ROBUST AND EFFICIENT FILTERBANK STRUCTURE FOR RECONSTRUCTION FROM RECURRENT NONUNIFORM SAMPLES

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ABSTRACT

In this paper, we present a robust and efficient structure for the reconstruction of uniform samples of a signal from its recurrent nonuniform samples. This structure makes use of the synthesis part of a uniform discrete Fourier transform (DFT) modulated filterbank. The proposed structure is a refinement over an existing filterbank structure that utilizes the inverse of a nonuniform DFT matrix, the numerical stability of which cannot be guaranteed. The poor numerical stability of the inverse is overcome in a modified structure by inverting a preconditioned matrix having an improved numerically stable inverse. Necessary and sufficient conditions that ensure numerical stability of the preconditioned matrix inverse have been presented.

1. INTRODUCTION

There are a variety of applications in which the signals are sampled nonuniformly and some of them have been discussed in [1, 2]. Different reconstruction schemes for obtaining uniform samples of a signal from its nonuniform samples have been proposed and a few of them are presented in [1]. A common type of nonuniform sampling is recurrent nonuniform sampling. When many parallel low speed A/D converters are operating in a time-interleaved manner [3], there will be a time offset among these A/D converters. This results in a sequence of recurrent nonuniform samples of the original signal. The filterbank interpretations of recurrent nonuniform sampling scheme have been proposed in [2, 4]. The problem of recurrent nonuniform sampling and its reconstruction for multiband signals have been studied in [5, 6]. In [2], multilevel piecewise constant filters have been used. A new alternative model has been proposed in such a manner that there is a simple mutual relation between different filters, as outlined in [7] . This mutual relation can be used to switch the order of the up-samplers and the filters, resulting in a realization with improved efficiency.

In [7], an alternative model has been described by the analysis part of a uniform DFT modulated filterbank, from which different uniformly distributed and down-sampled frequency bands are mixed in a particular manner. This alternative model description gives an efficient structure for reconstructing uniform samples of a signal from its recurrent nonuniform samples [7]. The limitations of this filterbank reconstruction structure have been explained in [7]. In [8], a new fast and efficient Adaptive weights-Conjugate gradient-Toeplitz matrix (ACT) algorithm was proposed for the reconstruction of uniform samples of a signal from its recurrent nonuniform samples. The iterative Conjugate Gradient (CG) algorithm is briefly described in Proposition 3 in [8]. The ACT algorithm in [8] utilizes a Toeplitz Hermitian matrix, which is inverted by means of the CG algorithm to obtain the vector of reconstructed uniform samples of the signal. If the time gap between any two adjacent samples is suitably upper bounded, then the condition number of the Toeplitz Hermitian matrix is also upper bounded (Proposition 2 in [8]). This means that the Toeplitz Hermitian matrix has a numerically stable inverse if the gap between any two adjacent time instants is suitably upper bounded (upper bound of which is given in [8]).

The statement of our problem is as follows: In this paper, we address one of the limitations of the alternative model filterbank reconstruction structure proposed in [7] (third point in Section VII of [7]). The reconstruction structure presented in [7] makes use of the inverse of a nonuniform DFT matrix. This inverse matrix will become numerically unstable if any two delays (nonuniform time instants) are almost equal. In this paper, we precondition the nonuniform DFT matrix used in [7] in a specific manner to obtain a Toeplitz Hermitian matrix similar to the one utilized in [8]. Since our preconditioned matrix and the Toeplitz Hermitian matrix in [8] are similar, the conditions under which the Toeplitz Hermitian matrix has a numerically stable inverse are also applicable for our preconditioned matrix. We utilize the same conditions provided in Proposition 1 and 2 in [8] to obtain the necessary and sufficient conditions for our preconditioned matrix to have a numerically stable inverse. Finally, we present a modified, robust and efficient reconstruction structure which makes use of the inverse of the Toeplitz Hermitian preconditioned matrix. For the recurrent nonuniform sampling sets satisfying the above conditions, our modified robust filterbank reconstruction structure is a better choice than the reconstruction structure proposed in [7]. However, a limitation of our modified reconstruction structure is that the numerical stability of the preconditioned matrix cannot be guaranteed for sampling sets violating the above conditions. In this paper, an example is provided to demonstrate a case where the modified structure gives accurate output in lower number of CG method iterations when compared to the original structure. Our proposed approach is suitable for applications where the maximum separation of the sampling points can be controlled but not the minimum separation.

