PILOT SYMBOL DESIGN FOR CHANNEL ESTIMATION IN MIMO-OFDM SYSTEMS WITH NULL SUBCARRIERS

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ABSTRACT

In this paper, pilot design for channel estimation in multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) systems with null subcarriers is considered, where the mean square error (MSE) is chosen as our optimization criterion. We design the placement of pilot symbols and their powers for multiple transmit antennas to minimize the MSE of the least square (LS) channel estimates. To reduce the interference of pilot symbols from other transmit antennas, an algorithm that ensures that the pilot symbols are disjoint from the ones of any other antenna is proposed. Simulation results based on IEEE 802.16e are presented to illustrate the superior performance of our proposed method over the existing standard and the partially equi-spaced pilot symbols.

1. INTRODUCTION

Robustness of OFDM systems in multipath environments together with the significant information capacity gain as well as improved BER performance of MIMO systems, highlight the substantial potential of MIMO-OFDM systems. However, in comparison to a single antenna system with only one channel to be estimated, a MIMO system with N_t transmit and N_r receive antennas necessitates $N_t \times N_r$ channels to be estimated. This increased number of channels to be estimated may reduce the higher data rate of a MIMO system if pilot subcarriers are not well optimized [1]. Therefore, the placement and power distribution to pilot symbols to efficiently track the channel variation both in time and/or frequency domains is crucial as the designed pilot symbols has impact on the channel estimation performance and the BER performance of the system.

In the literature, training signal design for channel estimation have been predominantly developed for single input single output (SISO)-OFDM systems [2–6], and the reference therein. Optimal pilot symbols for OFDM systems in the absence of null edges subcarriers are considered in [2–5] where equi-distant and equi-powered pilot symbols were found to be optimal with respect to several performance measures.

In [7], a novel method for optimal preamble and pilot symbols design for SISO-OFDM systems with null subcarriers is considered in a frequency-selective block-fading channel estimation. Both pilot power and placement were obtained by minimizing the MSE of channel estimate with convex optimization methods. The same problem is addressed in [6] where the placement of training signals is obtained by parametric optimization, while the pilot power is obtained by minimizing the infinite norm of the channel MSE with convex optimization. However, in [7] it has been reported that, formulation of the convex optimization problem in [6] uses some approximation in the objective function which may not accurately represent the infinity norm of the channel MSE. Furthermore, the accuracy of cubic function based optimizations in [6] depends on many parameters to be selected for every channel/subcarriers configuration, which complicate the design especially when the method is to be adopted in MIMO-OFDM system that requires pilot set of every transmit antenna to be disjoint from the ones of any other antenna.

A number of pilot design methods for MIMO-OFDM systems have been studied, e.g. in [8–13]. In [9], equi-powered pilot symbols are studied for channel estimation in multiple antenna OFDM system with null subcarriers. But they are not always optimal even for point-to-point OFDM system. Pilot sequences designed to reduce the channel MSE in multiple antenna OFDM system are also reported in [10] but they are not necessarily optimal. In [11], partially equi-spaced pilot symbols (PEP) for MIMO-OFDM with null edge subcarriers is proposed, however the pilot placements are not unique and may not result into good pilot set for some channel/subcarriers configurations.

In this paper, we utilize the method proposed in [7] for SISO systems where pilot symbols are obtained from the optimal preamble by iterative removal of pilot symbols with minimum power. We extend this technique to MIMO systems with some modifications to ensure that the pilot symbols of one antenna are disjoint from the pilot symbols of any other antenna. A modified algorithm is proposed to ensure that the composite pilot sequence from all antennas are positioned in the active subcarriers and are placed symmetrically about the center of the active subcarrier zone.

Our novel method can be used to easily design pilot symbols for MIMO-OFDM systems with different channel/subcarriers configurations. Furthermore, our approach introduces a new pilot design paradigm that supports a prominent number of transmit antennas with more tractability in terms of complexity as well as applicability to OFDM systems with different frame structures. Several design examples based on IEEE 802.16e are provided in Section 5 to demonstrate the efficacy of our impressive design.

