IMPROVEMENT OF THE ACCURACY OF LINEAR INTERPOLATION FOR DOUBLE EXPANSION OF IMAGES

Keita Inoue, Kota Murahira, and Akira Taguchi

Department of Biomedical Engineering, Tokyo City University Setagaya-ku, 158-8557, Tokyo, Japan phone/fax: +81 3 5707 2161, email: taguchi@bme.tcu.ac.jp web: http://www.bme.tcu.ac.jp/en/index.html

ABSTRACT

In this paper we propose a novel interpolation method which is a modified the Bi-linear interpolation. The Bilinear interpolation is computationally efficient, but it often generates jaggy-noise around the edges. We deal with the double expansion in this paper. In the case of the double expansion, the accuracy of the interpolation is improved and jaggy noise is disappeared by improving the interpolation method for the interior point. Here, we propose a novel edge-adaptive interpolation method for the interior point. By the experimental results, the effectiveness of the proposed method is verified from the subjective and objective points of view.

1. INTRODUCTION

When displaying digital images on various devices, we have to adjust their resolution according to each device. Therefore, image interpolation is an important technique for its application to image resolution conversion.

Typical interpolation methods such as Bi-linear interpolation or Bi-cubic interpolation are used for image resolution conversion [1]. However, it often generated jaggy-noise by using these interpolation methods, since the local features of images are not taken into account. Several methods improve this problem with locally-varied coefficients [2], [3]. These methods produce clearer expansion images compared with the conventional interpolations. However, the computational cost of these methods is very high. In this paper, we propose a novel interpolation method which is a modified the Bi-linear interpolation which maintains the Bi-linear interpolation method's advantages such as the computational cost with improvement to the expanded image quality.

We deal with the double expansion case in this paper. First, the cause why the jaggy-noise is generated is cleared. From this study, we can understand that the accuracy of the interpolation is improved and the jaggy-noise is disappeared by improving the interpolation method for the *interior* point in the double expansion case.

We will introduce a switching interpolation scheme for the *interior* point. The proposed method is based on the Bilinear interpolation. In this method, the two-dimensional interpolation or the diagonal direction one-dimensional interpolation is selected according to the local gradient and the local edge amount for the *interior* point's interpolation, thereby producing high-quality image interpolation with less noise. The proposed method has three parameters. We decide the suitable values for these parameters thorough the experiments.

2. THE CAUSE WHY THE JAGGY-NOISE IS GENERATED

2.1 Bi-linear Interpolation

In this paper, we deal with the double expansion case for digital images. In Figure 1, the solid circular dots indicate pixels from the original image of low resolution. We begin with the interpolation for the points marked by \square , the *aligned* points as follow;

$$f(i + \frac{1}{2}, j) = \{f(i, j) + f(i + 1, j)\}/2$$

$$f(i + \frac{1}{2}, j + 1) = \{f(i, j + 1) + f(i + 1, j + 1)\}/2$$

$$f(i, j + \frac{1}{2}) = \{f(i, j) + f(i, j + 1)\}/2$$

$$(1)$$

$$f(i + 1, j + \frac{1}{2}) = \{f(i + 1, j) + f(i + 1, j + 1)\}/2$$

where f(i,j) is an original image.

Then, we determine value at point marked \bigcirc , the *interior* point as,

$$f(i+\frac{1}{2}, j+\frac{1}{2}) = \{f(i, j) + f(i+1, j) + f(i, j+1) + f(i+1, j+1)\}/4$$
(2)



Figure 1 – Original Points and Interpolated points. The solid circular dots indicate pixels from original image

2.2 The Cause Why The Jaggy-Noise is Generated

In the conventional linear interpolation such as Bi-linear interpolation or Bi-cubic interpolation, it often generates the

jaggy-noise around the diagonal direction edges. We explain why the jaggy noise is generated as follow.

Figure 2(a) shows the diagonal edge in low resolution. Bilinear interpolation result for the low resolution diagonal edge is shown in Figure 2(b). The diagonal edge of double expansion image is degraded. Jaggy noise is generated. If points "A" and "B" in Figure 2(b) change black, jaggy noise is disappeared. Points "A" and "B" are corresponded to the interior point.