This paper is organized as follows: In Section 2, we briefly discuss the reconstruction equation and the reconstruction structure proposed in [7]. In Section 3, we provide an upper bound for the condition number of the preconditioned matrix. This section also lists the necessary and sufficient conditions under which the upper bound exists. We present a modified robust and efficient reconstruction structure which makes use of the inverse of the preconditioned matrix. In Section 4, we discuss the simulation results and Section 5 concludes the paper.

Notation Convention: We will follow the same notation convention as outlined in [7]. Lower case characters represent the signals in the time-domain and upper case characters represent the signals in the frequency-domain. Underlined boldface characters are used for vectors, boldface characters for matrices, $diag\{\}$ is a diagonal matrix, $\boldsymbol{W}^\dagger$ is the pseudo-inverse of the matrix $\boldsymbol{W}$, $cond(\boldsymbol{Q})$ represents the l_2-condition number of the matrix $\boldsymbol{Q}$ and $sqrt(.)$ is the square root function.

2. EFFICIENT DFT MODULATED FILTERBANK RECONSTRUCTION STRUCTURE [7]

In [7], a new alternative discrete-time analysis model of the recurrent nonuniform sampling scenario was presented. This new alternative model was introduced in order to avoid the phase jump in the Fundamental Interval (FI) of the aliased signal that results after the down-sampling operation. This alternative model structure was illustrated in Fig. 8 of [7]. This model was described by the analysis part of a uniform DFT modulated filterbank from which the uniformly distributed and down-sampled frequency bands were mixed in a specific manner. This new alternative model was used to obtain an efficient DFT modulated filterbank structure for reconstructing uniform samples of a signal from its recurrent nonuniform samples. The down-sampler and the up-sampler values of this structure are defined as K. Also, $\frac{1}{T_0}$ is the Nyquist rate and the recurrence period, $T_s = KT_0$. Each of the K inputs to this structure is taken at $\frac{1}{K}$ times the Nyquist rate. This implies that the recurrent nonuniform samples are taken at the Nyquist rate. Here, we consider the same structure with recurrent nonuniform samples taken at a rate greater than or equal to the Nyquist rate. Now, each recurrence period (T_s) consists of N nonuniform sampling points, where $N \geq K$. Thus, the recurrent nonuniform samples are considered here at $\frac{N}{K}$ times the Nyquist rate. The basic N nonuniform time instants in a recurrence period are $\{\tau_p T_0\}_{p=0}^{N-1}$, where $0 \leq \tau_0 < \tau_1 < \cdots < \tau_{N-1} < K$. The complete set of recurrent nonuniform time instants for $n \in (-\infty, \infty)$ is given by

$$\tau_p T_0 + nT_s,\ p = 0, 1, \ldots, (N-1).$$

Let the normalized nonuniform time instant be $t_i = \frac{\tau_i}{K}$, for $i = 0, 1, \ldots, (N-1)$.

