CODE APERTURE DESIGN FOR COMPRESSIVE SPECTRAL IMAGING

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ABSTRACT

Compressive sensing (CS) is an emerging field that exploits the underlying sparsity of a signal to perform sampling at rates below the Nyquist-criterion. This article presents a new code aperture design framework for compressive spectral imaging based on the Coded Aperture Snapshot Spectral Imaging (CASSI) system. Firstly, the methodology allows the CASSI system to use multiple snapshots which permits adjustable spectral and spatial resolution. Secondly, the measurement codeword matrices are generated using a pair of model equations, leading to code aperture patterns that permit the recovery of specific spectral bands of a given object. The developed methodology is tested using a real data cube and simulations are shown which illustrate that one can recover arbitrary spectral bands with high flexibility and performance.

1. INTRODUCTION

Compressive sensing has emerged as a promising research area that can enable the acquisition of signals at sampling rates below the Nyquist-criterion. In CS traditional sampling is replaced by measurements of inner products with random vectors. The signals are then reconstructed by solving an inverse problem such as a linear program or a greedy pursuit in a basis where these admit sparse representations. The key idea in CS is the realization that most signals encountered in practice are sparse in some sense and the theory of CS exploits such sparsity to dictate that far few sampling resources than traditional approaches are needed [4, 5, 7, 8]. More formally, given a T sparse signal $\underline{x} \in \mathscr{R}^n$ on some basis $\Psi = [\underline{\psi}_1, \underline{\psi}_2, \ldots, \underline{\psi}_n]$, such that $\underline{x}$ can be approximated by a linear combination of T vectors from Ψ with $T \ll n$, the theory of compressive sensing shows that $\underline{x}$ can be recovered from m random projections with high probability when $m \approx T \log n \ll n$. The projections are given by $\underline{y} = P\underline{x}$, where P is an $m \times n$ random measurement matrix with its rows incoherent with the columns of Ψ. Commonly used random measurement matrices for CS are random Gaussian matrices ($P_{ij} \in \{\mathscr{N}(0, 1/n)\}$), Rademacher matrices ($P_{ij} \in \{\pm 1/\sqrt{n}\}$) and partial Fourier matrices.

Recently, the Coded Aperture Snapshot Spectral Imaging (CASSI) architecture has made it possible to implement CS in spectral imaging [2, 3]. CASSI is indeed a remarkable imaging architecture that has been studied in [1, 2, 3, 6]. The single-shot CASSI architecture, however, suffers from the following limitations as it pertains to the goals of this work.

†This work was supported in part by the National Science Foundation under the Grants ECCS-0725422, CCF0915800 and by ONR under the Grant N00014-07-1-0393.

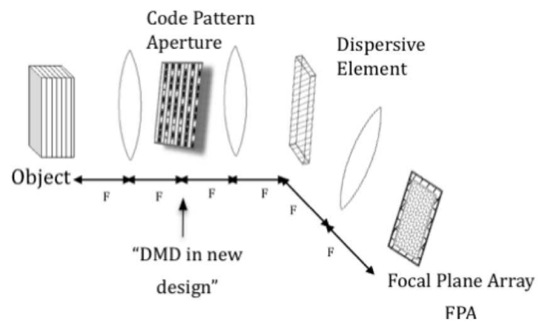

Figure 1: Diagram of the experimental CASSI system setup.The fixed aperture is replaced for a DMD in the new design and F is the focal distance.

Firstly, the single-shot system uses excessive compression to represent spectrally rich image cubes, which may result in poor-quality image reconstructions as well as low spectral resolution. Secondly, the reconstruction algorithms are rigid in that the entire spectral image cube is reconstructed at once; thus, not satisfying the agile spectrum sensing requirements of some applications. In this work, a new strategy is proposed in which aperture code designs are used to develop a multi-shot CASSI system. This new approach enables the extraction of specific bands. The mathematical model and the details of operation of the CASSI system are described in Sec. 2. A strategy to recover a periodically spaced group of bands is shown in Sec. 3. Finally, a general approach to recover a more flexible spaced group of spectral bands is derived in Sec. 4.1. Simulations illustrating the new techniques are presented in Sec. 5.

