FFT BASED SIGN MODULATED DWT FILTER BANK

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ABSTRACT

Discrete Wavelet Transform (DWT) and its generalization, Wavelet Packets (WPs) have acquired central position for signal representation. DWT provides good compaction for low pass signals only. On the other hand WPs offers good approximation property for arbitrary signal but the associated computational cost of finding an optimal WP basis is quite high. In this paper, we introduce a signal conditioning based modulated wavelet transform. The proposed transformation provides better approximation performance than that offered by DWT for signal with arbitrary spectra, which can be used in signal approximation, compression, de-noising etc. The proposed transformation in its original form requires computation of signal parity information for which a fast algorithm is proposed. The proposed transform can be implemented efficiently similar to wavelet transform. Simulation results to demonstrate the improved approximation performance are also provided.

1. INTRODUCTION AND MOTIVATION

In the last two decades the wavelet transform has become a vital tool for signal representation and processing [1] - [4]. The multiresolution features, embedded in the wavelet transform, are typically suited for a variety of applications, such as, including among others, signal approximation, analysis, compression, coding, storage and transmission [5] - [9]. It provides good compaction for signals which are low pass in nature. But the signal in practice can have arbitrary spectra [1] - [3], [10]. The DWT being a fixed transform is typically suited for low pass signals. At the same time efficient algorithm exits for their fast implementation, which not only rely on divide and conquer strategy but also on the other features like structured basis etc. [11] - [14]. Wavelet packets [15] - [19] are an elegant generalization of wavelets transformation and provide flexible subband decomposition for arbitrary signal. The process of selecting the best basis for a particular signal, for compactly representation, is computationally very intensive [17] - [19]. Hence, the issue of representing a signal having arbitrary spectra with improved compaction property and at reduced computational cost remains important.

In this article, a novel signal conditioning transformation is introduced based on the spectra of the given signal. Based on this a modification of DWT is presented with improved approximation performance. Compared to DWT, resulting transformation is an adaptive transformation and provides improved approximation performance for arbitrary signal. The signal conditioning information is compactly described by a parameter called signal parity, which is computational intensive activity. This limitation is addressed through a computationally efficient algorithm to find signal conditioning information based on the signal spectra. In this paper these topics will be described in the same order.

2. SIGNAL CONDITIONING BASED WAVELET TRANSFORM

Let x[n] $\in l_2(Z)$ be a given discrete time deterministic signal having arbitrary spectra. The J level decomposition by a DWT of x[n] results in approximation space, V_J and detailed spaces $W_J, W_{J-1}, W_{J-2}, \ldots, W_1$. The role of each stage of DWT decomposition is to compact the signal energy in few coefficients, and the emphasis has been on the decomposition low frequency bands, as shown in Fig. 1. If after j stages of decomposition, the signal in V_j has more energy in high frequency band than in the low frequency band, then on further decomposition, V_{j+1} space will have less signal energy compared to that in W_{j+1}. Hence, further decomposition of V_{j+1} space, which has less signal energy compared to that in W_{j+1}, would not provide any additional energy compaction. That is further efficiency in terms of approximation performance will not be achieved, and hence further decomposition of V_j as such is not required. With these considerations in mind, we propose a signal conditioning transformation, which ensures better decorrelation at each stage of signal decomposition. In the following subsections we introduce a modification of DWT based on the concept of signal conditioning [20], which provides an alternate and an efficient non-linear representation for signals with arbitrary spectra.

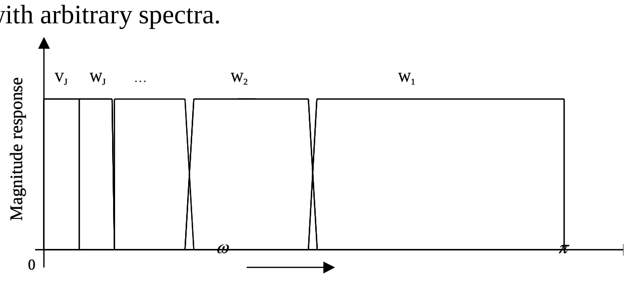

Figure 1 – Vector space for J-band decomposition by DWT in frequency domain.

