T(n), \mathbf{r}_2^T(n), \dots, \mathbf{r}_N^T(n), \mathbf{d}_1^T(n-\delta-1), \\ \mathbf{d}_2^T(n-\delta-1), \dots, \mathbf{d}_M^T(n-\delta-1)]^T$$

where

$$\mathbf{r}_{i}(n) = \left[r_{i}(n), r_{i}(n-1), \dots, r_{i}\left(n-L_{f}+1\right)\right]^{T}$$
  
$$i = 1, \dots, N$$

and

Here,  $d_i(n)$  is the detected symbol of the *i*th substream at time index *n* while  $L_f$  and  $L_b$  denote feedforward and feedback filter lengths for the MIMO-DFE, respectively. Consequently, the length of  $\mathbf{x}(n)$  is

$$K = N \times L_f + M \times L_h.$$

It is obvious that the past decisions,  $d_i(n-\delta-j)_{j=1,\dots,L_b}^{i=1,\dots,M}$ , are replaced by the known-for-the-receiver transmitted symbols,  $s_i(n-\delta-j)_{j=1,\dots,L_b}^{i=1,\dots,M}$ , during training.

### - ICI cancelation and symbol detection by the V-BLAST detector

The ISI-canceled signals,  $\hat{\mathbf{d}}(n-\delta)$ , together with the estimations of the virtual flat-fading channel and the noise/interference covariance matrix from the previous iteration  $-\hat{\mathbf{H}}(n-\delta-1)$  and  $\mathbf{Q}_u(n-1)$  respectively (see subsection 3.2)– are fed to the V-BLAST detector to render the ICI cancelation and detect the transmitted symbols. The detected symbols are arranged in an  $M \times 1$  vector as

$$\mathbf{d}(n-\delta) = [d_1(n-\delta), d_2(n-\delta), \dots, d_M(n-\delta)]^T.$$

The V-BLAST detector is implemented based on minimum mean-square error (MMSE) criterion where the MMSE filter, G(n), uses  $Q_u(n-1)$  for regularization and is expressed as

$$\mathbf{G}(n) = \left(\mathbf{\mathscr{H}}^*(n-\delta-1)\mathbf{\mathscr{H}}(n-\delta-1) + \mathbf{Q}_u(n-1)\right)^{-1}\mathbf{\mathscr{H}}^*(n-\delta-1).$$
<sup>(3)</sup>

# 3.2 Identification and tracking of the virtual flat-fading channel

The virtual non-frequency-selective channel,  $\mathcal{H}(n - \delta)$ , can be identified and tracked using the RLS algorithm in the adaptive filtering scenario of Fig. 3 via the following set of equations

$$\mathbf{e}(n) = \hat{\mathbf{d}}(n-\delta) - \hat{\mathcal{H}}(n-\delta-1)\mathbf{d}(n-\delta)$$
(4a)

$$\mathbf{q}(n) = \mathbf{P}(n-1)\mathbf{d}(n-\delta) \tag{4b}$$

$$\mathbf{k}(n) = \frac{\mathbf{q}(n)}{\lambda + \mathbf{d}^*(n - \delta)\mathbf{q}(n)}$$
(4c)

$$\mathbf{P}(n) = \lambda^{-1} \big( \mathbf{P}(n-1) - \mathbf{k}(n) \mathbf{q}^*(n) \big)$$
(4d)

$$\mathbf{\acute{H}}(n-\delta) = \mathbf{\acute{H}}(n-\delta-1) + \mathbf{k}(n)\mathbf{e}^{*}(n)$$
(4e)

where  $\lambda$  is the forgetting factor and  $\hat{\mathcal{H}}(n-\delta)$  is the estimate of  $\mathcal{H}(n-\delta)$ . The channel identification is carried out with a delay equal to decision delay  $\delta$ . The noise covariance matrix of the shortened system can also be estimated recursively via

$$\mathbf{Q}_{u}(n) = \lambda \mathbf{Q}_{u}(n-1) + (1-\lambda)\boldsymbol{e}(n)\boldsymbol{e}^{*}(n)$$
(5)

where

$$\boldsymbol{e}(n) = \boldsymbol{\mathscr{H}}(n-\delta)\mathbf{d}(n-\delta) - \boldsymbol{\widehat{\mathbf{d}}}(n-\delta). \tag{6}$$

Note that  $\mathbf{d}(n-\delta)$  is replaced by the vector of known transmitted symbols,  $\mathbf{s}(n-\delta)$ , during training.