2. CODED APERTURE SNAPSHOT SPECTRAL IMAGING (CASSI) SYSTEM

The CASSI system realizes a single shot compressive spectral imaging system [1, 3]. It encodes both 2D spatial and spectral information of objects through an aperture code projection that is captured after it propagates through a dispersive element (Figure 1). An array detector then collects all light passing through the aperture and the dispersive element [1]. Figure 1 shows the CASSI system and its principal components. It is important to emphasize that the code aperture pattern remains fix in the sampling process. Suppose that the scene or object is represented by $f(\lambda, x, y)$ where λ is the wavelength and x and y correspond to the spatial position, in discrete form it is denoted as f_{mnk}. Suppose that the code pattern is C_{mn} then the signal in front of the array detector can be expressed by [2],

$$S_{mn} = \sum_k f_{(m+k)nk} C_{(m+k)n} + \omega_{mn}. \tag{1}$$

The sum in (1) captures the single measurement (shot) of the CASSI system taking into count all spectral information. Each spectral band is weighted differently by $C_{(m+k)n}$. The term ω_{mn} takes into account all possible noise sources. S_{mn} is a compressed version of f_{mnk} modulated by C_{mn}. S_{mn} is thus a compressive sensing version of f_{mnk}. A reconstruction algorithm is thus necessary to recover f_{mnk} from S_{mn}. A number of strategies have been developed for CS signal reconstruction. All of them take into account the sparsity of the source f_{mnk} [2, 3, 6]. Accordingly, the spectral data cube f_{mnk} can be expressed as $f = \mathbf{W}\theta$ where $\mathbf{W}$ is the inverse wavelet transform and θ is the three dimensional coefficient wavelet decomposition of f_{mnk}. Equation (1) can then be rewritten as,

$$S_{mn} = \mathbf{HW}\theta + \omega_{mn}, \tag{2}$$

where the linear operator $\mathbf{H}$ represents the system forward model. The reconstruction of f_{mnk} is attained by solving the optimization problem,

$$\hat{f} = \mathbf{W}\left[\arg\min_{\theta'} \left\| S_{mn} - \mathbf{HW}\theta' \right\|_2^2 + \tau \left\| \theta' \right\|_1 \right]. \tag{3}$$

The first term minimizes the ℓ_2 difference between the model and the measurement S_{mn}. The variable $\tau > 0$ controls the level of sparsity attained in the reconstruction. The sparser the source f_{mnk} in $\mathbf{W}$, the better the performance of the reconstruction algorithm. In this work the l1-ls CS reconstruction algorithm was used to solve (3) [11].The above procedure tries to recover the overall data cube with only one measurement and hence it often yields a low SNR output performance [10].

This paper aims at generalizing the CASSI architecture allowing multishot measurements such that different subsets of spectral data cube can be separately recovered with higher SNR and lower reconstruction time. The new approach thus replaces the static code aperture in the CASSI system by a Digital Micromirror Device(DMD) that permits changes in the code pattern, enabling the design of a multi-shot CASSI system. The multishot system thus requires the design of a sequence of code aperture patterns. The contribution of this paper is precisely the design of a family of aperture codes that will provide the above mentioned advantages.

3. RECOVERING UNIFORMLY SPACED SPECTRAL BANDS

The method developed in [10] derives a set of spectral codes that allows the simultaneous recovery of L uniformly spaced bands. Suppose that the spectral information of f_{mnk} is formed by K different bands, $k \in [1, K]$. The spectral data cube can then be expressed as $\Omega = \{f_0, f_1, ..., f_{K-1}\}$, where f_i is an $N \times N$ spectral image. In this method, Ω is divided into L subsets, each one shifted by L spectral bands of each other as $\Omega_i = \left\{f_i, f_{L+i}, ..., f_{(M-1)L+i}\right\}$, where $M = K/L$ is the number of spectral sub bands in each group and $i = 0, ..., L-1$. In general, L CASSI snap-shots are necessary to extract the L subsets. Suppose that P_r is an $N \times N$ random matrix and P_g^i is given by,

