DESIGN OF NARROW-BAND AND WIDE-BAND FREQUENCY-RESPONSE MASKING FILTERS USING SPARSE NON-PERIODIC SUB-FILTERS

Zaka Ullah Sheikh and Oscar Gustafsson

Department of Electrical Engineering, Linköping University
SE-581 83 Linköping, Sweden
Email: {zaka, oscarg}@isy.liu.se

ABSTRACT

In this work a new technique for design of narrow-band and wide-band linear-phase finite-length impulse response (FIR) frequency-response masking based filters is introduced. The technique is based on a sparse FIR filter design method for both the model (bandedge shaping) filter as well as the masking filter using mixed integer linear programming optimization. The proposed technique shows promising results for realization of efficient low arithmetic complexity structures.

1. INTRODUCTION

Frequency-response masking techniques offer certain attractive benefits over conventional single-stage filters and a number of such techniques have evolved which are known to be highly efficient for achieving reduction of the number of multiplications and additions in narrow transition band linear-phase finite-impulse response (FIR) filters [1–5]. A narrow-band filter (narrow-band means here that for the passband edge, $\omega_c T$, we have $\omega_c T < \pi/2$) can be realized as a cascade of two filters, i.e., a periodic model filter and a masking filter as shown in Fig. 1(a). This is also known as an interpolated FIR filter. In the lowpass filter case, the masking filter $F(z)$ selects the passband in the baseband of $G(z^M)$, and eliminates the remaining passbands. This is outlined in Fig. 2, where Fig. 2(a) illustrates the initial model filter, Fig. 2(b) the periodic model filter, with period $2\pi/M$, Fig. 2(c) the masking filter, and Fig. 2(d) the overall cascaded frequency response.

The filter order of an FIR filter and therefore the arithmetic complexity (number of multiplications and additions) is inversely proportional to the width of the transistion band [3]. As the transition bands of the filters $F(z)$ and $G(z)$ are significantly wider that that of $H(z)$, one can expect a significant reduction of the arithmetic complexity for a correctly chosen M.

Selecting $F(z)$ as a highpass or a bandpass filter with the same model filter $G(z^M)$, will produce an overall highpass or bandpass filter, respectively. Similarly a wide-band filter (wide-band means here that for a lowpass filter $\omega_c T > \pi/2$) can be easily obtained by complementing a linear phase narrow-band filter as illustrated in Fig. 1(b).

Several techniques have been proposed for improving the computational efficiency of narrow-band frequency-response masking (NBFRM) which include single filter frequency masking filters [6], where the same filter is used with different periodicities, and the use of very simple subfilters, either as prefilters [7–9] or as masking filters [10]. Furthermore, designs using minimum-phase filters have been proposed [11].

In this work the application of sparse FIR filter design

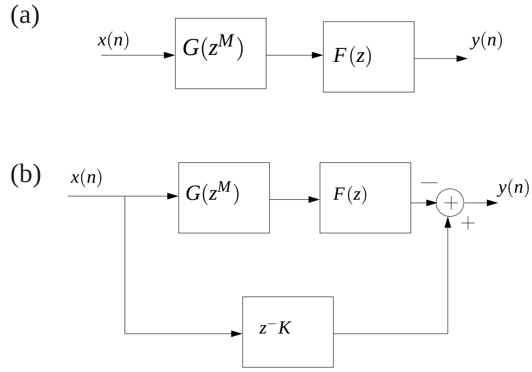

Figure 1: (a) Narrow-band FIR frequency response masking filter. (b) Wide-band FIR frequency-response masking filter.

based on mixed integer linear programming to NBFRM is presented. As we optimize for sparsity, we will in the ideal case not have a periodic model filter, instead the optimization procedure will find the sparsest possible model filter. The model and masking filters are designed in separate steps, and, hence, a globally optimal solution is not found. This is in general not surprising since the design of two cascaded FIR filters is a non-convex problem. Still, the results show a significant computational complexity reduction compared to conventional separate design.

In the next section we discuss the formulation of the mixed-integer linear programming problem used and describe our proposed design method. Then, in Section 3 some design examples are presented and elaborated to illustrate the properties and benefits of the proposed design method. Finally, some concluding remarks are given in Section 4.

