IMPROVEMENT OF TARGET DETECTION BASED ON TENSORIAL MODELLING

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ABSTRACT

In this paper a multichannel and multicomponent restoration scheme is introduced for hyperspectral images (HSI) with the aim of improving target detection. This noise reduction (NR) method takes advantage of the whole data along all dimension simultaneously by defining data as a tensor. The aim of this paper is to prove the improvement in considering the cross-dependency of spatial and spectral information. Using jointly spatial and spectral processing enables better spectral signature restoration and consequently increase the target discrimination. Defining a tensor model, our method is based on tensor decomposition without any dimensional splitting during the processing. The optimization criterion used is the minimization of the mean square error between the estimated and the desired signals. This minimization leads to some estimated *n*-mode filters for each dimension, which can be considered as the extension of the well-known Wiener filter in a particular mode (such that dimension). In order to take into account the mode cross-dependency, an Alternating Least Square (ALS) algorithm is proposed to jointly determine the *n*-mode Wiener filter. Comparative studies with the classical bidimensional filtering methods show that our algorithm presents better performances by improving the detection probability.

1. INTRODUCTION

Although hyperspectral images (HSI) exhibits a correct signal-to-noise ratio (SNR), in general the SNR is not sufficient to allow an *optimum* information extraction. Indeed, the noise corrupting hyperspectral images depends not only on the performance of sensors but also on the conditions during the data acquisition including illumination and atmospheric effects. Under such conditions, noise reduction (NR) is a necessary preprocessing step to increase the SNR in order to improve the detection or classification processing by both decreasing the target spectral variability and spatially smoothing homogeneous areas [5–10]. A basic estimation scheme processes all channels separately. It is a band-by-band processing, considering each one as an independent signal. This NR method does not take advantage of inter-channel relationships which is one of the principal hyperspectral characteristics. In order to make use of this inter-channel information, a Karhunen-Loeve domain orthogonalization that decorrelates the channels is proposed. Actually, the most common in NR when deal with multichannel data is to perform an hybrid filter which consists first in making a Principal Component Analysis (PCA) transform and then in removing noise with one spatial restoration for each decorrelated channel [12, 14–16]. But those classical processing techniques consist in splitting data set into matrices or vectors and operate in the spatial and spectral domains independently. The splitting reduces considerably the information quantity related to the all data without separate spatial and spectral information and hence as a result the possibility of studying the relations between components of different channels is lost. In this study, data are modeled as a tensor. Tensor models are used in a large range of fields such as data analysis or signal and image processing . Each mode (dimension) of the tensor is associated with a physical quantity. Hence, we propose a multiway filtering [1], for denoising hyperspectral images. This new approach implicitly implies the use of multilinear algebra and mathematical tools [1] that extend the singular value decomposition (SVD) to tensors.

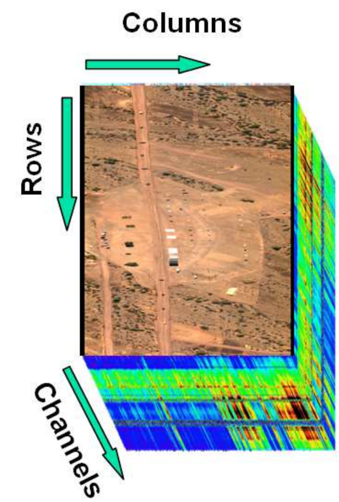


Figure 1: Tensor of hyperspectral images.

The remainder of the paper is organized as follows. Section 2 introduces the tensor formulation of the classical noise-removing problem. Section 3 presents a new version of Wiener filtering based on the *n*-mode signal subspace and tensor decomposition. Section 4 presents some comparative detection results concerning the multiway filtering, channel-by-channel based Wiener filtering and an hybrid Wiener filter. Finally, conclusions are presented in Section 5.

2. TENSOR MODEL FOR HYPERSPECTRAL IMAGES

2.1 Tensor modeling

Hyperspectral images can be modeled by a three-order tensor $\mathscr{X} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ (see Fig.1) where I_1 is the number of rows, I_2 the number of columns, and I_3 the number of spectral channels. Each dimension of the tensor is called *n*-mode where *n* refers to the n^{th} index. Using tensor model enables to generalize the theory to the N order corresponding to the number of dimensions.