$$P_g^i = \begin{cases} 1, & mod\,(n, L) = mod\,(i, L) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

where *mod* is the modulo operation, then each code aperture modulation pattern i is defined by

$$P_i = P_g^i \times P_r. \tag{5}$$

If the i^{th} snap-shot is taken using a different code pattern given by (5), then L matrices of $N \times (N + L - 1)$ elements are needed and these are given by

$$S_{mni} = \sum_{k=0}^{K-1} f_k(m, n+k) P_r(m, n+k) P_g^i(m, n+k). \tag{6}$$

Each of these matrices S_{mni} represents a compressive and combined version of the i^{th} subsets of Ω. In order to separate each group a decoding process is necessary. This process consists in reorganizing the measurements of the CASSI system so that only the information of each group appears in the $\hat{S}_{mni}$ matrix

$$\hat{S}_{mni} = S_{mni} \qquad if \qquad mod(i, L) = mod(n+k-1, L). \tag{7}$$

At this point, there are L matrices $\hat{S}_{mni}$, $i \in [0, L-1]$ of size $N \times (N + L - 1)$. Each $\hat{S}_{mni}$ matrix encodes M spectral bands that can be recovered using (3). The number of nonzero elements of the matrix P_r over its dimensions is called the compression rate r, which is an important parameter that establishes the percentage of points taken in each spectral band. This parameter will be analyzed in detail in the simulations section. It is important to note that the above coding procedure depends highly on the value $L \leq K$ used. In general, the larger L, the better SNR in the reconstruction; however, the trade off is more time consumming snap-shots as L increases.

4. SPECTRAL BAND SELECTIVITY

In some applications, it is desirable to recover only a specific subset of bands within the complete data cube Ω. For example, suppose that it is necessary to simultaneously recover two specific bands x_1 and x_2 of Ω. One option is to use the approach of the previous section. In order to use this method, however, it is necessary to find all factors of K and to use each factor as a possible value L. Next, for each L value, one must verify if there are any subsets $\hat{S}_{mni}$ that contains simultaneously the desired bands x_1 and x_2. If x_1 and x_2 are in different subsets $\hat{S}_{mni}$, then it would be necessary to solve (3) twice which is computationally expensive. The method of Sec. 3 can only recover subsets of bands uniformly spaced. A more general method is thus introduced next.

4.1 Multishot code aperture design

In order to find all allowed values of L to be used in the method of Sec. 3, a codeword measurement matrix C is created. This matrix contains all possible valid combinations of spectral bands that the method of Sec. 3 can recover effectively. First, it is necessary to define a factor that expresses the concentration of spectral information of a subset of spectral bands; this is called the density factor,

$$\eta = \frac{N_L}{K}, \tag{8}$$

where N_L is the number of sub bands recovered simultaneously and K is the number total of bands of the spectral data

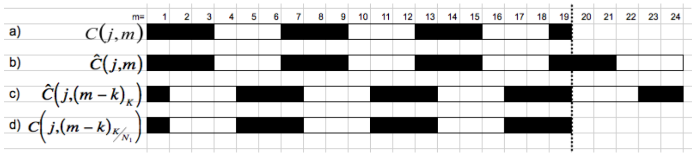

Figure 2: Example of **(a)** A representative row of matrix $C(j,m)$, **(b)** Periodical extension $\hat{C}(j,m)$ **(c)** Circular shift of (b): $\hat{C}(j,(m-k)_K)$ **(d)** $C\left(j,(m-k)_{K/N_j}\right)$ takes only K values of (c). $K=19$ and $N_j=6$.

cube Ω. The lower η, the easier it is to recover the N_L bands, in terms of computational complexity. Notice that this factor is bounded by $\frac{1}{K} \le \eta \le 1$.