2. PROPOSED DESIGN APPROACH

2.1 Design of Sparse FIR Filters

The transfer function of an N:th-order FIR filter can be written as [12]

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n} \tag{1}$$

where $h(n)$ are the impulse response coefficients. Usually, the computational complexity of an FIR filter is largely determined by the number of multiplications required to realize the transfer function, i.e, the number of non-zero h_i. Let us denote the desired magnitude function as $D(\omega T)$ where typically $D(\omega T)$ is one in passband and zero in the stopband. If we denote the maximum allowed deviation from the desired magnitude, i.e., the ripple as $\delta(\omega T)$, it can be rewritten as

 ISSN 2076-1465

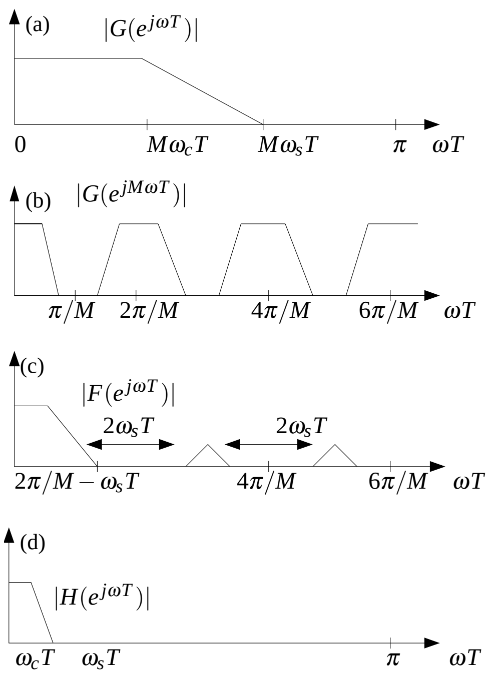

Figure 2: Frequency responses for: (a) model filter, (b) periodic model filter, (c) masking filter, and (d) overall narrowband filter.

two constraints

$$\sum_{i=0}^{L} h_i \mathrm{trig}(i,\omega T) \leq \delta(\omega T) + D(\omega T) \quad (2)$$

$$-\sum_{i=0}^{L} h_i \mathrm{trig}(i,\omega T) \leq \delta(\omega T) - D(\omega T) \quad (3)$$

where we use the zero-phase magnitude response to formulate linear constraints. Here trig is a weighted trigonometric function depending on whether N is even or odd and if the impulse response is symmetric or anti symmetric [13].

In order to minimize the number of non-zero coefficients, two more constraints are introduced which are based on non-zero binary variables $x_i \in [0,1]$ such that [12]

$$|x_i| = \begin{cases} 0 & h_i = 0 \\ 1 & h_i \neq 0 \end{cases} \quad (4)$$

The above equations can be written using linear constraints as

$$h_i \leq k_i x_i,\ \forall i \quad (5)$$

$$-h_i \leq k_i x_i,\ \forall i \quad (6)$$

Here k_i define the upper and lower bound of the variables h_i. These values can be determined using e.g. a bounding approach as proposed in [12], it is also possible to assign them a suitable value, e.g., $k_i = 1$.

As the aim is to find a solution with minimum number of multiplications the objective function is formulated as the sum of all x_i. The complete optimization problem is then formulated as

$$\text{minimize} \sum_{i=0}^{L} x_i \quad (7)$$

subject to (2), (3), (4), (5), and (6). This problem can be solved using standard MILP solvers employing techniques such as branch and bound or branch and cut.

2.2 Proposed Design Method

Ideally one would design the model filter without any period. However, since the solution of MILP problems are time consuming and the number of variables are roughly inversely proportional to the periodicity, we will consider a design problem with a period M_a, a for actual periodicity, for the model filter, while the masking filter is designed for a periodicity of M_d, d for designed periodicity.

Consider a narrow-band lowpass filter with passband ripple δ_c, stopband ripple δ_s, passband edge $\omega_c T$ and stopband edge $\omega_s T$. The minimum filter order for a single-stage FIR filter meeting the specification is N_S. The proposed design method is as follows for a given M_d and M_a (in general a search over possible $M_d \leq \pi/\omega_s T$ and $M_a \leq M_d$ is required):

1. Design an initial masking filter $F(z)$ based on the following specification:

$$\begin{array}{ll} \delta_{c,F} = \delta_c/2 & \delta_{s,F} = \delta_s \\ \omega_{c,F} T = \omega_c T & \omega_{s,F} T = 2\pi/M_d - \omega_s T \end{array} \quad (8)$$

   For this filter linear programming [14] or the Remez exchange algorithm [15] can be used. It can be noted that this filter is designed without any don't care bands, which is common for masking filters for a periodic model filter. However, as the masking filter will be reoptimized, the final masking filter will still have a low complexity.