2.1 Signal Conditioning

Let $X(e^{j\omega})$ be the discrete-time Fourier Transform of the signal x[n]. We define two parameters, namely, Low Pass Signal Energy (***LPSE***) and High Pass Signal Energy (***HPSE***) as follows:

$$LPSE = \frac{1}{2\pi}\int_{0}^{\pi/2} \left|X(e^{j\omega})\right|^{2} d\omega \ , \qquad (1)$$

$$HPSE = \frac{1}{2\pi}\int_{\pi/2}^{\pi} \left|X(e^{j\omega})\right|^{2} d\omega \ . \qquad (2)$$

We propose and define the following signal conditioning transformation T^a for the given signal as:

$$y[n] \equiv T^a(x[n]) \equiv \begin{pmatrix} x[n], & \text{if } LPSE \geq HPSE \\ (-1)^n x[n], & \text{if } LPSE < HPSE \end{pmatrix}, \qquad (3)$$

where, the superscript 'a' is used to denote the signal conditioning for analysis side. In the first case, in (3), the signal conditioning transformation is an identity transformation, and the signal y[n] becomes a low pass signal. For the second case, taking discrete time Fourier transform of both sides, we get

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{+\infty}(-1)^n x[n]e^{j\omega n} = X(e^{j(\omega+\pi)}) \ . \qquad (4)$$

where, we have used the fact that $e^{j\pi} = -1$. Hence the signal is transformed to low pass again. In the z-domain, the transformation in (3) becomes:

$$T^a(X(z)) \equiv \begin{pmatrix} X(z), & \text{if } LPSE \geq HPSE \\ X(-z), & \text{if } LPSE < HPSE \end{pmatrix}. \qquad (5)$$

That is, irrespective of the type of signal, the conditioning transformation always transforms the given signal into a dominantly low pass signal.

Based on this conditioning transformation, we propose a modification of DWT transform. Let the embedded DWT in the proposed transformation is implemented by J stages of perfect reconstruction (PR) filter bank, where analysis and synthesis filter pairs are $\{h_0(n), h_1(n)\}$ and $\{g_0(n), g_1(n)\}$ respectively. Here, $h_0(n), g_0(n)$ are low pass filters and $h_1(n), g_1(n)$ are high pass filters respectively. We propose one stage of conditioning based modulated wavelet transformation as a stage of signal conditioning followed by decomposition by a stage of DWT. The signal - conditioning step ensures that the signal corresponding to low pas filtering path has more signal energy compared to that in the high pass filtering path. Then for better decorrelation, it is meaningful to apply the next stage of decomposition on the low pass filtering path. We call the proposed transformation as **S**pectral **D**ensity **D**riven **W**avelet **T**ransformation (**SDDWT**). The resultant complete signal conditioning based J level SDDWT decomposition tree is as shown in Fig. 2(a), where, T_j^a, is the signal conditioning transformation at the j-th stage of decomposition by the proposed **SDDWT**. This process leads to decomposition of the signal in vector sub spaces V_J' and W_j' for j = 1 to J.

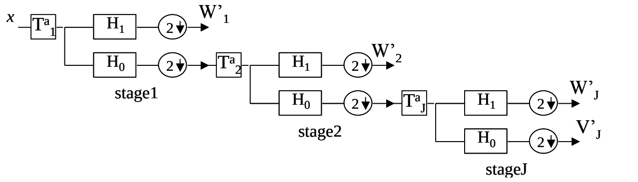

Figure 2(a) – SDDWT decomposition tree for J-stages SDDWT.

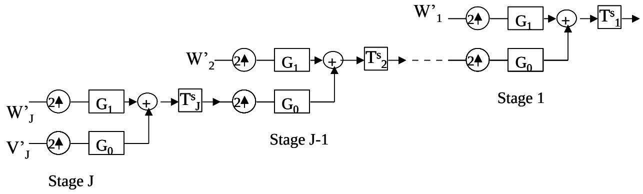

Figure 2(b) – SDDWT reconstruction tree for J-stages SDDWT.

