A FAST SECOND ORDER BLIND IDENTIFICATION METHOD FOR SEPARATION OF PERIODIC SOURCES

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ABSTRACT

In this paper a fast method for blind identification of periodic sources is presented. In the well-known second order blind identification method, the information is extracted from instantaneous mixtures by simultaneously diagonalizing several time-delayed covariance matrices, however, the delays are chosen arbitrarily. This imposes computational cost which is linearly related to the number of covariance matrices. Statistical characteristics of periodic sources are exploited here to develop a method to effectively choose the appropriate delays in which the diagonalization takes place. Detail theory together with the corresponding theorems have been presented. Software simulations verify the superior performance of the algorithm in the face of different noise and frequency variation levels over alternative methods.

1. INTRODUCTION

Blind source separation (BSS) has attracted many researchers during recent years and has been effectively applied in different fields including biomedical engineering, telecommunications, sound, and image processing. In general, it is impossible to solve a BSS problem unless some information is known a priori.

There are many processes in nature that originate from periodic phenomena and are studied or used in science and engineering. The knowledge about the periodicity of the signals can be exploited to separate the sources. In [1] and [2] a method based on generalized eigenvalue decomposition (GEVD) is used to diagonalize the covariance matrices of the observation vector at zero and a lag equal to the period of the source of interest. The method called periodic component analysis (πCA) maximizes a cost function which is a measure of periodicity of the estimated source. In the case of varying periods, the observations have to be adjusted to have perfect periods [2]. The performance of this method depends on proper detection of the cycles of the periodic source signal(s).

Second order statistics are widely used in source separation context. In [3] an average eigen structure of the data is obtained by simultaneous diagonalization of a set of covariance matrices each calculated at a different delay of the pre-whitened data. It has been shown that the sources can be estimated using the joint diagonalizer ([3] and [4]) of the covariance matrices. This method is called second order blind identification (SOBI). Whitening a nonzero delay covariance matrix is suggested in [5] to reduce white noise effects in the non-stationary data. In order to reduce the effects of spatially colored noise on the separation performance, the whitening is performed on a positive definite matrix in [6]. This matrix is a linear combination of covariance matrices at different delays. To minimize the effects of spatially colored noise on separation performance, a bank of subband filters is proposed in [7]. The method is based on reducing the covariance matrix of noise subband from the covariance matrix of the observations.

Despite the good performance of the methods in [5, 6, 3, 7], there is no guideline regarding the selection of the appropriate delays in order to achieve the best performance and the least computational cost in separation. Moreover, it is not known how many delayed covariance matrices are required such that the condition of essential uniqueness theorem [3] is met. In the simulations, the first $min(100, N/3)$ delayed covariance matrices are used as default, where N is the total number of samples. Although using this number of covariance matrices provides acceptable average eigenvalue decomposition, the computational cost is high (computational cost of jointly diagonalizing c matrices is proportional to c [3]).

Under the periodicity assumption of the sources, a method for selecting appropriate delays used in SOBI is presented in this paper. It is shown that for n periodic signals using just n delayed covariance matrices is enough to obtain a high quality estimation. This method is also robust to noise and performs well in those cases where the main frequency of the sources varies with time.

The outline of the letter is as follows. Problem formulation is detailed in the next section. The proposed algorithm and the simulated experiments are presented in sections III and IV, respectively. Section V contains concluding remarks.

2. PROBLEM FORMULATION

Assume a typical instantaneous BSS problem in which m mutually statistically independent unknown sources are mixed through an unknown medium and measured at n ($n \geq m$) sensors. Also, let the mixing medium be modeled by matrix $\mathbf{A}$. Such a system therefore can be formulated in a vector form as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) = [s_1(t)...s_m(t)]^T$ is the $m \times 1$ source vector, $\mathbf{n}(t) = [n_1(t)...n_n(t)]^T$ is an $n \times 1$ stationary zero mean, white noise vector independent of the source signals, $\mathbf{x}(t) = [x_1(t)...x_n(t)]^T$ is the $n \times 1$ measurement vector, $\mathbf{A}$ is an $n \times m$ unknown full column rank mixing matrix and superscript T represents the transpose operator.