The measurement codeword matrix C is defined as a $T \times K$ matrix, where T is a selectable parameter. Each column of C expresses a spectral band of Ω and each row of C is a possible combination of bands to be recovered using the method of Sec. 3. Furthermore, each row of C is periodic with period $N_j \neq 0$ that satisfies,

$$N_j = \min_{N_j} \{C(j,m) = C(j,m+N_j)\}, \tag{9}$$

where $j \in [0,T-1]$ and $m \in [0,K-1]$. Additionally, define $C\left(j,(m-k)_{K/N_j}\right)$ as having a circular shift of duration k and period N_j where only K values are taken into account. Thus, $C\left(j,(m-k)_{K/N_j}\right) = \hat{C}(j,(m-k)_K)$ where $m=0,...,K-1$ and

$$\hat{C}(j,m) = \begin{cases} C(j,m), & 0 \le m \le K-1 \\ C(j,m-N_j), & K \le m \le (([x]+1))N_j - 1 \\ 0, & \text{elsewhere} \end{cases}. \tag{10}$$

$\hat{C}(j,m)$ is called the periodic extension of $C(j,m)$. To clarify, a typical row of the matrix C, $C(j,m), m=0,...,K-1$ is illustrated in Fig. 2. Each square represents an element of row $C(j)$, if the square is black then this element is 1 and 0 elsewhere. In this example $K=19$ and $N_j=6$. Further, the correlation between two rows i and j in C obeys

$$\hat{r}_{ij}(l) = \sum_{k=0}^{K-1} C(i,k)\, C\left(j,(k-l)_{K/N_j}\right) \qquad i \neq j, \tag{11}$$

for $i,j \in [0,T-1]$ and $k,l \in [0,K-1]$. Now, it is defined that row i and j are shift-independent if they obey

$$\begin{gathered} \frac{\max(\hat{r}_{ij}(l))}{K} < \min(\eta_i,\eta_j) \\ \max(\eta_i,\eta_j) \neq \min(\eta_i,\eta_j) \qquad i \neq j, \end{gathered} \tag{12}$$

for $i,j \in [0,T-1]$ and η_i and η_j are the densities of $C(i)$ and $C(j)$ calcualted using (8) that are equal to $\eta_i = \frac{1}{K}\sum_{k=0}^{K-1} C(i,k)$ and $\eta_j = \frac{1}{K}\sum_{k=0}^{K-1} C(j,k)$.

Equations (9), (12) are used to generate the matrix C. The number T of rows of C can be limited to a given number or it can be established or bounded by 2^K. Computer simulations of these model equations are showed in Fig. 3. Next to the C matrix appears the factor of density (η_j) of each row and the number $L = N_j$ of snap-shots necessary to get this combination of spectral bands using the method of Sec. 3.

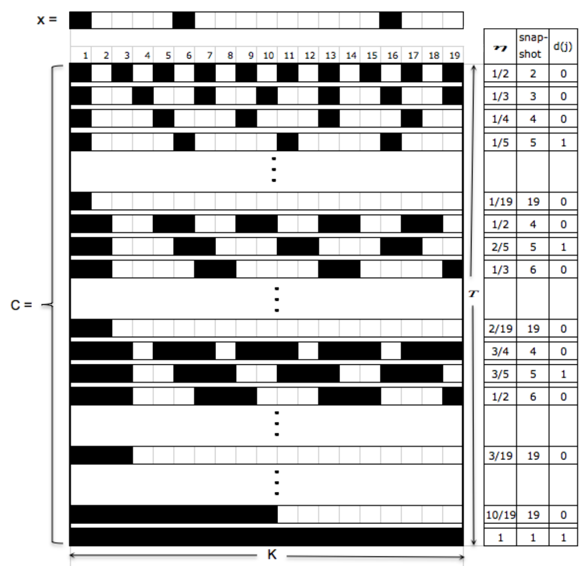

Figure 3: A typical matrix C. The density factor η is shown for each row as well as the number $L = N_j$ of snap-shots necessary to implement each codeword through the method of Sec. 3.