### 3.3 Updating filter coefficients of the adaptive MIMO-DFE

Finally, filter tap weights of the adaptive MIMO-DFE, W(n), can be updated using the RLS algorithm:

$$\boldsymbol{q}(n) = \boldsymbol{\mathcal{P}}(n-1)\mathbf{x}(n) \tag{7a}$$

$$\boldsymbol{k}(n) = \frac{\boldsymbol{q}(n)}{\boldsymbol{\lambda} + \mathbf{x}^*(n)\boldsymbol{a}(n)} \tag{7b}$$

$$\boldsymbol{\mathcal{P}}(n) = \lambda^{-1} \big( \boldsymbol{\mathcal{P}}(n-1) - \boldsymbol{k}(n) \boldsymbol{q}^*(n) \big)$$
(7c)

$$\mathbf{W}(n) = \mathbf{W}(n-1) + \boldsymbol{e}^*(n) \otimes \boldsymbol{k}(n)$$
(7d)

where  $\otimes$  stands for Kronecker product.

It should be noted that in order to prevent the adaptive algorithms from stalling, the virtual channel identification process should be initiated with an adequate delay after initiation of the MIMO-DFE coefficient update process in the training.



Fig. 4, Complexity comparison in terms of required floating point operations (FLOPS) for  $L_f = L_b = 2$ .

### 4. COMPUTATIONAL COMPLEXITY

The proposed equalizer using the RLS algorithm requires  $3K^2 + 2KM + K$  multiplications,  $2K^2 + 2KM - M$  additions and K divisions at each iteration for equalization and coefficient update of the MIMO-DFE,  $\frac{7}{6}M^3 + \frac{7}{2}M^2 - \frac{5}{3}M + 1$ multiplications and  $\frac{7}{6}M^3 + \frac{3}{2}M^2 - \frac{13}{6}M$  additions for the V-BLAST detector (using the algorithm of [11]) and  $8M^2 + M$ multiplications,  $5M^2$  additions and M divisions for identifying and tracking the virtual channel and estimating the noise covariance matrix. Therefore, the new equalizer requires a total of  $3K^2 + 2KM + K + \frac{7}{6}M^3 + \frac{23}{2}M^2 - \frac{2}{3}M + 1$  multiplications,  $2K^2 + 2KM + \frac{7}{6}M^3 + \frac{13}{2}M^2 - \frac{19}{6}M$  additions and K + M divisions at each iteration when using the RLS algorithm. It also requires  $3KM + \frac{7}{6}M^3 + \frac{21}{2}M^2 - \frac{5}{3}M + 3$  multiplications,  $2KM + \frac{7}{6}M^3 + \frac{11}{2}M^2 - \frac{19}{6}M + 2$  additions and 2 divisions when using the NLMS algorithm. The computational complexity of the proposed algorithm and the algorithms of [8] and [9] are compared in Table 1. Assuming six floating-point operations (FLOPs) for each multiplication or division and two floating-point operations for each addition or subtraction [13], the total number of required FLOPs for different algorithms is illustrated in Figs. 4 and 5 for different numbers of transmitter/receiver antennas and different temporal spans for feedforward and feedback filters of the MIMO-DFE. In both Figs. 4 and 5, M = N while in Fig. 4,  $L_f = L_b = 2$  and in Fig. 5,  $L_f = L_b = 6$ . It is seen that the longer the MIMO-DFE filters are, the more the computational saving by the new equalizer is.

### 5. SIMULATIONS

In order to verify the performance of the new equalizer, simulation results are provided here considering a  $4 \times 4$  MIMO wireless communication system. Subchannels between all transmitter and receiver antenna pairs are modelled independently according to the JTC indoor residential channel model A [14]. This channel model comprises three taps



Fig. 5, Complexity comparison in terms of required floating point operations (FLOPS) for  $L_f = L_b = 6$ .

and each tap independently undergoes Rayleigh fading with a normalized Doppler frequency of  $T_s f_D = 1 \times 10^{-5}$  according to Jakes model [15]. The temporal spans of  $L_f = 6$ and  $L_b = 3$  are considered for the MIMO-DFE. Following the standard practice for MIMO-DFE design, a fixed decision delay of  $\delta = L_f - 1$  for all the substreams is chosen [16]. Transmitted signals are uncoded QPSK and a forgetting factor of  $\lambda = 0.99$  is used. The V-BLAST algorithm is implemented according to the MMSE criterion. The noise vectors for different time indexes are independent and identically distributed (i.i.d.) complex circular Gaussian random vectors with a zero mean vector. In Fig. 6, learning curves of the new equalizer using RLS algorithm and the equalizers of [8] and [9] are compared. The curves have been obtained by averaging over 1000 independent runs and over all substreams. Signal-to-noise ratio has been set to SNR = 14 dB. First 100 transmitted symbol vectors are used for training; after that, algorithms switch to the decision-directed mode. Fig. 7 compares bit-error-rate performance of the new equalizer using the RLS and RLS-NLMS algorithms and the equalizers of [8] and [9]. The new equalizer with RLS-NLMS algorithm utilizes the RLS algorithm during training and switches to the NLMS algorithm in decision-directed mode.