Here, it is assumed that the source signals are periodic with distinct fundamental frequencies. Furthermore, to simplify the notation and with no loss of generality we assume that m=n.

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The covariance matrix of vector $\mathbf{v}(t)$ at time t and delay τ is defined as

$$\mathbf{R}_v(t,\tau) = \langle \mathbf{v}(t)\mathbf{v}^H(t+\tau)\rangle = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{t=-N}^{N} \mathbf{v}(t)\mathbf{v}^H(t+\tau) \quad (2)$$

where $\langle . \rangle$ is the expected value of the enclosed term, N is the total number of samples and superscript H denotes complex conjugate transpose of matrix. We refer the ijth element of $\mathbf{R}_v(t,\tau)$ as $r_v^{ij}(t,\tau)$.

In order to overcome the scaling problem, without loss of generality, we assume that the source signals are unit norm, which means

$$\mathbf{R}_s(t,0) = \langle \mathbf{s}(t)\mathbf{s}^H(t)\rangle = \mathbf{I} \quad (3)$$

where $\mathbf{I}$ is an $n\times n$ identity matrix . From this assumption we can easily conclude the following relations for ijth element of the covariance matrix (3):

$$|r_s^{ii}(t,\tau)| \le |r_s^{ii}(t,0)| \quad \forall t,\tau; \quad \forall i\; 1\le i\le n \quad (4)$$

$$|r_s^{ij}(t,\tau)| = 0 \quad \forall t,\tau; \quad \forall i,j\; 1\le i\ne j\le n \quad (5)$$

To estimate the original sources, the observations are firstly pre-whitened to obtain $\mathbf{z}(t) = \mathbf{W}\mathbf{x}(t) = \mathbf{C}\mathbf{s}(t)$, where $\mathbf{C} = \mathbf{W}\mathbf{A}$. $\mathbf{C}$ is a unitary matrix because $\mathbf{R}_z(t,0) = \langle \mathbf{z}(t)\mathbf{z}^H(t)\rangle = \mathbf{W}\mathbf{A}\langle \mathbf{s}(t)\mathbf{s}^H(t)\rangle \mathbf{A}^H\mathbf{W}^H = \mathbf{C}\mathbf{C}^H = \mathbf{I}$. The fundamental idea behind the method is to find a matrix $\mathbf{B}$ which estimates the source signals by a rotation. In other words, the sources can be identified by $\hat{\mathbf{s}}(t) = \mathbf{B}^H\mathbf{z}(t)$.

The covariance matrix of the whitened data at lag τ is:

$$\mathbf{R}_z(t,\tau) = \langle \mathbf{z}(t)\mathbf{z}^H(t+\tau)\rangle = \mathbf{C}\langle \mathbf{s}(t)\mathbf{s}^H(t+\tau)\rangle \mathbf{C}^H \quad (6)$$

which clearly is a normal matrix. We want to find a matrix $\mathbf{B}$ which is equal to $\mathbf{C}$ or essentially equal to $\mathbf{C}$. In this case $\hat{\mathbf{A}} = \mathbf{W}^\sharp\mathbf{B}$ where $\hat{\mathbf{A}}$ is the estimation of $\mathbf{A}$ and $\mathbf{W}^\sharp$ denotes Moore-Penrose pseudo inverse of $\mathbf{W}$.

It is known from linear algebra that all normal matrices are diagonalizable by some unitary matrices (spectral theorem in [8]) which may lead to separation. The unitary diagonalizer matrix of a whitened covariance matrix at some lag τ is the separating matrix if the covariance matrix has distinct eigenvalues. However, without a prior knowledge it is difficult to find the a time delay in which the covariance matrix is full rank. In order to reduce the probability that an unfortunate choice of time lag τ results in unidentifiability of $\mathbf{C}$ from $\mathbf{R}_z(t,\tau)$ the joint diagonalization of several covariance matrices is proposed in [3]. The consequent problem in joint diagonalization is the uniqueness of the unitary diagonalizer matrix. Here, the periodicity of the sources is used to obtain the unique unitary diagonalizer, which is the separator matrix.