4.2 Code aperture design algorithm

The basis of the code aperture design algorithm is to find one or more rows of matrix C that can recover the desirable bands simultaneously. Different rows recover unknown bands from different subsets with different density factor. The higher density factor, the poorer quality of reconstruction and the larger the time of recovery, but less snap-shots are necessary. The lower density factor, the higher quality and the lower time of recovering process, but more snap-shots are required. In some applications, the number of snap-shots is critical due to motion. In other applications, the object or scene can change extremely fast and there is not sufficiently time to take several snap-shots. On the other hand, certain applications require the highest possible SNR.

Suppose that is necessary to recover a given number L_x of spectral bands in positions $\{p_1, p_2, ..., p_{L_x}\}$ inside the spectral data cube. Then a binary vector x is derived with each element representing a spectral band. The element is a 1 if it is necessary to recover this band or 0 otherwise;

$$x(n) = \begin{cases} 1, & \text{if } n \text{ band is required} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

Next, it is necessary to calculate the correlation matrix between matrix C and the vector x. Thus, $\hat{r}_{jl} = x \circledK \hat{C}(j,(-l))$ for $j = 1,...,T-1$, where T is the number of rows of C and $\circledK$ is the circular convolution of period K. $\hat{r}_{jl}$ can be calculated through a Fast Fourier Transform(FFT) by, $\hat{r}_{jl} = IFFT\{X(f)\hat{C}^*(j,f)\}$, where $X(f)$ and $\hat{C}^*(j,f)$ are the FFT of x and $C(j)$ respectively. It is then necessary to calculate the variables y and d given by,

$$y(j) = \max\{\hat{r}_{xj}\} \tag{14}$$

$$d(j) = \begin{cases} 1, & if \quad y(j) = L_x \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

where L_x is the number of nonzero elements of x or the same the number of spectral bands that it wants to recover. The variable d is the decision variable, if $d(j)=1$ then the row j of matrix C can be used to recover the spectral bands indicated by x. Each selected row of C is a combination of spectral bands that can be effectively recovered using the method of Sec. 3. At this point, the user can select a row of C with period N_j. The method of Sec. 3 then separates the data cube into subsets $\Omega_i = \{f_i, f_{L+i}, \ldots, f_{(M-1)L+i}\}$ where $L = N_j$. In order to recover simultaneously the desired bands given by x, the measurement 2D vector is created

$$\hat{S}_{mnx} = \left\{\hat{S}_{mnp_1}, \hat{S}_{mnp_2}, \ldots, \hat{S}_{mnp_{L_x}}\right\}. \tag{16}$$

Using equation (3) with $S_{mn} = \hat{S}_{mnx}$ it is possible to recover the desired bands $\{f_{p_1}, f_{p_2}, \ldots, f_{p_{L_x}}\}$ simultaneously. Figure 3 shows an example of this procedure. In the top of this figure appears: a typical vector x, the codeword measurement matrix C, respective values of η_j, number $L = N_j$ of snapshots and the decision variable $d(j)$. In this example, it is desirable to recover bands 1, 6 and 16. In the column labeled with d, all rows or codeword of C appears that can be used to recover the spectral bands indicated.