The rest of this paper is organized as follows: The MIMO-OFDM system model is briefly described in Section 2. Channel estimation in MIMO-OFDM is concisely presented in Section 3, while the proposed multiple antennas pilot design is addressed in 4. In Section 5, simulation results demonstrating the performance of our proposed algorithm as compared to the standard and the PEP scheme in [11] are presented and finally, Section 6 concludes our paper.

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2. MIMO-OFDM SYSTEM MODEL

We consider a frequency selective MIMO-OFDM wireless system with N_t transmit and N_r receive antennas. We assume that the discrete-time baseband equivalent channel between each transmit-receive antenna has FIR of maximum length L, and remains constant in at least one OFDM block, i.e., is quasi-static. Let us denote the channel from the ith transmit antenna to the mth receive antenna as

$$\mathbf{h}_{im} = [h_{im}[0], h_{im}[1], \ldots, h_{im}[L-1]]^T. \tag{1}$$

Our OFDM symbol is assumed to have N subcarriers. We consider one OFDM symbol duration and denote the transmitted OFDM symbol from the ith transmit antenna as

$$\begin{aligned} \mathbf{s}_i &= [s_i[0], s_i[1], \ldots, s_i[N-1]]^T & (2)\\ &= \mathbf{d}_i + \mathbf{p}_i, & (3) \end{aligned}$$

where $\mathbf{d}_i$ consists of data symbols, while $\mathbf{p}_i$ pilot symbols. We assume that $\mathbf{d}_i$ and $\mathbf{p}_i$ are in disjoint subcarrier positions.

At the transmitter, each $\mathbf{s}_i$ undergoes serial-to-parallel (S/P) followed by an N-points inverse discrete Fourier transform (IDFT) to produce an OFDM symbols. Each OFDM symbol is parallel-to-serial (P/S) converted and a cyclic prefix (CP) of length N_p is appended to mitigate the multipath effects. Then, our discrete-time baseband equivalent transmitted signals can be expressed as

$$\tilde{s}_i[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_i[k] e^{j\frac{2\pi kn}{N}}, \quad n \in [0, N-1]. \tag{4}$$

Assume that $N_p \geq L$ so that there is no inter-symbol interference (ISI) between consecutive OFDM symbols. At the receiver, we assume perfect timing synchronization.

After removing CP, the received time-domain signal at the mth receive antenna is given by

$$\tilde{\mathbf{Y}}_m = \sum_{i=1}^{N_t} \mathbf{D}(\tilde{s}_i)\mathbf{h}_{im} + \tilde{\mathbf{W}}_m, \tag{5}$$

where $\mathbf{D}(\tilde{s}_i)$ represents the diagonal matrix whose diagonal entries are $\tilde{\mathbf{s}}_i = [\tilde{s}_i[0], \tilde{s}_i[1], \ldots, \tilde{s}_i[N-1]]^T$ and $\tilde{\mathbf{W}}_m$ is assumed to be i.i.d. circular Gaussian vector with zero mean and variance $\sigma_w^2 I$.

Applying discrete Fourier transform (DFT) to the received time-domain signal $\tilde{\mathbf{Y}}_m = [\tilde{y}_m[0], \tilde{y}_m[1], \ldots, \tilde{y}_m[N-1]]^T$ we obtain

$$Y_m[k] = \sum_{i=1}^{N_t} H_{im}[k] s_i[k] + W_m[k], \tag{6}$$

where $H_{im}[k]$ is the channel frequency response of the (i,m)th channel at frequency $2\pi k/N$ given by

$$H_{im}[k] = \sum_{l=0}^{L-1} h_{im}[l] e^{-j\frac{2\pi kl}{N}}, \tag{7}$$

and the noise $\{W_k\}$ is the DFT of $\tilde{\mathbf{W}}_m$.