The reconstruction process takes into account the signal conditioning and provides for the synthesis filter bank pair of a stage of DWT followed by a synthesis signal conditioning transformation T_j^s at each stage of reconstruction. The signal conditioning at the j-th stage of reconstruction is defined in terms of signal conditioning at the j-th stage of decomposition T_j^a as follows:

$$T_j^s \equiv \begin{pmatrix} \text{Identity}, & \text{if } T_j^a \text{ is identity transformation,} \\ \text{Change of spectra by } \boldsymbol{\pi} \text{ radians}, & \text{otherwise} \end{pmatrix} \qquad (6)$$

That is, the transformation T_j^s at j-th stage of reconstruction is same as that at the stage j of analysis tree and does not depend on the statistics of the transform coefficients, i.e. it is an identity if the T_j^a at that level of decomposition is identity and frequency shifting otherwise. The signal reconstruction process by the synthesis tree is shown in Fig. 2(b).

For J-level SDDWT based decomposition, we need to compute signal-conditioning information J times. This information is compactly represented by a ***parity vector p*** of the decomposition, which we define as follows: the j-th component's value, *p(j)* is '1' if the signal conditioning transformation T_j^a is frequency shifting and '0' if T_j^a is identity at the j-th level i.e.

$$p(j) = \begin{Bmatrix} 0 \text{ if } LPSE(j) \geq HPSE(j) \\ 1 \text{ if } LPSE(j) < HPSE(j) \end{Bmatrix}. \qquad (7)$$

Here *LPSE(j)* and *HPSE(j)* are the signal energies, in the low pass and high pass bands, respectively, before the j-th level of decomposition as defined by (1) and (2). The proposed scheme for the *J* level of decomposition is as shown in Figs. 2(a) and 2(b). This is the only extra cost compared to DWT.

In a number of applications, we would like infer something about signal using a subset of information. In wavelet representation this role is played by signal corresponding to V_J space. This signal provides the 'average' information about signal. This is particularly good for signals which are low pass and not for others. We call the signal corresponding to subspaces V_J' as 'representative signal' as it is representative of the given signal independent of the spectra. This is because, irrespective of type of given signal, this subspace carries more energy per coefficient. This has been ensured because of signal conditioning. Figure 3 shows 4-level decomposition of frequency plane for parity p = [0 1 0 1]. Note the corresponding emphasis on the particular band.

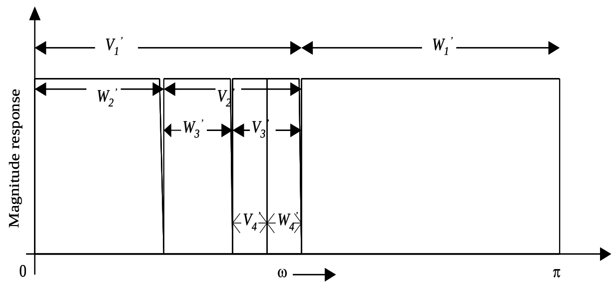

Figure 3 – Vector space for 5-band decomposition in frequency domain by SDDWT with parity p = [0 1 0 1].

2.2 Properties of DWT

The proposed SDDWT transform has a number of positive features. The fundamental difference between the DWT and the proposed SDDWT is that while DWT is blind to signal characteristics, the SDDWT is an adaptive transformation and utilizes the spectral information of the signal at each stage of decomposition for decorrelation. That is, it is able to provide better or equal approximation performance when compared to DWT. Signal conditioning is an integer, unary transformation, which is particularly important attribute from implementation point of view; the complete SDDWT scheme is invertible as the filter bank is also assumed to have perfect reconstruction (PR) property. The complete specification of the SDDWT decomposition requires the information about the SDDWT coefficients along with the parity information. The SDDWT also keeps the nice properties like successive approximation, structured basis, and computational regularity etc. of the conventional DWT intact. Hence, existing fast algorithm for DWT implementation in software or hardware can be also used for SDDWT implementation.