We know that the source signal $s_i(t)$ is periodic for all $1\le i\le n$. This requires that for every source s_i, we have:

$$r_s^{ii}(t,kT_i) = r_s^{ii}(t,0) \quad \forall t,i \quad 1\le i\le n \quad (7)$$

where T_i is the period of source s_i, and k is an arbitrary integer. $r_s^{ii}(t,0)$ is the maximum allowed value for the covariance of the ith source. It means that the value of this function is less than $r_s^{ii}(t,0)$ for all delays except those which are integer multiples of T_i. Since the original sources are unit norm, the covariance matrix of $\mathbf{s}$ in delay kT_i obeys the following structure:

$$\begin{aligned} &\mathbf{R}_s(t,kT_i) = diag(r_1,\dots,r_{i-1},1,r_{i+1},\dots,r_n) \\ &|r_l| < 1,\; l\ne i,\; kT_i \ne T_j,\; 1\le i,j,l\le n,\; k\in\mathbb{N} \end{aligned} \quad (8)$$

Assume a unitary matrix $\mathbf{B}$ diagonalizes the covariance matrix $\mathbf{R}_z(t,\tau)$ at lag τ such that $\mathbf{B}^H\mathbf{R}_z(t,\tau)\mathbf{B} = \mathbf{B}^H\mathbf{C}\mathbf{R}_s(t,\tau)\mathbf{C}^H\mathbf{B} = \Lambda$. Both $\mathbf{B}$ and $\mathbf{C}$ are unitary matrices, so $\mathbf{D} = \mathbf{B}^H\mathbf{C}$ is also a unitary matrix and the diagonal matrix $\Lambda = diag(\lambda_1,\lambda_2,\dots,\lambda_n)$ is the eigenvalue matrix of $\mathbf{R}_s(t,\tau)$ which is equal to $\mathbf{R}_s(t,\tau)$. Therefore, for each delay T_i, the covariance matrix $\mathbf{R}_z(t,T_i)$ is diagonalizable and only one of its eigenvalues is equal to 1. This means for each T_i we have:

$$r_i = 1 \ne r_j \quad \forall j\;\; 1\le j\le n,\; j\ne i \quad (9)$$

This fact is used in the following theorem to guarantee uniqueness of the the unitary diagonalizer.

Theorem 1. *Assume that $\mathbf{z}(t)$ is a white mixture of periodic sources with distinct periods and the covariance matrices of the source vector $\mathbf{s}(t)$ satisfy* (8). *If a unitary matrix $\boldsymbol{B}$ simultaneously diagonalizes the set of covariance matrices $\mathscr{R} = \{\boldsymbol{R}_z(t,T_i)\ \forall i\; 1\le i\le n\}$, (i.e. for all i $\boldsymbol{R}_z(t,T_i) = \boldsymbol{B}\boldsymbol{D}_i\boldsymbol{B}^H$, where $D_i = diag(d_1(i),d_2(i),\dots,d_n(i))$) then any joint diagonalizer of elements of $\mathscr{R}$ is essentially equal to $\boldsymbol{B}$.*

Proof. To prove the sufficiency of the theorem, we assume that a linear combination of the columns of $\mathbf{B}$ (i.e. $\mathbf{e} = \sum_{i=1}^n \alpha_i\mathbf{b}_i$) is a common eigenvector of the members of $\mathscr{R}$. Therefore, for all $1\le j\le n$

$$\mathbf{R}_z(t,T_j)\mathbf{e} = \lambda_j\mathbf{e} = \sum_{i=1}^{n} \lambda_j\alpha_i\mathbf{b}_i \quad (10)$$

where $\mathbf{b}_i$ is the ith column of $\mathbf{B}$, λ_j is an eigenvalue of $\mathbf{R}_z(t,T_j)$ and α_i's are complex coefficients. We arbitrarily assume that $\alpha_p \ne 0$. Then, j can be found in a way that $d_p(j) = \mathbf{b}_p^H\mathbf{R}_z(t,T_j)\mathbf{b}_p = 1$, $1\le p\le n$. We also know that

$$\mathbf{R}_z(t,T_j)\mathbf{e} = \sum_{i=1}^{n} \alpha_i\mathbf{R}_z(t,T_j)\mathbf{b}_i = \sum_{i=1}^{n} \alpha_i d_i(j)\mathbf{b}_i \quad (11)$$