3. CHANNEL ESTIMATION IN MIMO-OFDM

For a discrete set $\mathscr{I}$, we denote $|\mathscr{I}|$ as the number of elements of $\mathscr{I}$. Let $\mathscr{K}_s$ be the set of active subcarriers. We assume that the number of pilot symbols in each OFDM symbol to be N_p. For OFDM symbol transmitted from the ith transmit antenna, we put pilot and data symbols at subcarrier sets denoted as $\mathscr{K}_{p_i}$ and $\mathscr{K}_{d_i}$, respectively.

To simplify the LS estimation, we set $\mathscr{K}_{p_i}$ for $i = 1, 2, \ldots, N_t$ to be disjoint such that

$$\mathscr{K}_{p_i} \cap \mathscr{K}_{p_n} = \emptyset \text{ for } i \neq n. \tag{8}$$

We also assume that there are no pilot symbols at $\mathscr{K}_{d_i}$, i.e.,

$$\mathscr{K}_{d_i} \subseteqq \mathscr{K}_s \setminus \left(\mathscr{K}_{p_1} \cup \mathscr{K}_{p_2} \cdots \cup \mathscr{K}_{p_{N_t}}\right) \tag{9}$$

where $\setminus$ denotes set difference.

Since the same channel estimation process is performed at each receive antenna, we only need to consider N_t transmit antennas and one receive antenna in designing pilot symbols, that is, the channel is modeled as a superposition of multiple-input single-output (MISO) channels, as in [11, 12]. Thus, without loss of generality, we can describe the first receive antenna and omit the receive antenna index.

Suppose that we estimate the channels for coherent detection with pilot sets $\mathscr{K}_{p_1}, \mathscr{K}_{p_2}, \ldots, \mathscr{K}_{p_{N_t}}$, then, to transmit data symbols, it is necessary to meet $|\mathscr{K}_s| - N_p N_t > 0$

Let us define the frequency-domain channel gain at

$$\mathbf{H}_i = [H_i[k_1], \ldots, H_i[k_{|\mathscr{K}_s|}]]^T, \tag{10}$$

where $k_n < k_{n'}$ if $n < n'$.

We define $\mathbf{F}$ as an $N \times N$ DFT matrix whose $(m+1, n+1)$th entry is $e^{-j2\pi mn/N}$, and

$$\mathbf{F}_L = [\mathbf{f}_0, \ldots, \mathbf{f}_{N-1}]^{\mathscr{H}} \tag{11}$$

as an $N \times L$ matrix consisting of N rows and first L columns of a DFT matrix $\mathbf{F}$, where $(\cdot)^{\mathscr{H}}$ is the complex conjugate transpose operator. We also define an $N_p \times L$ matrix $\mathbf{F}_{p_i}$ having $\mathbf{f}_{k_n}^{\mathscr{H}}$ for $k_n \in \mathscr{K}_{p_i}$ as its nth row.

Then, the received signals in (6) having pilot symbols from the ith transmit antenna is expressed as

$$\tilde{\mathbf{Y}}_i = \mathbf{D}_{p_i} \mathbf{F}_{p_i} \mathbf{h}_i + \tilde{\mathbf{W}}_i, \tag{12}$$

where $\mathbf{D}_{p_i}$ is a diagonal matrix constructed from pilot symbols from the ith transmit antenna and $\tilde{\mathbf{W}}_i$ is the corresponding sub-vector of $\tilde{\mathbf{W}}_m$.