$$0 \leq t_0 < t_1 < \cdots < t_{N-1} < 1 \ . \tag{1}$$

From [7], let $X(e^{j\theta})$ represent the discrete-time Fourier transform (DTFT) of the uniform samples of the original continuous-time signal $x(t)$. The output discrete-time signals of the alternative discrete-time model in [7] combinedly represent the recurrent nonuniform samples of $x(t)$. Their frequency responses are given by $Y_0(e^{j\theta})$, $Y_1(e^{j\theta})$, $\ldots$,and $Y_{N-1}(e^{j\theta})$, i.e., $\{Y_p(e^{j\theta})\}_{p=0}^{N-1}$. For $p = 0, 1, \ldots, (N-1)$, define $Y_{s,p}(e^{j\theta})$ as $Y_{s,p}(e^{j\theta}e^{j(K-1)\pi}) = Y_p(e^{j\theta})$. Let $\boldsymbol{F}$ be a $K \times K$ DFT matrix with elements $(\boldsymbol{F})_{p,q} = W_K^{pq}$, for $p, q = 0, 1, \ldots, (K-1)$, where the twiddle factor $W_K = e^{-j2\pi/K}$. The frequency shift matrix is defined as $\boldsymbol{S} = diag\{W_K^{-\frac{K-1}{2}\cdot 0}, \ldots, W_K^{-\frac{K-1}{2}\cdot(K-1)}\}$. Therefore, the shifted DFT matrix is represented by $\boldsymbol{F}_s = \boldsymbol{SF}$.

The matrix-vector equation describing the alternative discrete-time model presented in [7], is given by

$$\underline{\boldsymbol{Y}}_s(e^{j\theta}) = \frac{1}{K} \cdot \boldsymbol{\Delta}(e^{j\theta/K}) \cdot \boldsymbol{W} \cdot \underline{\boldsymbol{X}}(e^{j\theta/K}) \ , \tag{2}$$

where $\underline{\boldsymbol{X}}(e^{j\theta/K}) = [X(e^{j\theta/K} \cdot W_K^{\frac{K-1}{2}}), \ldots, X(e^{j\theta/K} \cdot W_K^{-\frac{K-1}{2}})]^T$, $\boldsymbol{\Delta}(e^{j\theta/K}) = diag\{e^{-j\tau_0\theta/K}, \ldots, e^{-j\tau_{N-1}\theta/K}\}$, $\boldsymbol{W}$ is a $N \times K$ nonuniform DFT matrix with entries $(\boldsymbol{W})_{k,p} = W_K^{(p-\frac{K-1}{2})\tau_k}$, for $k = 0, 1, \ldots, (N-1)$ and $p = 0, 1, \ldots, (K-1)$. Also, we have $\underline{\boldsymbol{Y}}_s(e^{j\theta}) = [Y_{s,0}(e^{j\theta}), \ldots, Y_{s,N-1}(e^{j\theta})]^T$. The frequency responses of the outputs representing the recurrent nonuniform samples are given by the following vector: $\underline{\boldsymbol{Y}}(e^{j\theta}) = [Y_0(e^{j\theta}), \ldots, Y_{N-1}(e^{j\theta})]^T = [Y_{s,0}(e^{j\theta} \cdot e^{j(K-1)\pi}), \ldots, Y_{s,N-1}(e^{j\theta} \cdot e^{j(K-1)\pi})]^T$.

The description of the alternative discrete-time model of the recurrent nonuniform sampling scenario in [7] consists of a mixture of the K uniformly distributed down-sampled frequency bands: $X(e^{j\theta/K} \cdot W_K^{\frac{K-1}{2}})$, $X(e^{j\theta/K} \cdot W_K^{\frac{K-3}{2}})$, $\ldots$, $X(e^{j\theta/K} \cdot W_K^{-\frac{K-3}{2}})$ and $X(e^{j\theta/K} \cdot W_K^{-\frac{K-1}{2}})$. From (2), the reconstruction equation can be presented as

$$\frac{1}{K} \cdot \underline{\boldsymbol{X}}(e^{j\theta/K}) = \boldsymbol{W}^\dagger \cdot \boldsymbol{\Delta}^{-1}(e^{j\theta/K}) \cdot \underline{\boldsymbol{Y}}_s(e^{j\theta}) \ . \tag{3}$$

Based on [7], the efficient DFT modulated filterbank reconstruction structure for recurrent nonuniform samples taken at $\frac{N}{K}$ times the Nyquist rate is given in Figure 1.