3. FAST SIGNAL PARITY COMPUTATION ALGORITHM

The computation of parity at each stage of decomposition is the additional computational burden of the proposed transformation. Hence, it is clear that any computationally efficient scheme for the implementation of SDDWT would hinge upon the scheme to find signal parity information. To this effect, we propose an algorithm for efficiently computing the parity information, leading to an efficient implementation of SDDWT.

Let x[n] be a given signal of length N_s and let $X(z)$ and $X(e^{j\omega})$ respectively, be the z-transform and discrete time Fourier transform of the given signal. Also for the analysis filters, $H_i(z)$ and $H_i(e^{j\omega})$ for i = 0 and 1, be, respectively, the z-transform and discrete time Fourier transform. The key insight about the proposed algorithm is obtained by observing the dependence of LPSE and HPSE, at each stage, on the given signal and the wavelet used for the decomposition. This algorithm obtains the signal conditioning information without doing the actual decomposition at each stage by the SDDWT.

Let $X_j(z)$ be the signal in the low pass path after j stages of decomposition by the SDDWT scheme, with $X_0(z) \equiv X(z)$ the original signal. Referring to the Fig. 4, one can easily write $X_j(z)$ as:

$$X_j(z) = \frac{1}{2}\begin{bmatrix} X_{j-1}((-1)^{pj} z^{1/2}) H_0(z^{1/2}) + \\ X_{j-1}(-(-1)^{pj} z^{1/2}) H_0(-z^{1/2}) \end{bmatrix} \qquad (8)$$

Equivalently, in the frequency domain, the signal at different stages is given as:

$$X_0(e^{j\omega}) = X(e^{j\omega}), \qquad (9(a))$$

$$X_j(e^{i\omega}) = \frac{1}{2}\begin{bmatrix} X_{j-1}((-1)^{pj} e^{j\omega/2}) H_0(e^{j\omega/2}) + \\ X_{j-1}(-(-1)^{pj} e^{j\omega/2}) H_0(-e^{j\omega/2}) \end{bmatrix}, \qquad (9(b))$$

$$for\ j = 1, 2, \ldots, J.$$

Equation (9) relates the spectrum of the signal at the (j-1)-th stage to that of the j-th stage, in the low frequency filtering path. Hence using (9), the spectra, and hence the parity can be computed for each stage of decomposition, from the spectra of the given signal and the analysis side low pass filter.

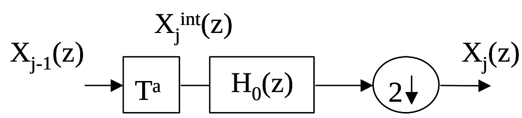

Figure 4 – Signal dependence in lowpass filtering path of SDDWT decomposition.

Parity Calculation Algorithm

Step 1: Initialization Evaluate the DFT of the signal as well as that of analysis filter h_0 at $N_0 = N+1$ equidistant frequency points. Here N is chosen as some power of 2, i.e. $N=2^q$, where q is greater than the number of stages J of decomposition i.e. $q \geq J$ and $N \leq Ns$.

Evaluate $\omega(k) = \frac{(k-1)\pi}{N}$, for $k=1$ to N_0 and set j=1.

Also define $N_j = \frac{N}{2^j} + 1$ as the number of frequency points after j stage of decomposition.

Step 2: Calculate the *LPSE(j)* and *HPSE(j)*, the low pass and high pass energy prior to the j-th stage of decomposition, using N_{j-1} frequency points by :

$$LPSE(j) = \sum_{i=1}^{(N_{j-1}-1)/2} \left| X_{j-1}(e^{j\omega(i)}) \right|^2,$$

and

$$HPSE(j) = \sum_{i=(N_{j-1}-1)/2}^{N_{j-1}} \left| X_{j-1}(e^{j\omega(i)}) \right|^2.$$

Then find the parity *p(j)* using (7) and if j = J then STOP.

Step 3: Calculation of $\left\lfloor X_{j-1}((-1)^{p_j} e^{j\omega/2}) H_0(e^{j\omega/2}) \right\rfloor$ for N_j points.

Use the value of $HN_k \equiv H_0(e^{j\omega(k)})$ for N_j frequency points given by $\omega(k)=\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1* to N_j.