Similar to $\mathbf{F}_{p_i}$, we define a $|\mathscr{K}_s| \times L$ matrix $\mathbf{F}_s$ having $\mathbf{f}_k^{\mathscr{H}}$ for $k \in \mathscr{K}_s$ as its kth row, where $k_n < k_{n'}$ if $n < n'$. Then, we obtain

$$\mathbf{H}_i = \mathbf{F}_s \mathbf{h}_i. \tag{13}$$

From (12) and (13), the LS estimate $\hat{\mathbf{H}}_i$ of $\mathbf{H}_i$ is given by

$$\hat{\mathbf{H}}_i = \mathbf{F}_s (\mathbf{F}_{p_i}^{\mathscr{H}} \mathbf{\Lambda}_{p_i} \mathbf{F}_{p_i})^{-1} (\mathbf{D}_{p_i} \mathbf{F}_{p_i})^{\mathscr{H}} \tilde{\mathbf{Y}}_i, \tag{14}$$

where

$$\mathbf{\Lambda}_{p_i} = \mathbf{D}_{p_i}^{\mathscr{H}} \mathbf{D}_{p_i} = \text{diag}\left(\lambda_{i,1}, \ldots, \lambda_{i,N_p}\right). \tag{15}$$

Let us define the sum of the mean-square error (MSE) of the channel gain at $\mathscr{K}_s$ as

$$\eta_i = E\{||\hat{\mathbf{H}}_i - \mathbf{H}_i||^2\}, \tag{16}$$

where $||\cdot||$ is the Euclidean norm. i.e. ℓ_2 norm. Then, the channel MSE η_i can be expressed as [6, 7]

$$\eta_i = \mathbf{F}_s \left[\frac{1}{\sigma_w^2} \left(\mathbf{F}_{p_i}^{\mathscr{H}} \mathbf{\Lambda}_{p_i} \mathbf{F}_{p_i} \right) \right]^{-1} \mathbf{F}_s^{\mathscr{H}} \tag{17}$$

For a given pilot set, the optimal pilot power $\lambda_{i,1}, \ldots, \lambda_{i,N_p}$ that minimizes the channel MSE η_i can be found numerically by resorting to convex optimization technique [7].

Since we have N_r receive antennas, the average of the LS channel MSE of each receive antenna is given by

$$\xi = \frac{\sigma_w^2}{N_t} \sum_{i=1}^{N_t} tr \left[\mathbf{F}_s \left(\mathbf{F}_{p_i}^{\mathscr{H}} \mathbf{\Lambda}_{p_i} \mathbf{F}_{p_i} \right)^{-1} \mathbf{F}_s^{\mathscr{H}} \right] \tag{18}$$

In the following, based on (18), we determine the sets $\mathscr{K}_{p_1}, \mathscr{K}_{p_2}, \ldots, \mathscr{K}_{p_{N_t}}$ and power distributions to pilot subcarriers by using convex optimization technique.

4. PILOT DESIGN FOR MIMO-OFDM

To determine pilot sets and power distributions to pilot subcarriers, we modify the algorithm in [7] to accommodate multiple antennas while guaranteeing that the designed pilot sets are disjoint from each transmit antenna. The main objective of disjoint pilot sequences in each transmit antenna is to ensure appropriate separation of pilot sequences in the receiver.

The pilot set for the first transmit antenna is obtained from the designed optimal preamble with semidefinite programming (SDP) by iterative removal of N_m minimum subcarriers symmetrically, followed by optimization of the remaining subcarriers as in [7].

Once the pilot set for the first transmit antenna is found, the set is excluded from the active subcarrier set and the pilot set for the second transmit antenna is obtained from the remaining active subcarriers using the iterative algorithm until the second pilot set is obtained. The algorithm is executed until pilot sets for all N_t transmit antennas are obtained.

The modified pilot placement and power design procedure for N_t transmit antennas is summarized as follows:

1. *Initialize $\mathscr{K}_r = \mathscr{K}_s$, where $\mathscr{K}_r$ stands for the set of available subcarriers.*
2. *while $i = 1, \ldots, N_t$*
   (a) *Define the temporary set $\mathscr{K}_t = \mathscr{K}_r$ and optimize $\mathscr{K}_t$ subcarriers using convex optimization*
   (b) *Save the obtained position and power of the subcarriers*
   (c) *If $N_p < |\mathscr{K}_t|$, remove N_m minimum subcarriers symmetrically to the zeroth subcarrier, else go to step* f)
   (d) *Update $\mathscr{K}_t$ ($|\mathscr{K}_t| = |\mathscr{K}_t| - N_m$).*
   (e) *Optimize the power of the remaining subcarriers using SDP and go to step* b)
   (f) *Save pilot position as $\mathscr{K}_{p_i}$ and its power distribution*
   (g) *Update $\mathscr{K}_r = \mathscr{K}_r \setminus \mathscr{K}_{p_i}$, $i \leftarrow i+1$ and return step* a) *until $i \geq N_t$*