3. A MODIFIED ROBUST AND EFFICIENT DFT MODULATED FILTERBANK RECONSTRUCTION STRUCTURE

We have, $\boldsymbol{W}^\dagger = (\boldsymbol{W}^H\boldsymbol{W})^{-1}\boldsymbol{W}^H$ and the matrix $\boldsymbol{W}^H\boldsymbol{W}$ will have a bounded condition number if a subset of K sampling instants (from the total N sampling instants in a recurrence period) are sufficiently separated. But, in practice, different delays (nonuniform time instants) can almost be equal to each other. In such cases, $\boldsymbol{W}^\dagger$ used in the reconstruction structure in Figure 1 will have poor numerical stability. This leads to inaccurate reconstruction of uniform samples of the signal or results in a slow convergence of the output in Figure 1 towards the original signal uniform samples. One common technique to overcome this problem is to precondition the nonuniform DFT matrix $\boldsymbol{W}$, such that the resultant matrix has an improved condition number. In this paper, we precondition $\boldsymbol{W}$ in a specific way (as described below) to obtain a Toeplitz Hermitian matrix whose properties have been

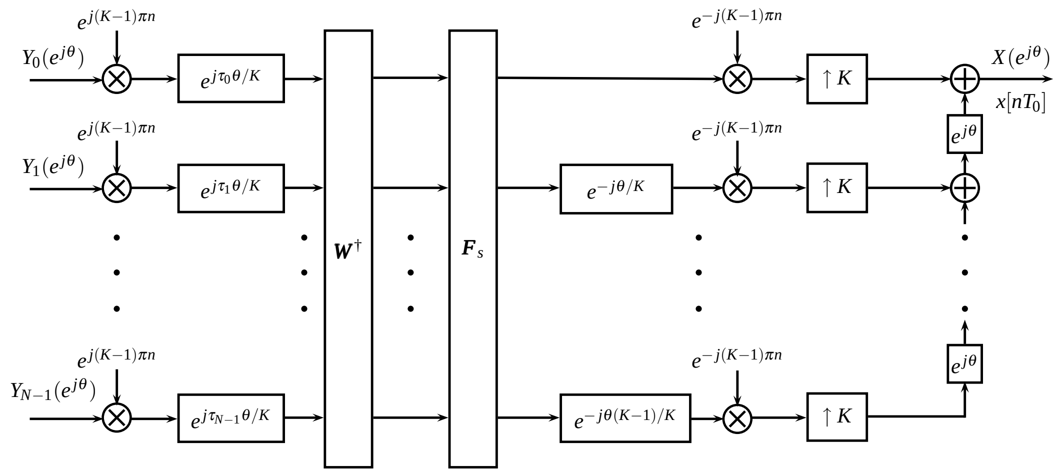

Figure 1: Efficient DFT modulated filterbank structure for reconstruction from recursive nonuniform samples taken at $\frac{N}{K}$ times the Nyquist rate (from [7])

discussed in [8]. From (2),

$$\frac{1}{K}\cdot\boldsymbol{W}\cdot\underline{\boldsymbol{X}}(e^{j\theta/K}) = \boldsymbol{\Delta}^{-1}(e^{j\theta/K})\cdot\underline{\boldsymbol{Y}}_s(e^{j\theta}) \ .$$

Let the diagonal matrix $\boldsymbol{G} = diag\{\alpha_0, \alpha_1, \ldots, \alpha_{N-1}\}$, where $\{\alpha_p\}_{p=0}^{N-1}$ are positive adaptive weights defined in [8]. Multiplying the above equation by $\boldsymbol{W}^H\boldsymbol{G}$ on both sides, we obtain

$$\frac{1}{K}\cdot\boldsymbol{W}^H\boldsymbol{G}\boldsymbol{W}\cdot\underline{\boldsymbol{X}}(e^{j\theta/K}) = \boldsymbol{W}^H\boldsymbol{G}\cdot\boldsymbol{\Delta}^{-1}(e^{j\theta/K})\cdot\underline{\boldsymbol{Y}}_s(e^{j\theta}) \ .$$