It is evident from Figs. 6 and 7 that the new equalizer outperforms the previously-proposed adaptive MIMO equalizers. In addition, the new equalizer using RLS-NLMS algorithm outperforms the conventional MIMO-DFE using the RLS algorithm (equalizer of [8]) where utilizing the NLMS algorithm in decision-directed mode makes it less computationally demanding compared to the equalizer of [8].

### 6. CONCLUSION

A new equalizer for frequency-selective and time-varying MIMO channels was proposed by coupling an adaptive MIMO-DFE and a V-BLAST detector. The frequency-selective MIMO channel concatenated with the MIMO-DFE



Fig. 6, Learning curves of different equalizers for a  $4 \times 4$ MIMO system with JTC indoor residential channel model A,  $T_s f_D = 1 \times 10^{-5}$ ,  $L_f = 6$ ,  $L_b = 3$ ,  $\delta = 5$ ,  $\lambda = 0.99$ , SNR = 14 dB, uncoded QPSK, and 100 symbol vectors used for training.

is considered as a virtual flat-fading channel and this virtual channel is identified and tracked adaptively. The new equalizer achieves superior performance with less computational requirements compared to the existing adaptive wideband MIMO channel equalizers. Since the proposed equalizer can straightforwardly utilize any adaptive filtering algorithm, it has the potential to provide a sensible trade-off between performance and complexity. Furthermore, it can exploit the benefits of partial updating [17].

### REFERENCES

[1] H. Jafarkhani, *Space-Time Coding: Theory and Practice*, Cambridge, U.K.: Cambridge University Press, 2005.

[2] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Sel. Areas Commun.*, vol. 17, no. 11, pp. 1841–1852, Nov. 1999.

[3] G. Stuber, J. Barry, S. Mclaughlin, Y. Li, M. Ingram, and T. Pratt, "Broadband MIMO-OFDM wireless communications," *Proc. IEEE*, vol. 92, pp. 271–294, Feb. 2004.

[4] S. Haykin and M. Moher, *Modern Wireless Communications*, Englewood Cliffs, NJ: Prentice-Hall, 2003.

[5] S. Sesia, I. Toufik, and M. Baker, *LTE - The UMTS Long Term Evolution: From Theory to Practice*, Wiley, 2009.

[6] T. L. Marzetta, "BLAST training: Estimating channel characteristics for high capacity space-time wireless," in *Proc. 37th Annual Allerton Conf. Commun., Contr., Comput.*, Monticello, IL, Sept. 1999.

[7] S. J. Grant and J. K. Cavers, "Multiuser channel estimation for detection of cochannel signals," *IEEE Trans. Commun.*, vol. 49, pp. 1845–1855, Oct. 2001.



Fig. 7, BER performance of different equalizers for a  $4 \times 4$ MIMO system with JTC indoor residential channel model A,  $T_s f_D = 1 \times 10^{-5}$ ,  $L_f = 6$ ,  $L_b = 3$ ,  $\delta = 5$ ,  $\lambda = 0.99$ , uncoded QPSK, and 100 symbol vectors used for training.

[8] A. Maleki-Tehrani, B. Hassibi, and J. M. Cioffi, "Adaptive equalization of multiple-input multiple-output (MIMO) channels," in *Proc. IEEE Int. Conf. Communications*, New Orleans, LA, Jun. 2000, pp. 1670–1674.

[9] A. A. Rontogiannis, V. Kekatos, and K. Berberidis, "An adaptive decision feedback equalizer for time-varying frequency selective MIMO channels," in *Proc. Signal Process. Advances Wireless Commun.*, Cannes, France, July 2006, pp. 1–5.

[10] A. S. Lalos, V. Kekatos, and K. Berberidis, "Adaptive conjugate gradient DFEs for wideband MIMO systems," *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2406–2412, June 2009.

[11] Y. Shang and X.-G. Xia, "On fast recursive algorithms for V-BLAST with optimal ordered SIC detection," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2860–2865, June 2009.

[12] J. He, G. Gu, and Z. Wu, "MMSE interference suppression in MIMO frequency selective and time-varying fading channels,"*IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3638– 3651, Aug. 2008.

[13] J. Benesty, Y. Huang, and J. Chen, "A fast recursive algorithm for optimum sequential signal detection in a BLAST system," *IEEE Trans. Signal Process.*, vol. 51, no. 7, pp. 1722–1730, Jul. 2003.

[14] "Technical report on RF channel characterization and system deployment modeling," JTC (Air) Standards Contribution, Tech. Rep. JTC(AIR)/94.09.23-065R6, Sep. 1994.

[15] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.

[16] N. Al-Dhahir and A. H. Sayed, "The finite length multiinput multi-output MMSE-DFE," *IEEE Trans. Signal Process.*, vol. 48, no. 10, pp.2921–2936, Oct. 2000.

[17] K. Doğançay, *Partial-Update Adaptive Signal Processing: Design, Analysis and Implementation,* Academic Press, Oxford, UK, 2008.