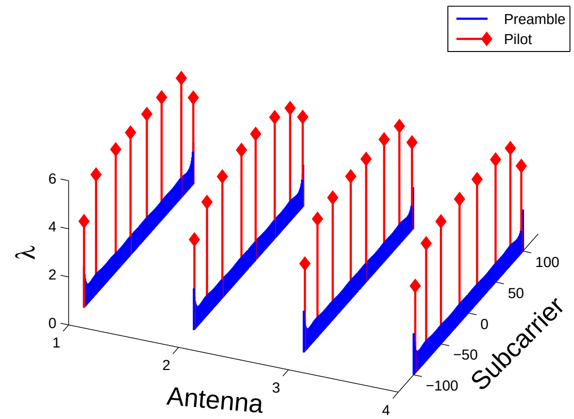

Figure 1: Pilot position and power distribution for four transmit antennas

In the algorithm, once the set $\mathscr{K}_{p_i}$ is obtained, it is excluded from the the remaining active subcarriers $\mathscr{K}_r \setminus \mathscr{K}_{p_i}$. This assures that the optimized pilot symbols from all transmit antennas are located in disjoint pilot set in any non null subcarriers, while the symmetrical removal of N_m subcarriers after every optimization, check for the disjoint pilot sets to be placed symmetrically about the center of the signal band.

When the algorithm exit, we will obtain the pilot positions and the normalized pilot powers for each antenna. To optimally distribute power between pilot symbols and data subcarriers, we can also modify the method in [7] depending on the data transmission scheme. If one adopts OFDMA for data transmission, the method in [7] can be directly applied, while if one prefers space time block coding for data transmission, the method in [7] should be modified accordingly to the data transmission scheme.

5. SIMULATION RESULTS

In this section, we demonstrate the effectiveness of our proposed pilot design through computer simulations, where we set $\sigma_w^2 = 1$. The parameters of the transmitted OFDM signal studied in our design examples are as in the IEEE 802.16e standard in [14, p.429], where an OFDM frame with $N = 256$ is considered. Out of 256 subcarriers, 200 are used as data subcarriers. Of the remaining 56 subcarriers, 28 are null in the lower frequency guard band while 27 are nulled in the upper frequency guard band and one is the central DC null subcarrier. Of the 200 used subcarriers, 8 are allocated as pilot subcarriers, while the remaining 192 are used for data transmission or null for pilot symbols of other antennas.

To design disjoint pilot tones to multiple transmit antennas, we construct a composite pilot sequence with index sets $\{\mathscr{K}_p\}$ having $N_t N_p$ subcarriers with significant pilot power and reasonable position. The pilot set for the first antenna is obtained as in [7], then by utilizing our algorithm in Section 4, which exclude the designed pilot set from the preamble and repeat the same procedure for the remaining subcarriers, we can obtain the pilot sets for all N_t transmit antennas.

Through our modified algorithm, we obtain the normalized optimal pilot symbols for the N_t transmit antennas, then we utilize the method proposed in [7] to distribute power to pilot and data subcarriers for a given OFDM power per

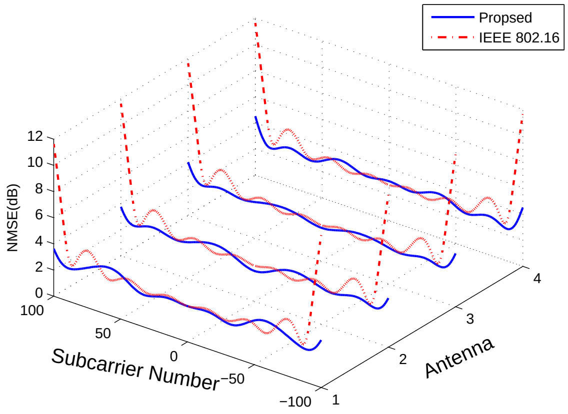

Figure 2: Comparison of channel estimate MSE between the proposed and the standard