2. Based on the discussion in the previous section formulate a MILP design problem designing an optimized model filter $G_o(z)$ which in combination with the masking filter $F(z)$ meets the filter specification. The constraint equations can be written as

$$|F(z)G_o(z^{M_a}) - 1| \leq \delta_c \quad (9)$$

$$|F(z)G_o(z^{M_a})| \leq \delta_s \quad (10)$$

   where the filter coefficients of $F(z)$ are fixed. The filter order of the model filter can be set to approximately N_S/M_a.

3. Redesign the masking filter minimizing the number of non-zero multiplication coefficients using a similar formulation as in the previous step, but with the coefficients of the model filter $G_o(z)$ fixed. Denote the resulting filter $F_o(z)$.

The two final steps can be iterated to possibly reduce the complexity further. However, based on our experience this rarely provides any improvements in practice.

It can be noted that for two different M_a values $M_{a,1}$ and $M_{a,2}$ such that one is an integer multiple of the other, say $M_{a,1} = KM_{a,2}$ for an integer K, the complexity of the optimized model filter can never be smaller for the $M_{a,1}$ case as those solutions are just a subset of the $M_{a,2}$ design. However, due to the non-convexity of the problem the total complexity may not always be minimized for smaller $M_{a,2}$.

3. DESIGN EXAMPLES

In our experiments we model the design problem using GLPK [16] and solve the problems using either GLPK or

Table 1: Complexity for narrow-band lowpass case

Period.		Complexity						
		Proposed				Conventional		
M_d	M_a	C_F	C_{Go}	C_{Fo}	Total	C_F	C_G	Total
10	7	39	5	22	27	39	6	45
9	3	28	5	19	24	28	6	34
8	7	22	5	13	18	22	7	29
7	1	15	5	11	16	15	8	23
6	1	12	7	11	18	12	10	22
5	1	10	8	9	17	10	12	22
4	1	7	11	5	16	7	14	21
3	3	5	16	4	20	5	19	24
2	1	3	21	3	24	3	28	31

M_d = Design periodicity
M_a = Actual used periodicity
C_F = Complexity of initial masking filter
C_{G_o} = Complexity of optimized model filter
C_{F_o} = Complexity of optimized masking filter
C_G = Complexity of conventional model filter

SCIP [17]. For comparison we use separate design of the masking and periodic model filters where the passband ripple is equally divided between the model and the masking filters, following [1]. The number of non-zero multiplications for a filter $H(z)$, utilizing symmetry, is denoted C_H.

3.1 Example 1 – Narrow-Band Lowpass Filter

In this first example we consider a narrow-band lowpass filter with the following specifications:

$$\begin{array}{ll} \delta_c = 0.01 & \delta_s = 0.01 \\ \omega_c T = 0.05\pi \text{ rad} & \omega_s T = 0.09\pi \text{ rad} \end{array} \tag{11}$$

A single-stage design requires a minimum filter order of 97, leading to a complexity of 49 multiplications. An exhaustive search is performed for all possible M_d with $2 \leq M_d \leq 10$ and all possible M_a with $1 \leq M_a \leq M_d$. The best results for each M_d is reported in Table 1. For comparison we have also performed a conventional design and these results are also presented in Table 1. As can be seen, the proposed design method reduce the complexity for all cases. The lower complexity is reduced from 21 for the conventional design to 16 for the proposed design method. This corresponds to a reduction of about 24%. The magnitude responses for the designed filters are shown in Fig. 3. As can be seen, the magnitude response for $M_a = 1$ did in fact end up to be periodic with a periodicity of seven. However, this was not enforced by the problem formulation.