Use the value of $XN_k \equiv X_{j-1}(e^{j\omega(k)})$ evaluated in previous stage for N_j frequency points depending on the value of parity *p(j)*.

If *p(j) = 0*, then frequency points are given by $\omega(k)=\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1 to* N_j.

If *p(j) = 1*, then the frequency points are given by $\omega(k)=\pi-\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1* to N_j.

Therefore the above expression is evaluated by multiplying HN_k and XN_k for *k=1* to N_j.

Step 4: Calculation of $\left\lfloor X_{j-1}(-(-1)^{p_j} e^{j\omega/2}) H_0(-e^{j\omega/2}) \right\rfloor$ for N_j points.

Use the value of $HA_k \equiv H_0(e^{j\omega(k)})$ for N_j frequency points given by $\omega(k)=\pi-\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1* to N_j.

Use the value of $XA_k \equiv X_{j-1}(e^{j\omega(k)})$, evaluated in previous stage, for N_j frequency points depending on the parity *p(j)*.

If *p(j) = 0*, then for following frequency points are given by $\omega(k)=\pi-\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1 to* N_j.

If *p(j) = 1*, then the frequency points are given by $\omega(k)=\dfrac{(k-1)\pi}{2(N_j-1)}$ for *k=1:*N_j.

Therefore the above expression is evaluated by multiplying HA_k and XA_k for *k=1* to N_j.

Step 5: Evaluate $X_j(e^{j\omega(k)})=\frac{1}{2}\left((HN_k\, XN_k)+(HA_k\, XA_k)\right)$ by (18) for k = 1 : N_j. Set *j = j+1*. And go to step 2.

Computational Complexity and Comments:

The efficiency, of the algorithm, is primarily due to two reasons. Firstly, the algorithm requires only two evaluations of *'N+1'* point Discrete Fourier Transform (DFT). This is a drastic reduction in comparison to direct computation where DFT's of size $N_{j-1}=\dfrac{N}{2^{j-1}}+1$ point are to be computed for each stage *'j'* from *1* to *J*. Secondly, as described above, the number of DFT points N can be chosen much smaller than the signal's length N_s. This is because, for accurate comparison of LPSE and HPSE at each stage of decomposition, LPSE and HPSE need not be absolutely accurate. So a small number of DFT points are sufficient for these calculations. Further, this choice and (9) dictate that the number of frequency points decrease by a ratio of 2, as we go from a stage to next. On these two counts, the proposed algorithm provides a computationally efficient and fast solution. This is the only additional computational cost of proposed scheme, when compared to the conventional DWT. Importantly; the cost of reconstruction is same as that for DWT.

4. SIMULATION RESULTS AND DISCUSSIONS

In this section we present the numerical simulation results to compare the performance of the SDDWT with that of DWT. Various clippings/frames of real life signals are chosen as test signals. The simulations are done on Matlab software. First we take up the simulation results to demonstrate the 'average' and 'representative' signal interpretation. In this regard, we have chosen a 'speech' frame as shown in Fig. 5. We have taken wavelet 'bior 3.5' for simulations and did 6 stages of decomposition. For this signal, the parity vector is p = [0 0 0 1 1 1], which shows that even though signal is dominantly low pass at low scale but at higher scale it is deviating from the low pass nature. The signal is reconstructed from the V_6 and V_6' subbands for DWT and for SDDWT respectively. Note that both the bands have the same number of coefficients. The PSNR is 7.6277 dB for the 'average signal' and 9.3052 dB for 'representative signal'. Simulation results clearly demonstrate that the signal reconstructed from SDDWT scheme is a better replica of the given signal than from the wavelet scheme.