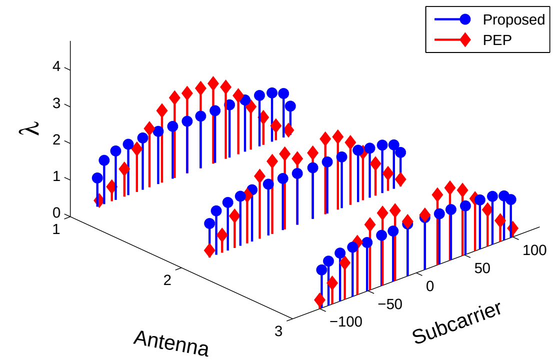

Figure 3: Comparison of pilot design for three transmit antennas

frame. The proposed method in [7] plots the BER as a function of the power allocation ratio α. Then optimal value of power allocation ratio α which minimizes the BER and make practical significance is obtained directly from the plot.

Fig. 1 shows the designed disjoint optimal pilot set for 4 transmit antennas when $N_p = L = 8$, and the total transmitted power per OFDM frame is $\mathscr{E} = 200$. We use the optimal value of $\alpha = 0.8623$ obtained in [7] for all transmit antennas. This implies that, the total pilot power for each antenna is the same for all transmit antennas.

For all antennas the pilot power and location are well distributed within the in-band region which promises better estimation of the channel even at the edge of the band. For the optimal preamble where all active subcarriers are considered as pilot symbols thereby $\alpha = 0$ and the total power dedicated to one OFDM frame is distributed to the pilot symbols according to their normalized optimal power.

In the following we compare each of the designed pilot set with the existing IEEE 802.16e standard pilot symbols separately i.e SISO-OFDM mode. The aim is to observe the performance of the designed pilot symbols in each antenna with respect to the standard one to ensure that each designed pilot set have better performance. A noteworthy fact is that, when some SISO-OFDM methods are adopted in MIMO-OFDM pilot designs the performance of some designed pilot sets deteriorates with increased number of transmit antennas. That is only few pilot sets yields a significant performance.

In Fig. 2, the normalized channel estimate MSE of the designed disjoint pilot symbols in Fig. 1 is compared with the existing standard which places the eight subcarriers at $\{\pm 13, \pm 38, \pm 63, \pm 88\}$. The total pilot power for each antenna is taken to be N_p, for both the standard (equally spaced, equi-powered pilot symbols) and our proposed method. From the plot it is clear that the performance of each antenna outperforms the standard. The standard pilot design does a poor job of estimating channel at the subcarriers near the guard band, this is due to lack of the pilot subcarriers at the edge of OFDM symbols in the IEEE 802.16e standard, and there by the estimation via the extrapolation for the edge subcarriers results in a higher error [8]. The possible solution would be to increase the number of pilot subcarriers at the edge subcarriers as proposed in [15], however this would decrease the spectral efficiency of the system. Our proposed design illuminates the improvement obtained by rearranging the pilot symbols without any addition of pilot subcarriers at the edge as suggested in [15]. This clarify that the uniform-spaced and equal power pilot symbols are suboptimal for an OFDM system with null subcarriers.

In [11], it is stated that, the power of pilot symbols decreases when the pilot symbols are close to the null/virtual carriers zone due to the fact that there are less data carriers, this might be true, however the power allocated to these subcarriers need to be significant, otherwise the problem of channel estimation via the extrapolation for the edge subcarriers will still persist.