To illustrate one of the properties of the proposed approach, we consider the detailed results obtained for the $M_d = 8$ case, as shown in Fig. 4, where the actual periodicity used in the model filter design, M_a, varies from eight to one. Based on the previous discussion, we would expect a non-increasing behavior for the model filter complexity when numbers are integer multiples. For example, for $M_a = 8, 4, 2, 1$ we can see this. In this case it also holds that the total complexity is non-increasing. However, the best result for this case is obtained for $M_a = 7$, despite that one could expect that $M_a = 1$ should give as good or better results. The reason for this is that the model filter solutions found for $M_a = 1$ and $M_a = 7$ are as good, with a complexity of five multiplications (as are the solutions for $M_a = 2$ and $M_a = 4$). However, the solution for $M_a = 7$ leads to a

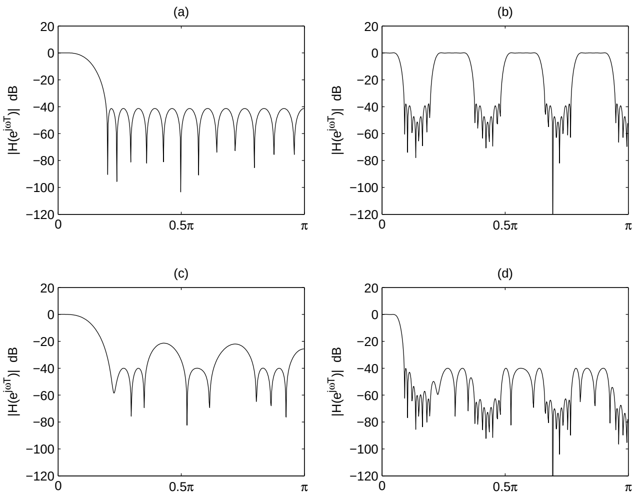

Figure 3: Magnitude responses for Example 1: (a) masking filter, (b) optimized model filter, (c) optimized masking filter, and (d) overall narrow-band filter.

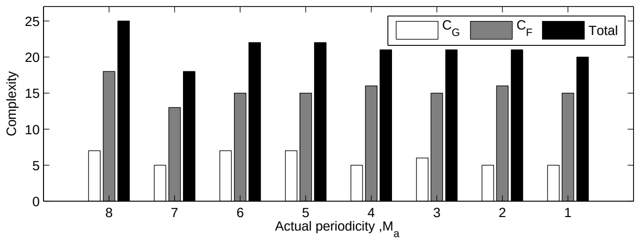

Figure 4: Masking filter, model filter, and total complexity for $M_d = 8$.

optimized masking filter with lower complexity. The reason that this can happen is obviously the non-convex underlying problem. Hence, it can make sense to try different M_a even if the filter order is low enough to allow a solution for $M_a = 1$. On the other hand, as seen from Table 1, for most of the different M_d values the solution for $M_a = 1$ is the best.

3.2 Example 2 – Wide-Band Lowpass Filter

As already mentioned the technique is equally suitable for wide-band filters as well, by utilizing the complement of the corresponding narrow band as shown in Fig. 1b. In order to further elaborate the design method and demonstrate the applicability of the technique for wide-band filters, the following specification of a wide-band lowpass filter is considered

$$\begin{array}{ll} \delta_c = 0.005 & \delta_s = 0.01 \\ \omega_c T = 0.90\pi \text{ rad} & \omega_s T = 0.91\pi \text{ rad} \end{array} \tag{12}$$

The filter order for a single-stage FIR filter meeting this specification is 430. Hence, 216 multiplications are required. This also gives that an optimization using $M_a = 1$ will require about 216 variables. Using the current optimization approach with too many coefficients, and to be able find a solution in reasonable time we used $M_a = M_d$ in this case in the design. The best result was found for $M_d = M_a = 6$ and the resulting magnitude responses are shown in Fig. 5. For this case the number of multiplications are found to be 32 for the model

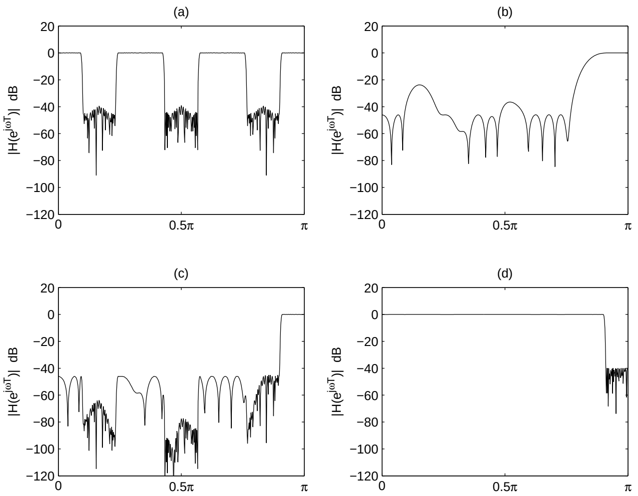

Figure 5: Magnitude responses for Example 2: (a) optimized model filter, (b) optimized masking filter, (c) narrow-band highpass filter, and (d) overall wide-band lowpass filter.

filter and 16 for the masking filter leading to a total complexity of 48 multiplications. A conventional design require 58 multiplications [10], and, hence, a complexity reduction of 17% is obtained.