The interest of tensor modeling is the multilinear alge-

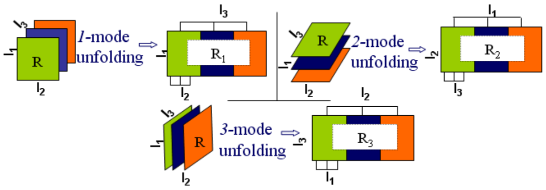


Figure 2: n-mode unfolding of tensor $\mathscr{X}$.

braic tools associated which allow for example to study the properties of data tensor $\mathscr{X}$ in a given n-mode. An illustration of the n-mode unfolding of a tensor is represented in Fig. 2.

In each unfolding of the tensor data are rearranged along all tensor dimension and the whole information contained in the tensor is present.

2.2 Problem setting

We assume that the hyperspectral tensor $\mathscr{R}$ is the sum of the desired information $\mathscr{X}$ and an additive white Gaussian noise $\mathscr{N}$:

$$\mathscr{R} = \mathscr{X} + \mathscr{N}. \tag{1}$$

Our aim is to estimate the desired signal $\mathscr{X}$ thanks to a multidimensional filtering of the data:

$$\widehat{\mathscr{X}} = \mathscr{R} \times_1 H_1 \times_2 H_2 \times_3 H_3 \ldots \times_N H_N, \tag{2}$$

where $\times_n$ is the n-mode product. The n-mode product generalize the product between both the data tensor $\mathscr{R}$ and the matrix H_n along the n-mode. From a signal processing point of view, the n-mode product is a n-mode filtering of data tensor $\mathscr{R}$ by n-mode filter H_n. In the following, we review the expression of the multiway Wiener filtering for a tensor of order N [1].

The optimization criterion chosen to determine the optimal n-mode filters $\{H_n, n = 1, \ldots N\}$ is the minimization of the the mean square error between the estimated signal $\widehat{\mathscr{X}}$ and the initial signal $\mathscr{X}$:

$$e(H_1, \ldots, H_N) = \mathrm{E}\left(\|\mathscr{X} - \mathscr{R} \times_1 H_1 \cdots \times_N H_N\|^2 \right). \tag{3}$$

In extension of the first order case, n-mode filters H_n correspond to n-mode Wiener filters.
In the classical multidimensional and multi-mode signal processing assumptions, $E^{(n)}$ is the superposition of two orthogonal subspaces: the signal subspace $E_1^{(n)}$ of dimension K_n, and the noise subspace $E_2^{(n)}$ with dimension $I_n - K_n$, such as $E^{(n)} = E_1^{(n)} \oplus E_2^{(n)}$.

3. WIENER MULTIWAY FILTERING

3.1 Expression of n-mode Wiener filters

Following [1] by developing the squared norm of equation (3), and unfolding it over the n-mode and after some computations, the final expression of H_n n-mode filter associated to fixed H_m m-mode filters, $m \neq n$, expression (3) becomes:

$$H_n = V_s^{(n)} \Lambda^n V_s^{(n)^T} \tag{4}$$

where, $V_s^{(n)}$ is the K_n largest eigenvectors originally from the eigenvalue decomposition along the n-mode unfolding of $\mathscr{R}$
and where,

$$\Lambda^n = diag\left\{ \frac{\lambda_1^\gamma - \sigma_\gamma^{(n)^2}}{\lambda_1^\Gamma}, \ldots, \frac{\lambda_{K_n}^\gamma - \sigma_\gamma^{(n)^2}}{\lambda_{K_n}^\Gamma} \right\} \tag{5}$$

in which $\{\lambda_i^\gamma, \forall i = 1, \ldots, K_n\}$ and $\{\lambda_i^\Gamma, \forall i = 1, \ldots, K_n\}$ are the K_n largest eigenvalues, respectively of matrices
$\mathrm{E}\left[X_n \mathbf{q}^{(n)} R_n^T\right]$ and $\mathrm{E}\left[R_n \mathbf{Q}^{(n)} R_n^T\right]$
with

$$\mathbf{q}^{(n)} = H_1 \otimes \cdots H_{n-1} \otimes H_{n+1} \cdots \otimes H_N, \tag{6}$$

$$\mathbf{Q}^{(n)} = H_1^T H_1 \otimes \cdots H_{n-1}^T H_{n-1} \otimes H_{n+1}^T H_{n+1} \cdots \otimes H_N^T H_N. \tag{7}$$

The symbol $\otimes$ defines the Kronecker product.