From (10) and (11) one can conclude that for all i, $\alpha_i(\lambda_j - d_i(j)) = 0$. As the sources are periodic, we know that $d_j(j) = 1 \ne d_i(j)$. Therefore, $\lambda_j = d_p(j)$ and $\alpha_i = 0$ for all $i\ne p$.

For the necessity condition assume that for two arbitrary indices (p,q) $d_p(j) = d_q(j)$ for all j. It's clear that any linear combination of the columns of $\mathbf{B}$ is a common eigenvalue of the members of $\mathscr{R}$.

□

Although the above analysis is based on the assumption that the periods of the signals are exactly known, the analysis is still true for some delays close to the exact periods. In other words, when there is uncertainty about the fundamental periods of the sources or fundamental periods vary with time the method can still successfully be used. In (7) and (8) we showed that for each periodic signal there is a set of delays in which the source covariance has a maximum value. A rough estimation of the source frequencies may be obtained by different time and frequency domain methods. It is very likely that the maximum covariance value can be found in a delay close to the estimated period. Hence, to best cover the estimation indeterminacy or the frequency variations a window W centered at the delay corresponding to the estimated frequency is used. A suitable window length L (which depends on the nature of the sources) meets the condition of Theorem 1 and so can lead to separation of the source signals. It is also noteworthy that choosing the appropriate window length is not our major concern here.

Remarks:

- Periodic component analysis [1] is a special case of the presented method in which the diagonalization is done in only two lags (i.e. zero and the one corresponding to the frequency of periodic signal). However, the accuracy of this method is subject to the frequency variations. The presented method in [2] for adjusting the period is useful as long as the cycles of the periodic signals can be accurately recognized by some means.
- The proposed method can be considered as a special case of the well-known SOBI method. SOBI is a widely used method and has an approved performance, however, a large number of matrices is usually used in this method. As it is shown in section IV selecting a small number of covariance matrices does not provide a correct separation by SOBI and a large number of matrices require higher computational cost.

3. SEPARATION ALGORITHM

Based on Theorem 1 our objective is to find a unitary matrix $\mathbf{B}$ which jointly diagonalizes the set of selected delayed covariance matrices. In other words, the desired $\mathbf{B}$ is the one which minimizes the squared off-diagonal elements of the set of all $\mathbf{B}^H\mathbf{R}_z(t,T_i)\mathbf{B}$ for all periods T_i. The implementation of the proposed method is presented in Algorithm 1.

Algorithm 1

1) Estimate periods of the sources
2) Pre-whiten the data by $\mathbf{W}$ as $\mathbf{z}(t) = \mathbf{W}\mathbf{x}(t)$. $\mathbf{W} = \Lambda^{-1/2}\mathbf{E}^T$ where Λ is the eigenvalue matrix of $\mathbf{x}(t)$ and $\mathbf{E}$ is the corresponding eigenvector matrix.
3) Calculate $\mathscr{R} = \bigcup_{i=1}^{n}\mathscr{R}_i$ where $\mathscr{R}_i = \{\mathbf{R}_z(t,T_i),\mathbf{R}_z(t,T_i\pm 1),\ldots,\mathbf{R}_z(t,T_i\pm\lfloor L/2\rfloor)\}$ *
4) Find $\mathbf{B}$, the joint diagonalizer of the set of covariance matrices R.
5) The estimated sources are formed by $\mathbf{B}$ and $\mathbf{W}$ as $\hat{\mathbf{s}}(t) = \mathbf{B}^H\mathbf{W}\mathbf{x}(t)$.

where $\lfloor . \rfloor$ is the floor operator, i.e. the largest integer not greater than the operand.
* The set of covariance matrices $\mathscr{R}_i$ can be formed for any delay kT_i instead of T_i as long as $kT_i \neq T_j$, $j \neq i$ and $k \in \mathbb{N}$.