The preconditioner used for $\boldsymbol{W}$ is $(\boldsymbol{W}^H\boldsymbol{G})^{-1}$ and the resultant matrix obtained after preconditioning is $\boldsymbol{Q} = \boldsymbol{W}^H\boldsymbol{G}\boldsymbol{W}$. The modified reconstruction equation is given by

$$\frac{1}{K}\cdot\underline{\boldsymbol{X}}(e^{j\theta/K}) = \boldsymbol{Q}^{-1}\boldsymbol{W}^H\boldsymbol{G}\cdot\boldsymbol{\Delta}^{-1}(e^{j\theta/K})\cdot\underline{\boldsymbol{Y}}_s(e^{j\theta}) \ . \quad (4)$$

The preconditioned matrix $\boldsymbol{Q}$ is a $K\times K$ Toeplitz Hermitian matrix with elements,

$$(\boldsymbol{Q})_{k,q} = (\boldsymbol{W}^H\boldsymbol{G}\boldsymbol{W})_{k,q} = \sum_{p=0}^{N-1}\alpha_p W_K^{-(k-q)\tau_p}$$

$$\implies (\boldsymbol{Q})_{k,q} = \sum_{p=0}^{N-1}\alpha_p e^{j\frac{2\pi(k-q)\tau_p}{K}} = \sum_{p=0}^{N-1}\alpha_p e^{j2\pi(k-q)t_p} \ , \quad (5)$$

for $k,q = 0,1,\ldots,(K-1)$. The dimensions of $\boldsymbol{Q}$ are independent of N (number of nonuniform sampling points per recurrence period), which is not so in the case of the $N\times K$ matrix $\boldsymbol{W}$. The adaptive weights [8] are defined as

$$\alpha_p = \frac{t_{p+1}-t_{p-1}}{2}, \ p = 0,1,\ldots,(N-1) \ , \quad (6)$$

where $t_{-1} = t_{N-1}-1$ and $t_N = t_0+1$. This implies $\tau_{-1} = \tau_{N-1}-K$ and $\tau_N = \tau_0+K$. We can observe that the matrix $\boldsymbol{Q}$ is same as the transpose of the matrix $\boldsymbol{T}_w$ defined in [8]. The factors r and $2M$ defined in [8] correspond to the factors N and $K-1$, respectively. Since $\boldsymbol{T}_w$ is a Hermitian matrix, $\boldsymbol{T}_w$ and $\boldsymbol{T}_w^T$ will have the same eigenvalues and, therefore the same condition number. This implies that $\boldsymbol{T}_w$ defined in [8] and $\boldsymbol{Q}$ have the same condition number. In [8], an upper bound for the condition number of $\boldsymbol{T}_w$ was obtained, when the maximal gap is less than $\frac{1}{2M}$. We define a similar upper bound for the condition number of $\boldsymbol{Q}$ and the conditions under which the bound exists. For $p = 0,1,\ldots,N$, let maximal gap be $\delta = \max_p(t_p - t_{p-1})$ and $d = \max_p(\tau_p - \tau_{p-1})$.

$$\delta = \frac{d}{K} \quad (7)$$

Based on [8], the equivalent condition on the maximal gap is

$$\delta < \frac{1}{K-1} \implies d < \frac{K}{K-1} \quad (8)$$

$$0 \le \tau_0 < \tau_1 < \cdots < \tau_{N-1} < K \ . \quad (9)$$

Hence, (8) and (9) are the two conditions to be satisfied in order to have a finite upper bound for $cond(\boldsymbol{Q})$. From these two assumptions, we obtain

$$cond(\boldsymbol{Q}) \le \left(\frac{1+\delta(K-1)}{1-\delta(K-1)}\right)^2$$

$$\implies cond(\boldsymbol{Q}) \le \left(\frac{1+\frac{d}{K}(K-1)}{1-\frac{d}{K}(K-1)}\right)^2 \quad (10)$$

As d moves closer to $\frac{K}{K-1}$, the upper bound for $cond(\boldsymbol{Q})$ goes towards infinity. Such sampling sets should be avoided for obtaining a good performance. For $d = 1$, we have $cond(\boldsymbol{Q}) \le (2K-1)^2$ and therefore, $d \le 1$ can be considered to be a safe range for obtaining a good condition number.