Also, $\sigma_\gamma^{(n)^2}$ can be estimated by determining the $I_n - K_n$ smallest eigenvalues mean of $\gamma_{RR}^{(n)}$:

$$\sigma_\gamma^{(n)^2} = \frac{1}{I_n - K_n} \sum_{i=K_n+1}^{I_n} \lambda_i^\gamma. \tag{8}$$

Note that this expression requires the unknown parameter K_n. To apply it on real data, without *a priori* knowledge, we have to estimate it. We propose in the following section one criterion for this purpose.

3.2 Estimation of the lower n-mode rank

The K_n parameter is the lower n-mode rank approximation. In other word, K_n is the useful n-mode signal subspace dimension of the noisy image $\mathscr{R}$. Actually,

- if K_n is too low , information is lost
- if K_n is too elevated , noise is included in the restoration.

In those two cases, the necessary number of eigenvalues is not well approximated and the estimated tensor is not *optimum*. In this paper, we extend the well-know detection criteria [17, 18] in order to estimate K_n for each n-mode. Thus, the estimated signal subspace dimension is obtained merely by minimizing one of AIC criterion.
Consequently, for each n-mode unfolding of $\mathscr{R}$, the detection criterion AIC can be expressed as

$$AIC(k) = -2N \sum_{i=k+1}^{i=I_n} \log \lambda_i$$
$$+ N(I_n - k) \log\left(\frac{1}{I_n - k} \sum_{i=k+1}^{i=I_n} \lambda_i \right) + 2k(2I_n - k) \tag{9}$$

where $(\lambda_i)_{1 \leq i \leq I_n}$ are I_n eigenvalues of the covariance matrix of the n-mode unfolding $\mathscr{R}$: $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{K_n} > \lambda_{K_n+1} = \lambda_{K_n+2} = \ldots = \lambda_{I_n} = \sigma^2$, and N is the number of columns of the n-mode unfolding $\mathscr{R}$.
The n-mode rank K_n is the value of k ($k = 1, \ldots, I_n - 1$) which minimizes AIC.

In this step, we have defined the n-mode filters which now require no *a priori* knowledge. But our aim is to propose a multiway filtering, it means the n-mode filters must not be estimated independently. We have to take advantage of the n-mode cross-dependency to propose a coherent multiway filter. The way to overcome it is summarized in the next subsection.

3.3 ALS algorithm

An Alternative Least Square algorithm needs to be used to jointly find H_n n-mode Wiener filters that enables to reach the global minimum of mean square error $e(H_1, \ldots H_N)$ given by (3). This algorithm overcome a non linear optimization. One ALS algorithm can be summarized in the following steps:

1. **initialization** $k = 0$: $\mathscr{R}^0 = \mathscr{R} \Leftrightarrow H_n^0 = I_{I_n}$ for all $n = 1$ to N.
2. ALS loop: while $\left\|\mathscr{X} - \mathscr{R}^k\right\|^2 >$ *a priori* fixed threshold
   (a) K_n **estimation**, for $n = 1$ to N:
      i. $K_n = \arg\min_k AIC(k)$, k=1,...,I_n , eq.(9)
   (b) H_n **estimation**, for $n = 1$ to N:
      i. $\mathscr{R}_n^k = \mathscr{R} \times_1 H_1^k \cdots \times_{n-1} H_{n-1}^k \times_{n+1} H_{n+1}^k \cdots \times_N H_N^k$
      ii. $H_n^{k+1} = \arg\min \left\|\mathscr{X} - \mathscr{R}_n^k \times_n Q^{(n)}\right\|^2$ subject to $Q^{(n)} \in \mathbb{R}^{I_n \times I_n}$.
   (c) **Multiway filtering**,
      $\mathscr{R}^{k+1} = \mathscr{R} \times_1 H_1^{k+1} \cdots \times_N H_N^{k+1}$, $k \leftarrow k+1$.
3. output: $\widehat{\mathscr{X}} = \mathscr{R} \times_1 H_s^k \cdots \times_N H_N^{ks}$, where k_s is the convergence iteration index.