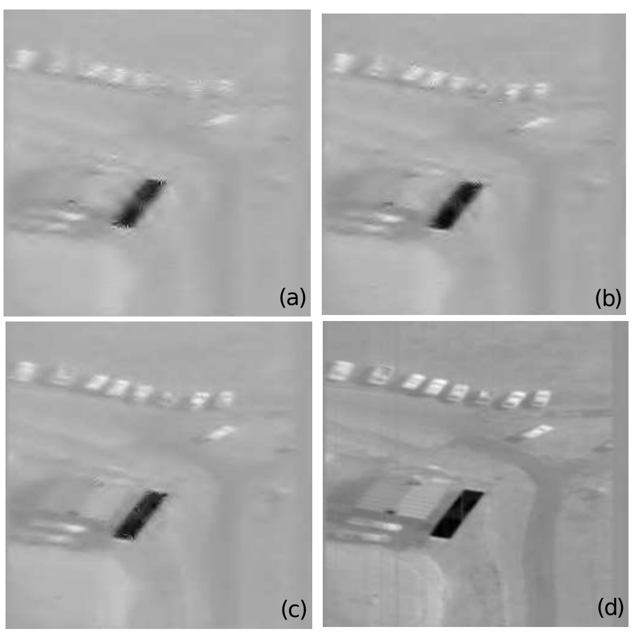

Figure 4: Examples of the same spectral band recovered using diferent density factors and the method of section 3, a) $\eta = 1$ PSNR=24.45dB b) $\eta = \frac{1}{2}$ PSNR=26.6dB c) $\eta = \frac{1}{4}$ PSNR=27.21 d) Original spectral band.

5. SIMULATIONS AND RESULTS

Simulations were performed using a 24 channel real data cube of size 256×256, the algorithm was implemented in Matlab using a 3GHz Intel processor with 4GB RAM. Other parameters were, wavelet basis *Symmlet* order 8 and maximum number of iterations equal to 2000. Figure 4 shows the implementation of the method of Sec. 3 for different values of η and their respective peak signal-to-noise ratios (PSNR). In all cases the spectral band number 1 was recovered, however, the method of Sec. 3 recovers other spectral bands simultaneously spaced periodically that are not shown there.

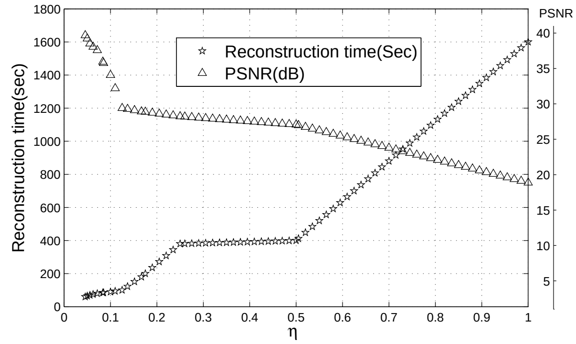

Figure 5: Recovering time and PSNR in function of density factor.

For example in Fig. 4 with $\eta = \frac{1}{4}$ indicates that $L = \eta \times 24 = 6$, thus the spectral bands 1, 7, 13 and 19 are recovered at the same time. It is observed that the lower η, the higher the expected *PSNR*. This method improves on

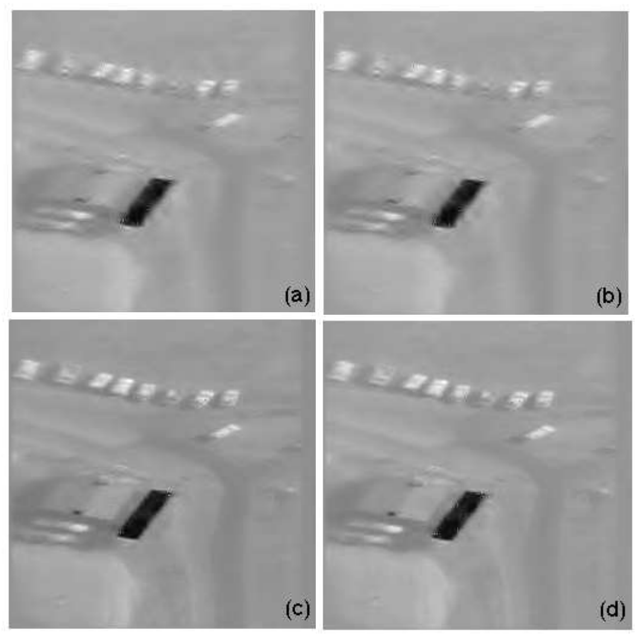

Figure 6: Examples of spectral bands 1 and 2 recovered simultaneously, (a) and (b) $\eta = \frac{1}{2}$ PSNR=24.37dB and 33.4dB respectively b) and c) $\eta = \frac{1}{4}$ PSNR=28.88dB and 37.67dB respectively.

other similar works in the area because it can recover the same spectral band with different values of η and therefore with diferent PSNRs. The Fig. 5 depicts the PSNR and time reconstruction time as a function of η. Results indicate that it is possible to recover a specific or subset of spectral bands with different PSNR values and reconstruction time. These results show the flexibility of the new method developed in this paper. As expected, high PSNR values and lower reconstruction times are obtained for low density factors. These results verify the importance of taking into count the density factor η in the codeword selection.