Figure 3(a) shows the diagonal line of 2x2 pixels. The Bilinear interpolation result of Figure 3(a) is shown in Figure 3(b). Expanded result is not line image. If Figure 3(a) is a black diagonal line (i.e., 135 degree line), the desired result is shown in Figure 3(c-1). On the other hand, if Figure 3(b) is a white diagonal line (i.e., 45 degree line), the desired result is shown in Figure 3(c-2). In both cases, the interpolation of the interior point is wrong.

From these results, we can understand that the accuracy of the interpolation is improved and the jaggy noise is disappeared by improving the interpolation method for the interior point.





(a)





(c-2) Desired result

(In the case of white line)

Figure 3 – The cause why the jaggy-noise is generated

A NOVEL INTERPOLATION METHOD FOR 3. THE INTERIOR POINT

If we improve the interpolation for the interior point, the jaggy-noise must be disappeared. A novel interpolation method for the *interior* point is proposed.

(Step 1)

The gradient of the original image f(i,j) is calculated by the Sobel operator. The gradient vector magnitude g(i,j) and direction $\theta(i, j)$ are given by

$$g(i,j) = \sqrt{g_i^2(i,j) + g_j^2(i,j)}$$
(3)

$$\theta(i,j) = \tan^{-1} \frac{g_i(i,j)}{g_j(i,j)} \tag{4}$$

Where

$$g_i(i, j) = \{f(i+1, j+1) + 2f(i+1, j) + f(i+1, j-1)\} - \{f(i-1, j+1) + 2f(i-1, j) + f(i-1, j-1)\}$$
(5)

$$g_{j}(i, j) = \{f(i-1, j+1) + 2f(i, j+1) + f(i+1, j+1)\} - \{f(i-1, j-1) + 2f(i, j-1) + f(i+1, j-1)\}$$
(6)

(Step 2)

Edge map e(i,j) is given by

$$e(i, j) = \begin{cases} 1 : g(i, j) > \varepsilon \\ 0 : otherwise \end{cases}$$
(7)

And the edge amount around (i+1/2,j+1/2) is calculated by

$$ER(i+\frac{1}{2},j+\frac{1}{2}) = \sum_{k=-1}^{2} \sum_{l=-1}^{2} e(i+k,j+l)$$
(8)

Therefore the range of $ER(i + \frac{1}{2}, j + \frac{1}{2})$ is from 0 to 16.

(Step 3)

1

We determine the value at the *interior* point as follows:

$$f(i + \frac{1}{2}, j + \frac{1}{2}) = \begin{cases} \{f(i, j) + f(i + 1, j + 1)\}/2 : \text{If the condition (A) is satsfied} \\ \{f(i, j + 1) + f(i + 1, j)\}/2 : \text{If the condition (B) is satsfied} \\ \{f(i, j) + f(i + 1, j) \\ + f(i, j + 1) + f(i + 1, j + 1)\}/4 : \text{If the condition (C) is satsfied} \end{cases}$$
(9)

Condition (A): $45^{\circ} - \mu <$

$$\begin{bmatrix} \text{More than three elements of} \\ \{\theta(i, j), \theta(i+1, j), \theta(i, j+1), \theta(i+1, j+1)\} \end{bmatrix} < 45^{\circ} + \mu$$

and $ER(i + \frac{1}{2}, j + \frac{1}{2}) < \delta$

Condition (B):

$$135^{\circ} - \mu <$$

$$\left[\begin{array}{c} \text{More than three elements of} \\ \left\{ \theta(i, j), \theta(i+1, j), \theta(i, j+1), \theta(i+1, j+1) \right\} \end{array} \right] < 135^{\circ} + \mu$$

and
$$ER(i+\frac{1}{2}, j+\frac{1}{2}) < \delta$$

Condition (C): Other than condition (A) and condition (B)

The jaggy-noise is only appeared on the clear diagonal edges. If the Condition (A) is satisfied, around (i+1/2,j+1/2)is regarded as the clear around 45° directional edge or line. And if the Condition (B) is satisfied, around (i+1/2,j+1/2) is regarded as the clear around 135° directional edge or line. On the other hand, $ER(i+\frac{1}{2}, j+\frac{1}{2})$ is larger than δ , the surrounding the (i+1/2,j+1/2) is regarded as the detail region. At the detail region, the conventional interpolation method shows better results compared to the one-dimensional interpolation.