Since $\boldsymbol{W}$ is not a square matrix, we have to interpret $cond(\boldsymbol{W})$ as $sqrt(cond(\boldsymbol{W}^H\boldsymbol{W}))$. Hence $cond(\boldsymbol{W})$ has a proper upper bound if a subset of K nonuniform sampling instants (from the N nonuniform sampling instants in a recurrence period) are sufficiently separated. But, if any two sampling instants in the subset of K nonuniform sampling instants are almost equal, then $cond(\boldsymbol{W})$ will have a poor upper bound and hence numerical stability of $\boldsymbol{W}^\dagger$ will be poor.

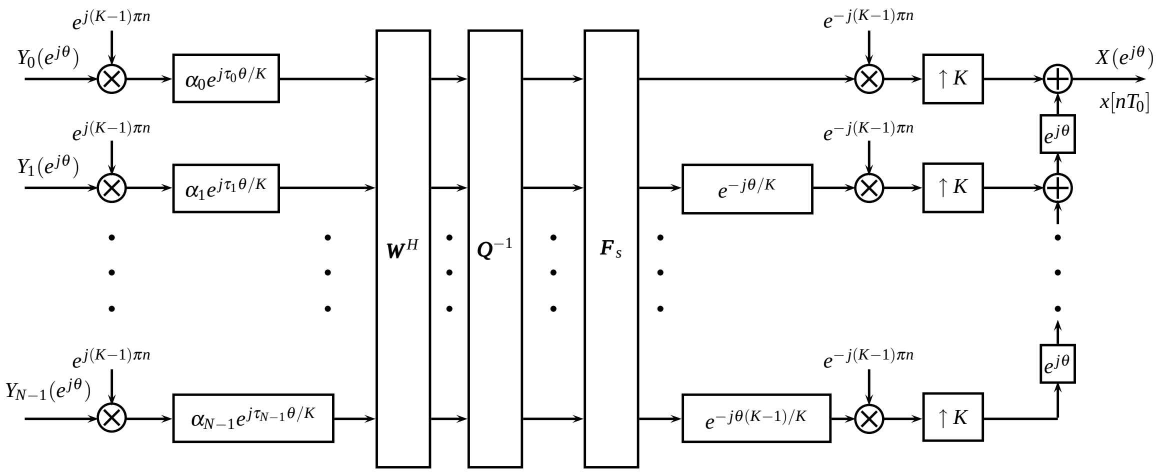

Figure 2: Modified robust and efficient DFT modulated filterbank realization of reconstruction of uniform samples from recursive nonuniform samples

Whereas, the upper bound for $cond(\boldsymbol{Q})$ is independent of the clustering effects of the nonuniform time instants. Hence, for sampling sets satisfying (8) and (9), the matrix $\boldsymbol{Q}$ will have a good numerically stable inverse irrespective of the shortest distance between the nonuniform time instants. This does not mean that $cond(\boldsymbol{Q})$ is smaller than $cond(\boldsymbol{W})$ for all sampling sets satisfying (8) and (9). But, utilizing $\boldsymbol{Q}^{-1}$ instead of $\boldsymbol{W}^{\dagger}$ in Figure 1 guarantees better and improved numerical stability for sampling sets satisfying (8) and (9), which leads to accurate reconstruction. Also, matrix $\boldsymbol{Q}$ being Toeplitz Hermitian, there are many fast direct and iterative Toeplitz solvers for solving (4), some of which are presented in [9, 10]. One efficient iterative method for solving (4) is the Conjugate Gradient (CG) acceleration method briefly described in Proposition 3 in [8]. A poor numerically stable matrix will require much larger number of CG method iterations when compared to a numerically stable matrix in order to converge to an approximate inverse solution.

By implementing (4), we obtain a modified, robust and efficient DFT modulated filterbank reconstruction structure presented in Figure 2. Since there are many fast Toeplitz solvers for inverting the Toeplitz Hermitian matrix $\boldsymbol{Q}$, we can say that our proposed reconstruction structure is more efficient than the reconstruction structure proposed in [7]. Also, since the matrix $\boldsymbol{Q}$ has a better numerically stable inverse for sampling sets satisfying (8) and (9), we can say that our proposed reconstruction structure is more robust than the reconstruction structure proposed in [7].