4. CONCLUSIONS

In this paper we proposed a new technique for complexity reduction in narrow-band and wide-band frequency-response masking filters by introduction of sparse filters resulting in possibly non-periodic model filters. The technique is based on subsequent design of the model and masking filter, which introduces sub-optimality as the overall problem is non-convex. However, from the example designs it can be seen that the proposed design technique decreases the complexity compared to conventional frequency-response masking filters.

REFERENCES

[1] Y. Neuvo, D. Cheng-Yu, and S. Mitra, "Interpolated finite impulse response filters," *IEEE Trans. Acoust. Speech Signal Processing*, vol. 32, pp. 563–570, May 1984.

[2] T. Saramäki, Y. Neuvo, and S. Mitra, "Design of computationally efficient interpolated FIR filters," *IEEE Trans. Circuits, Syst.,*, vol. 35, pp. 70–80, Jan. 1988.

[3] T. Saramäki, "Finite impulse response filter design," in *Handbook for Digital Signal Processing*, S. Mitra and J. Kaiser, Eds. New York: Wiley, 1993, ch. 4, pp. 155–277.

[4] R. Aboul-Hosn and S. Bozic, "Multirate techniques in narrow band FIR filters." *International Journal of Electronics*, vol. 71, no. 6, pp. 939–949, 1991.

[5] M. Hoshino, M. Ikehara, and S.-I. Takahashi, "Design of narrow-band FIR filters using interpolation technique," *Electronics and Communications in Japan (Part III: Fundamental Electronic Science)*, vol. 72, pp. 75–85, 1989.

[6] O. Gustafsson, H. Johansson, and L. Wanhammar, "Narrow-band and wide-band single filter frequency masking FIR filters," *Circuits, Syst., Signal Processing*, no. 2, pp. 181–184, May 2000.

[7] J. Cabezas and P. Diniz, "FIR filters using interpolated prefilters and equalizers," *IEEE Trans. Circuits, Syst.,*, vol. 37, no. 1, pp. 17–23, Jan. 1990.

[8] Y. Lian and C. Yang, "A new structure for design narrow band lowpass FIR filters," in *Proc. IEEE TENCON*, Aug. 2001, pp. 274–277.

[9] C. Yang and Y. Lian, "Efficient prefilter structure for narrow-band bandpass FIR filter design," in *Proc. IEEE TENCON*, Oct. 2002, pp. 893–896.

[10] C. Z. Yang, Y.-C. Lim, and Y. Lian, "The design of computationally efficient narrowband and wideband sharp FIR filters," in *Proc. IEEE Int. Symp. Circuits Syst.*, May 2009, pp. 281–284.

[11] L. Rosenbaum and H. Johansson, "On low-delay frequency-response masking FIR filters," *Circuits, Syst., Signal Processing*, vol. 26, no. 1, pp. 1–25, Feb. 2007.

[12] O. Gustafsson, L. S. Debrunner, V. DeBrunner, and H. Johansson, "On the design of sparse half-band like FIR filters," in *Proc. Asilomar Conf. Signals Systems Computers*, Nov. 2007, pp. 1098–1102.

[13] L. Wanhammar and H. Johansson, *Digital Filters*. Department of Electrical Engineering, Linköping University, 2007.

[14] L. Rabiner, "Linear program design of finite impulse response (FIR) digital filters," *IEEE Trans. Audio Electroacoust*, vol. AU-20, pp. 280–288, Oct. 1972.

[15] J. H. McClellan, T. W. Parks, and L. Rabiner, "A computer program for designing optimum FIR linear phase digital filters," *IEEE Trans. Audio Electroacoust.*, vol. AU-21, pp. 506–526, Dec. 1973.

[16] *GNU Linear Programming Kit 4.43*. http://www.gnu.org/software/glpk, 2010.

[17] T. Achterberg, "Constraint integer programming," Ph.D. dissertation, Berlin, 2007.