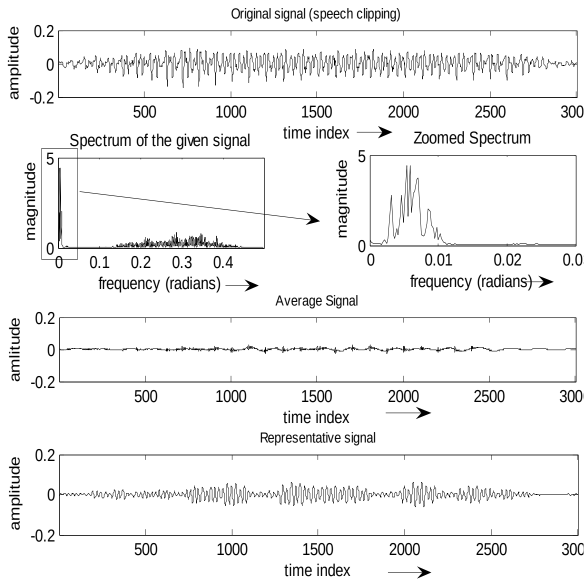

Figure 5 – This figure shows the given signal (speech), its magnitude spectrum and the average signal and representative signal. J = 6. PSNR(dB) for average is 7.6277 dB and for representative is 9.3052 dB. Parity vector for the signal is p = [0 0 0 1 1 1].

In Figs. 6(a and b), different frames of speech and music clipping from different sources are taken. The approximation/compaction performance is measured via reconstructed signal for a fixed percentage of coefficients (pN) having maximum magnitude across all the scales. From the simulation results we can clearly see that SDDWT performs better than the conventional DWT. The parity vector is calculated using the proposed fast signal parity computation algorithm.

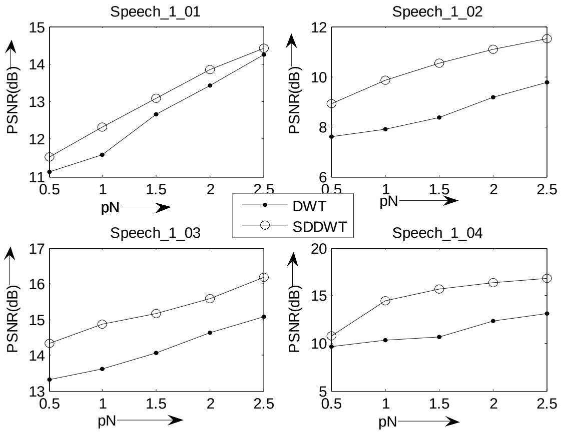

Figure 6(a) – This figure shows PSNR(db) for different signal clippings of speech signals for DWT and SDDWT, when signal reconstructed from pN percentage of coefficients.

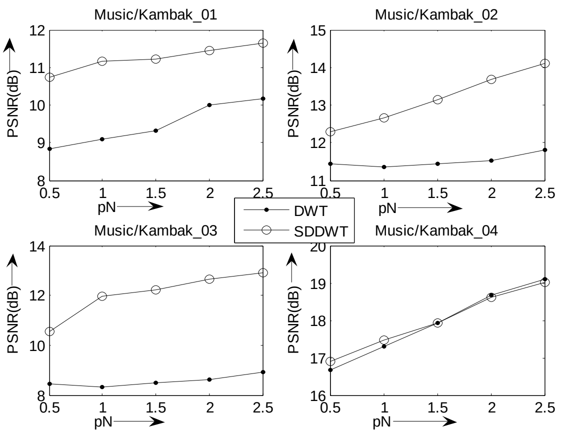

Figure 6(b) – This figure shows PSNR(db) for different signal clippings of song/music signals for DWT and SDDWT, when signal reconstructed from pN percentage of coefficients.

5. CONCLUSIONS

In this article, an adaptive SDDWT transform based on a new concept of signal conditioning is introduced. The SDDWT transform provides much improved approximation than the conventional DWT. Larger the deviations from low pass nature, better is the approximation performance compared to that for DWT. The structure of implementing the decomposition and reconstruction transformation is similar to that of DWT; hence prevalent fast algorithm to implement DWT can be used. The signal conditioning information is compactly represented by J bits parity vector for J levels of SDDWT decomposition and is computed by a fast algorithm. Then an important concept of "representative" signal, which is a better replica of the given signal, is also given. The proposed transformation can represent signal with arbitrary spectra efficiently and hence can be exploited for compression, de-noising and analysis etc.

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