Fig. 3 compares our proposed pilot symbols and the partially equi-spaced pilot (PEP) symbols proposed in [11] for $L = N_p = 16$. In the two designs, the total pilot power from the different transmit antennas are equal. For our proposed design power allocated to the edge pilot symbols is slightly lower than that of the mid pilot symbols, however the difference is not as large as in the PEP design. In [11], pilot placement does not consider any performance criterion, however the power allocation is based on minimizing the channel MSE to the designated pilot subcarriers. This reduces the computation complexity of the design but does not guarantee optimal pilot set. In our proposed design both pilot position and power are taken into consideration and thereby ensuring better performance under different performance criteria.

In Fig. 4, we made a comparison of the channel estimate MSE to each active subcarrier symbol for the designed disjoint pilot symbols in Fig. 3. From the plot, it is clear that the performance of our proposed design outperforms the PEP for some antennas. The PEP design does a poor job of estimating channel at the subcarriers near the guard band, this is not due to lack of the pilot subcarriers at the edge of OFDM symbols but insignificant power allocated to the pilot symbols close to the null subcarrier zone. This further suggest that both pilot powers and placements need to be carefully considered in the design.

To further demonstrate the potential of our proposed design, we made a comparison of the average channel estimate MSE vs channel length L. To obtain the channel MSE of our proposed design as well as the PEP scheme, we varied

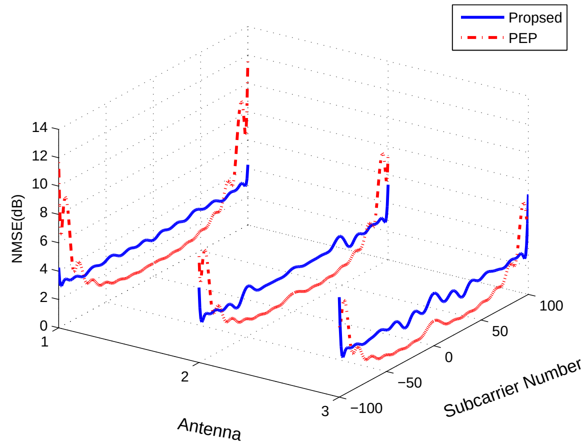

Figure 4: Comparison of channel MSE between the proposed and the PEP

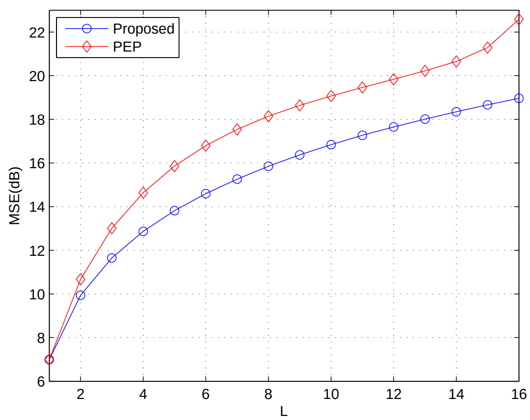

Figure 5: Channel MSE of pilot symbols for three transmit antennas

the channel length L, from 1 to 16. Fig. 5 presents the average channel MSE. The proposed optimized pilot symbols exhibit lesser channel MSE than the PEP symbols. This further demonstrate the efficiency of our proposed design.

6. CONCLUSION

In this paper we addressed the problem of channel estimation for MIMO-OFDM systems with null subcarriers. Specifically, we extended the optimization method for designing pilot symbols in a SISO-OFDM system in [7] to MIMO systems. Through numerical simulations, we have verified that the designed pilot subcarrier set for each transmit antenna has a better channel estimate performance than the existing equally spaced and equi-powered IEEE 802.16e standard and the partially equi-spaced pilot symbols. The results verify that the proposed algorithm is a prominent candidate for the design of disjoint pilot sequences in each transmit antenna that ensures appropriate separation of sequences at the receiver, while attaining a superior channel estimation over the equally spaced equal powered pilot symbols. We have also verified that the proposed method can be used to design pilot symbols in MIMO-OFDM systems with different channel/subcarriers configurations.

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