4. EXPERIMENTS

To evaluate the performance of the proposed method, different experiments were designed for both synthetic and real world data. The first data set used here includes four periodic sources, each composed of sine waves with normalized frequencies of 0.023, 0.01, 0.037, and 0.017 Hz and few harmonics. To evaluate the performance of the method in such

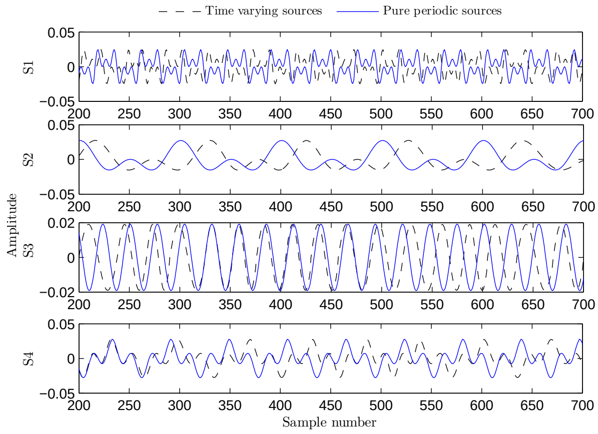


Fig. 1: Four periodic sources used in the experiments. For some experiments the frequencies of the sources are changed by time. The black dashed lines show the distorted sources when change of up to 10% in frequencies is permitted in each cycle.

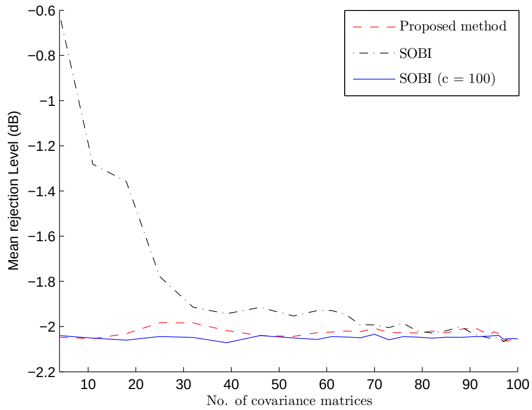


Fig. 2: Mean rejection level vs. the number of covariance matrices.

cases that the main frequency of the signals vary in time, a random coefficient is also applied to the frequencies in each cycle. Fig. 1 illustrates 500 samples of pure periodic sources along with their distorted versions. The main frequency of the distorted sources have a random variation of up to 10%. The second data set is a mixture of voice and music signals. Experiments 1-4 are performed on the synthetic data, and in the last experiment the proposed method is applied to real world data.

In the ideal case of the separation, $\hat{\mathbf{A}}$ should be equal to $\mathbf{A}$, or in other words $\hat{\mathbf{A}}^\sharp\mathbf{A} = \mathbf{I}$. Therefore, the sum of the squared off-diagonal elements of $\hat{\mathbf{A}}^\sharp\mathbf{A}$ which is called *mean rejection level* (*MRL*) is used here as a quantitative measure to evaluate the algorithm [3]. The lower the value of MRL is, the better performance from the algorithm is expected.

In the following experiments, 2000 samples of the peri-

Table 1: Simulation time vs. number of covariance matrices for the proposed method. The average time for SOBI with c = 100 is 94.1ms. (All times are in milliseconds.)