4. DECISION OF PARAMETERS

In this section, we would like to decide suitable parameters of the proposed method (i.e., ε, μ , and δ). We prepare four images, "Lena", "Barbara", "parrots" and "lighthouse" for examination (Figure 4). Each original image has 256x256 pixels and 8bits. 128x128 pixels' image is made by half-band filtering and decimation from the original image. Each image with 128x128 pixels is expanded by Bi-linear interpolation and the proposed method.



$$RMSE = \frac{MSE}{MSE_Bilinear}$$
(10)

where *MSE* and *MSE_Bilinear* indicate the mean squared error between original 256x256 pixels' image and the expanded 256x256 pixels' image which is obtained by the proposed method and the Bi-linear interpolation, respectively. Thus, if the performance of the proposed method shows better than that of the Bi-linear interpolation, *RMSE* value is smaller than 1.

From Figure 5, the proposed method is effective for "Lena" and "parrots". These images have many clear diagonal edges. All images show best result when $\mu = 45$. Therefore, we set μ of 45.

Next, we examine the decision ε and δ . We show parameters' region of 1.01 times (black region) and 1.015 times (gray region) from minimum MSE for each image in Figure 6. In this case, δ is fixed to 45.

"Lena" and "parrots" show a similar result. On the other hand "Barbara" shows the result that is considerably different from "Lena" and "parrots". And, the common part of Lena's region and Barbara's region is the smallest. (ε , δ)=(80, 15)



Figure 6– Suitable region of parameters (ε , δ)

	Bi- linear	Proposed _opt.	Proposed fix.	Cubic
Lena	73.25	67.71	68.56	58.01
Barbara	295.4	295.4	299.7	280.2
parrots	71.42	68.21	69.10	62.56
lighthouse	273.2	272.0	272.8	242.7

Table 1 - MSE results.

is center position of the common part. Thus, we set (ε , δ) of (80, 15).

5. EXPERIMENT RESULTS

The performance of the proposed method with $(\varepsilon, \mu, \delta) =$ (80,45,15) (Proposed_fix.) compare to that of the Bi-linear interpolation, Cubic interpolation and the proposed method with optimal parameters (Proposed_opt.). 128x128 pixels' images which are made by original images are also used in this simulation.

Table 1 shows the MSE results between the original image and the expanded image. The proposed method with optimal parameters is superior to the Bi-linear interpolation in the all images. The effectiveness of the proposed method is clear from the results. However, the performance of the proposed method is inferior to that of the Cubic interpolation.

The proposed method with $(\varepsilon, \mu, \delta) = (80,45,15)$ shows good results generally. However, the proposed method with fixed parameters is slightly inferior in performance only for Barbara in comparison with the Bi-linear interpolation.

The double expanded images are shown in Figure 7 and 8. From Figure 7(a) which is the Bi-linear interpolation result, the jaggy noise is generated at the brim of hat. On the other hand, the results of the proposed methods don't cause the jaggy noise. The jaggy noise is appeared on the result of Cubic interpolation (Figure 7(d)).

The similar result is observed by the part of the roof of the lighthouse (Figure 8). In the Bi-linear interpolation and Cubic interpolation, the striped pattern is observed on the part of the roof and the jaggy noise is also observed the edge of the roof. Natural expansion results are provided by the proposed methods.

6. CONCLUSION

We have introduced a new interpolation method which is based on Bi-linear interpolation method. In the double expansion case, the accuracy of the interpolation is improved and the jaggy noise is disappeared by improving the interpolation method for the *interior* point. We have proposed a novel switching interpolation scheme for the *interior* point. This method has tree parameters. We clear that these three parameters can be fixed. Furthermore, suitable parameters are derived. The effectiveness of the proposed method is clarified through many examples.



(a) Bi-linear interpolation



(b) Proposed _opt.



(c) Proposed _fix.



(d) Cubic interpolationFigure 7 – The double expansion images (Lena)



Figure 8 - The double expansion images (lighthouse)

REFERENCES

[1] A. K. Jain, *Fundamental of Digital Image Processing*, Prentice-Hall, Englewood Cliffs, NJ, 1988.

[2] X. Li and M. T. Orchard, "New Edge-Directed Interpolation," *IEEE Transactions on Image Processing*, vol. 10, pp. 1521-1527, October. 2001.

[3] D. D. Muresan and T. W. Parks, "Adaptive Quadratic Image Interpolation," *IEEE Transactions on Image Processing*, vol. 13, pp. 690-698, May 2004.