For sampling sets violating (8) and (9), the improved numerical stability of $\boldsymbol{Q}^{-1}$ cannot be guaranteed. This is a limitation of the modified reconstruction structure presented in Figure 2. It should be noted that all the recurrent nonuniform samples are available simultaneously at the inputs of the structure in Figure 2.

4. SIMULATION RESULTS

The continuous time signal considered is $x(t) = 2\sin(0.1\pi t) + \sin(0.4\pi t) + 3\sin(0.75\pi t)$. The sampling interval is $T_0 = 1$ s and 3600 uniform samples of $x(t)$ have been considered. We assume $K = 18$ and $N = 24$. There are many ways to implement non-integer delays. We have followed the same implementation scheme as given in [7]. Resolution factor of fractional delay is $\frac{1}{L} = \frac{1}{20} = 0.05$. The length of the fractional delay FIR filters used is $L_p = 38$ coefficients. The nonuniform time instants considered are [0, 0.05, 0.1, 0.2, 0.8, 1.2, 2, 3, 4, 5, 6, 7, 8, 8.9, 9.9, 10, 11, 12, 13, 14, 15, 16, 16.9, 17.9]. We can observe that the assumed sampling set satisfies (8) and (9). We compare the original reconstruction structure (Figure 1) output with our modified robust reconstruction structure (Figure 2) output. The parameter used for comparison is the Error-to-Signal-Ratio (ESR) defined by

$$ESR = \frac{\sum(x(nT_0) - \hat{x}(nT_0))^2}{\sum(x(nT_0))^2} \; ,$$

where $\hat{x}(nT_0)$ represents the reconstructed uniform samples. For computing ESR, we have used a fragment of length 2500 samples which only excludes few samples at both the ends, i.e., from $n = 501$ to $n = 3000$. We use the CG iterative method to compare the ESR (in dB) of both the structures with respect to the number of iterations. Since $\boldsymbol{W}$ in the original reconstruction structure is a rectangular matrix, CG method is applied for inverting the square matrix $\boldsymbol{W}^H\boldsymbol{W}$ (size $K \times K$). For the proposed modified reconstruction structure, CG method is applied to invert the $K \times K$ matrix $\boldsymbol{Q}$.

From Table 1, we can observe that the modified reconstruction structure proposed in this paper gives accurate output using much lower number of iterations. This demonstrates that our proposed reconstruction structure has a more numerically stable inverse compared to the reconstruction structure proposed in [7].

5. CONCLUSION

In this paper, we presented a robust and efficient structure for reconstructing uniform samples of a signal from its recurrent nonuniform samples. The structure presented has been obtained by preconditioning the matrix $\boldsymbol{W}$ used in the reconstruction structure discussed in [8]. Equations (8) and (9) provide the necessary and sufficient conditions for the condition number of the preconditioned matrix $\boldsymbol{Q}$ to have a finite upper bound. For nonuniform sampling sets satisfying (8)

Table 1: Comparison of convergence rate of outputs of original and modified reconstruction structures using CG method

Number of iterations	Error-to-Signal ratio (dB)	
	Original structure	Modified structure
1	-6.2876	-21.1847
2	-16.6271	-38.3623
3	-34.4162	-61.8226
4	-46.1789	-67.7101
5	-61.4257	-67.6265
6	-67.5299	-67.6239
7	-67.6184	-67.6233

and (9), the matrix $\boldsymbol{Q}$ of our modified reconstruction structure (Figure 2) has an improved numerically stable inverse compared to the matrix $\boldsymbol{W}$ used in the original reconstruction structure (Figure 1) given in [7]. However, a limitation of the modified reconstruction structure is that the numerical stability of $\boldsymbol{Q}^{-1}$ cannot be guaranteed for sampling sets violating (8) and (9).

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