	No. of covariance matrices (c)							
	11	18	39	53	64	79	94	100
time	19.1	23.0	38.1	50.0	57.0	70.5	87.3	94.3

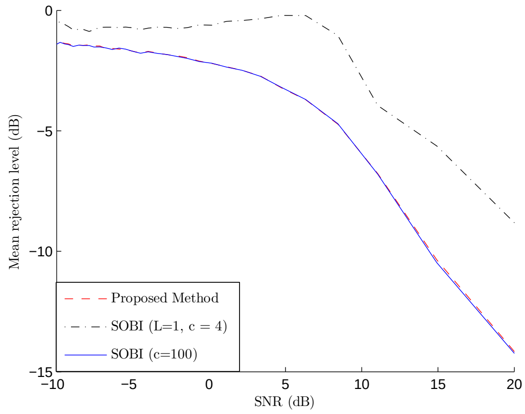


Fig. 3: Mean rejection level vs. SNR. The proposed method and SOBI (c=100) perform similarly.

odic sources are mixed through linear mixtures while Gaussian noise is added to the mixtures. MRL (when used) is averaged over 100 independent trials for each value of the horizontal axis. L covariance matrices are calculated for each estimated period and after omitting the overlapping delays a set of c matrices are jointly diagonalized.

Experiment 1: In order to compare the execution time of the proposed method to that of SOBI, both methods were applied to the first data set (while the sources are pure periodic sources, Fig. 1). The experiments were conducted on a PC with 3.2 GHz Pentium IV CPU and 1.5GB of RAM. The average execution time of 100 independent trials of the original SOBI and the proposed method for different number of covariance matrices have been reported in Table 1. Lower number of covariance matrices, yields lower execution time. At the same time that the algorithm converges fast, by analyzing the mean rejection levels of the experiments it is verified that the separation quality is very close to that of SOBI (see Fig.2).

Experiment 2: MRL against the total number of delayed covariance matrices (c) is shown in Fig. 2. The horizontal axis represents the total number of covariance matrices. The results of the proposed algorithm along with the results of SOBI, when the same number of covariance matrices are used are presented in this figure. Result of SOBI with 100 covariance matrices is also presented as a reference. In this experiment the signal-to-noise ratio (SNR) defined as SNR = $-10\log_{10}\sigma^2$ is set to -1dB, where σ^2 is the noise variance. For almost all values of c the proposed method performs very close to SOBI with 100 covariance matrices.

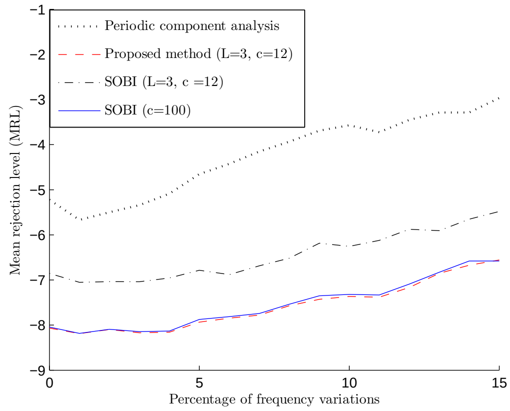


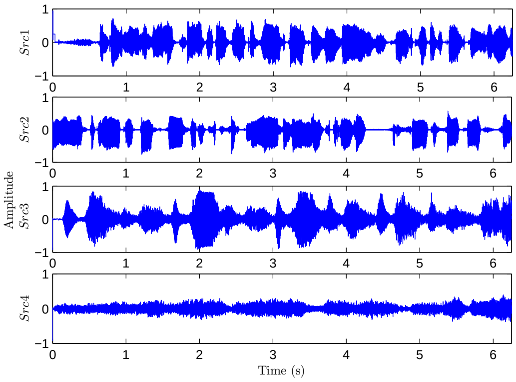
Fig. 4: Mean rejection level vs. variations in frequency.

Experiment 3: In this experiment the effects of SNR on the performance of the methods were investigated and the results have been depicted in Fig. 3. Different levels of noise were added to the linear mixtures of the same sources used in previous experiments. The results of the proposed method are shown by red dashed line. Although only 4 covariance matrices are diagonalized by the proposed method, the performance is very close to SOBI with 100 covariance matrices (blue solid line). This performance is the result of choosing appropriate time delayed covariance matrices.