Iteration after iteration the multiway filtering improves the SNR of the estimated tensor. Indeed, for example some signal-independent white Gaussian noise is added to a 150x150x158 tensor with a power noise resulting in a $0.9dB$, we apply the ALS algorithm and for the iteration $\{1, 2, 10, 24\}$ we obtained a estimated tensor with a SNR equal to $\{16.12dB, 16.98dB, 18.15dB, 19dB\}$. These results prove the fitting of the n-mode filters together.
The Figure **??** shows the improvement of the SNR output when the values of the spatial and spectral dimensions of HSI are well estimated using AIC or MDL criteria.

4. IMPROVEMENT OF TARGET DETECTION

A high spatial resolution HYperspectral Digital Imagery Collection Experiment (HYDICE) is considered in all our experiments. To highlight the advantages of multiway filtering, we compare it detection result with those given by the classical signal subspace based methods. The first one basically consists of a consecutive Wiener filtering of each two-dimensional spectral channel, that we denote hence after by *2D-Wiener*. The second one consists of a preprocessing by projection on the spectral mode to decorrelate the channels, then Wiener filtering is applied on each two-dimensional spectral channel, denoted by *PCA-2D Wiener*.

This noise, $\mathscr{N}$, can be modeled by

$$\mathscr{N} = \alpha \cdot \mathscr{G} \tag{10}$$

in which every element of $\mathscr{G} \in \mathbb{R}^{I_1 \times I_2 \times I_3}$ is an independent realization of a normalized centered Gaussian law, and where

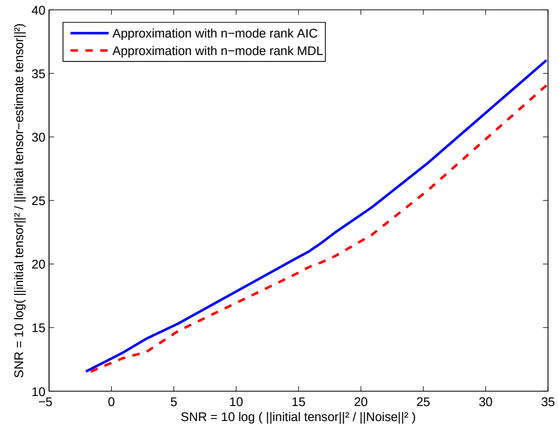


Figure 3: SNR output versus SNR input with the optimal ranks estimated by AIC or MDL.

α is a coefficient that permits to set the SNR in noisy data tensor $\mathscr{G}$.

Fig. 4a) shows the target test (the second one) in the initial HYDICE image we want to detect. This target is difficult to discriminate as the spectral signatures of the 1^{st}, 2^{nd} and 3^{rd} targets are similar, as shown on Fig.4b). To perform the target detection we use the adaptive coherence / cosine estimator (ACE) detector [13] a well-known constant false alarm rate (CFAR) and adaptive matched filter (AMF).

The ACE can be expressed as :

$$D_{ACE}(\mathbf{x}) = \frac{\left(\mathbf{s}^T \hat{\mathbf{\Gamma}}^{-1}\mathbf{x}\right)^2}{(\mathbf{s}^T \hat{\mathbf{\Gamma}}^{-1}\mathbf{s})(\mathbf{x}^T \hat{\mathbf{\Gamma}}^{-1}\mathbf{x})} \tag{11}$$