The Fig. 6 shows the results of the procedure developed in section 4.1 to recover bands 1, 2 together for diverse values of η. It is important to note that the method of Sec. 3 by itself cannot recover these bands together with high SNR because are spaced only one spectral band apart. This figure

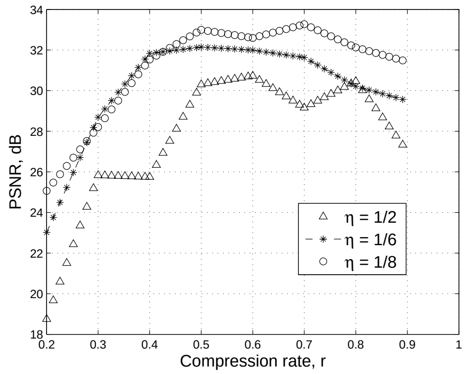

Figure 7: PSNR as function of compresion rate r, $\eta = \frac{1}{2}, \frac{1}{6}, \frac{1}{8}$.

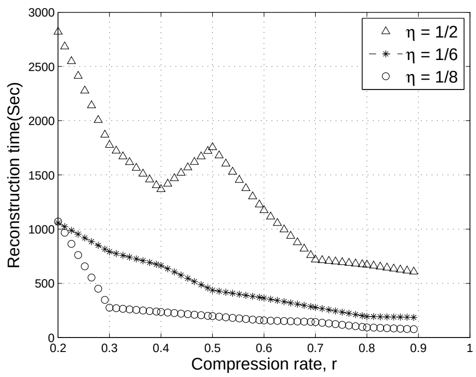

Figure 8: Reconstruction time as function of compresion rate r, $\eta = \frac{1}{2}, \frac{1}{6}, \frac{1}{8}$.

shows that applying code aperture design algorithm is possible to recover whatever subsets of bands with high flexibility and high PSNR. Additionally, it is important to analyze the compression performance of this method, namely the number the bits used to save the information. The compression rate r used in Fig. 4 was 0.5 then the total number of pixels of information was $N \times (N+L-1) \times r$ and the real information contained was $24 \times \eta \times N \times N$ pixels. This provide a compression factor of 44, 22 and 11 times for Fig. 4 (a), (b) and (c), respectively. Notice that the compression factor is important in applications where the spectral information is needed to be sent through a band limited channel. The Fig. 7 shows the PSNR for diverse values of compression rate. Notice that for lower compression rates, near to 0.2, low PSNR is obtained. Surprisingly, simulations showed that for compression rates near to 1 there are no good performances. The higher performance is attained setting the compression rates to values near to 0.5. This result is consistent for diverse values of η. On the other hand, Fig. 8 shows the reconstruction time for diverse values of η as a function of compression rate. At lower compression rates more computationally efforts are necessary to recover the original signal, for this reason Fig. 8 shows that at lower compression rates more reconstruction time is required.

6. CONCLUSIONS AND FUTURE WORK

A method to recover spectral bands selectively from a scene was developed. A code aperture design method was presented to recover a group of bands uniformly spaced. Using this method and a measurement codeword matrix, it is possible to recover arbitrary combinations of bands, simultaneously. Simulations showed that this method improves the PSNR and reconstruction time significantly. Furthermore, it was discovered that compression rates near to 0.5 appears to be the optimal value for the CS system. The methodology exposed here can be used to sensing spectral information at high ratios of compression that enables transmissions of this information through band limited channels. The method developed shows high flexibility in terms of user options, namely the method allows the recovery of a group of bands using different density factors.

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