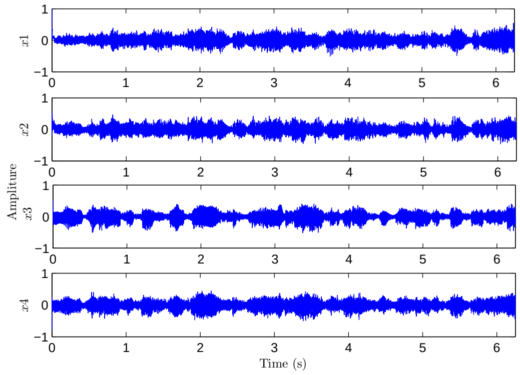
Experiment 4: Fig. 4 demonstrates the performance of the proposed algorithm for a set of periodic signals with time varying frequencies. Again, the main frequencies of the signals are equal to those of the signals used in experiment 1, but for each cycle of the ith source, $f_i + \beta f_i$ is used as the main frequency, where f_i is the main frequency and $-0.15 < \beta < 0.15$ is a random coefficient. As expected, the proposed method with c=12 and SOBI with c=100 provide better performance compared to periodic component analysis and SOBI with c=12.

Experiment 5: The second data set includes linear mixture of voice and music signals. Both the original sources and the mixtures were obtained form the ICA demo page at Helsinki University of Technology ICA research group website[1]. The sources 2, 4, 5, and 7 were selected and the proposed method was applied to their linear mixtures. Figures 5.a and b show the original sources and the mixtures respectively. The proposed method is applied to the mixtures while only 12 covariance ($L = 3$, $c = 12$) matrices are carefully chosen to be diagonalized simultaneously. The result of application of the method is presented in Fig. 5.c. Although there are scaling and permutation ambiguities, the estimated sources are very similar to the original ones.

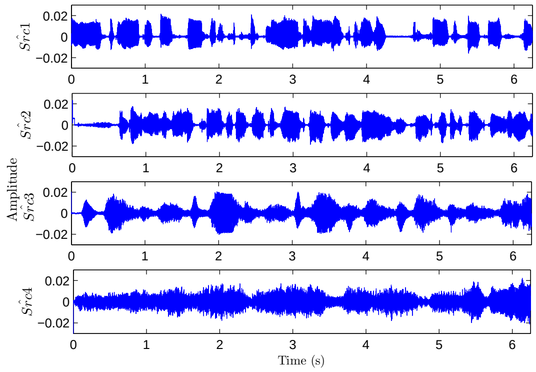
[1]The demo page is accessible from: `http://www.cis.hut.fi/projects/ica/cocktail/cocktail_en.cgi`



(a)



(b)



(c)

Fig. 5: A set of voice and music signals; (a) original sources, (b) linear mixtures, and (c) output of the proposed method.

The period of the original sources have to be known *a priori*. Here, the appropriate time delays are chosen using the power spectral density (PSD) of the sources. In Fig. 6 a portion of the PSD of the sources are presented. The periodicity of the sources can be detected from the high peaks of the PSDs, as for this case 34.9, 97.5, 134.6, and 164.4 hertz are selected empirically.

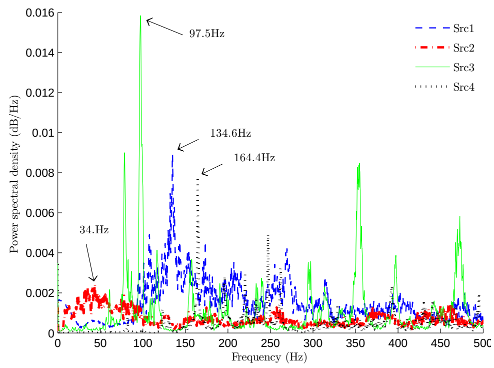


Fig. 6: Part of power spectral densities of the original sources in Fig. 5.a. Arrows point to some of the frequencies corresponding to the appropriate time delays to be used in the proposed algorithm.

5. CONCLUSION

In this paper an efficient method for selecting the optimal delays for second order blind identification of periodic signals has been presented. The cost of computations for simultaneous diagonalization of covariance matrices in the second order blind identification method is a linear function of the number of covariance matrices, however in the proposed method using considerably small set of covariance matrices results in a fast and still precise separation. Different experiments show that the results of the proposed method are the asymptotic results of SOBI with a significantly lower computational cost.

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