the adaptive matched filter (AMF) is given by

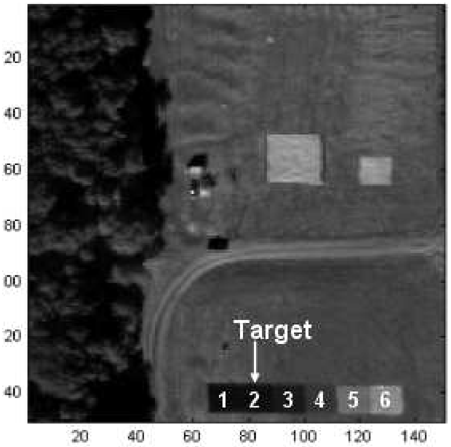
$$D_{AMF}(\mathbf{x}) = \frac{\mathbf{s}^T \hat{\mathbf{\Gamma}}^{-1}\mathbf{x}}{\mathbf{s}^T \hat{\mathbf{\Gamma}}^{-1}\mathbf{s}}, \tag{12}$$

where $\hat{\Gamma}$ is the estimated covariance matrix, $\mathbf{s}$ and $\mathbf{x}$ are respectively the target and test spectra. $\mathbf{s}$ is assumed *a priori* known from a supervised method directly on the initial hyperspectral tensor. So when,

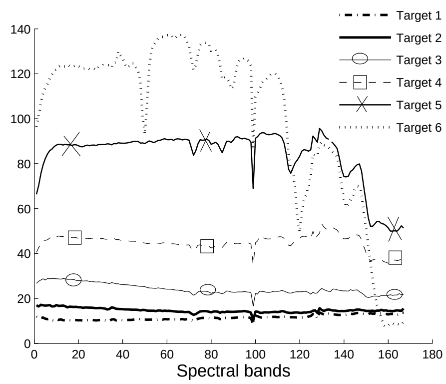
$$\begin{cases} \mathrm{D}_{ACE} \text{ or } \mathscr{D}_{AMF} > \eta, & \text{the target is present;} \\ \mathrm{D}_{ACE} \text{ or } \mathscr{D}_{AMF} < \eta, & \text{the target is absent.} \end{cases} \tag{13}$$

Where η is a detection threshold which allows the probability of detection and of false alarms estimation.

Figs. 5 and 6 represent the receiver operating characteristic (ROC) curves for the second target detection on an average of ten noise realizations with a standard deviation of 25. The zone of interest corresponds to false alarm probabilities from 10^{-4} up to 10^0. The multiway Wiener filter gives better results than the other NR methods. This tendency is confirmed in Fig. 7 with respect to the SNR_{in} varying from –3 to 13 dB and with a probability of false alarm fixed at 10^{-3}. Whatever noise power the multiway Wiener filter improves the detection performance of Hyperspectral tensor. This can be explained by its good spectral signature



a) HYDICE image at wavelength 1718 nm.



b) Spectral signatures.

Figure 4: Targets and spectral signatures.

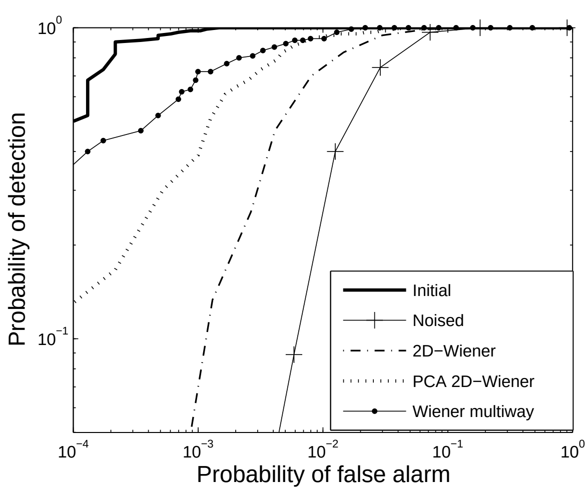


Figure 5: ROC curves obtained by ACE for the initial and estimated hyperspectral tensors.

restoration by considering simultaneously spectral and spatial processing. Figures 8 and 9 show examples of the results obtained on spectral pixel vectors.

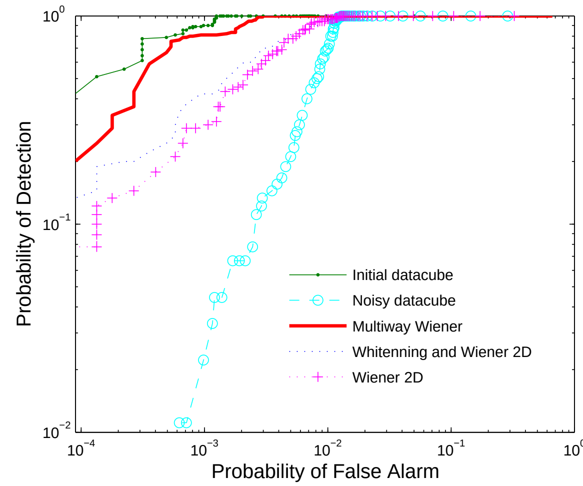


Figure 6: ROC curves obtained by AMF for the initial and estimated hyperspectral tensors.

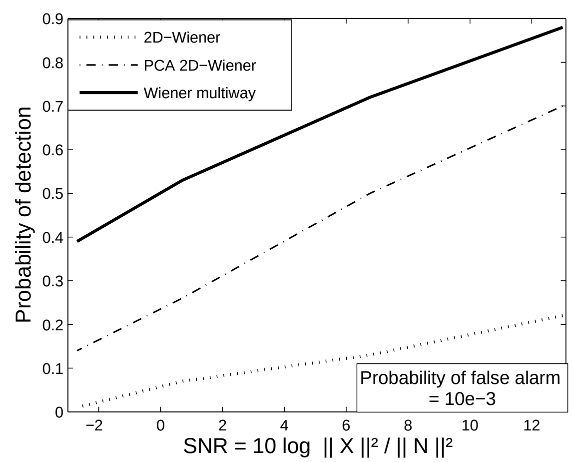


Figure 7: Probability of detection with respect to the SNR_{in} and with a fixed probability of false alarm equal to 10^{-3}

5. CONCLUSION

In this paper, we described a new algorithm for multidimensional and multicomponent restoration in order to improve the target detection. For hyperspectral images, we proposed a tensor model to consider all data as a whole tensor. The proposed multiway filtering is an extension of the bidimensional wiener filtering to tensor signal which is applied on n-mode unfolding of the noisy tensor. In order to estimate the signal subspace for each mode we have extended the well-known AIC criteria to the tensor signal. Since filters that minimize the mean squared error need to be determined simultaneously, an ALS algorithm was developed: both spatial and spectral information are jointly taken into account. The importance of the non-separability of both spatial and spectral information is highlighted and it impact on target detection detection was demonstrated. We conclude that multiway filtering realizes valuable target detection of HSI by restoring the spectral signature.

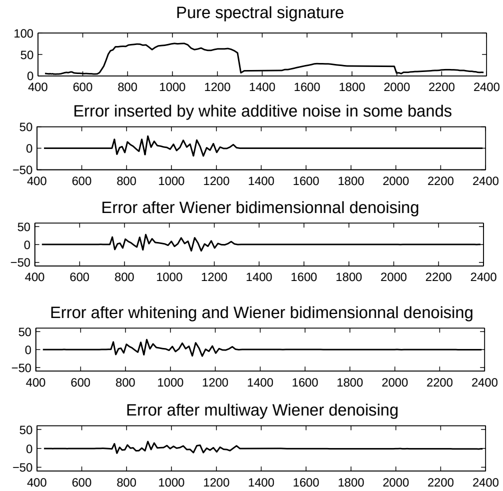


Figure 8: Error after denoising the spectral signatures with different methods

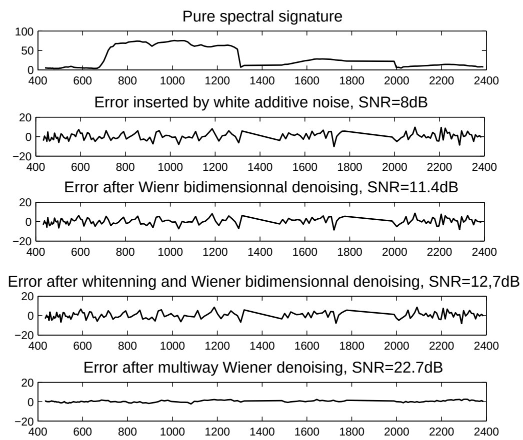


Figure 9: Error after denoising the spectral signatures with different methods

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