SUM-RATE MAXIMIZATION IN TWO-WAY RELAYING SYSTEMS WITH MIMO AMPLIFY AND FORWARD RELAYS VIA GENERALIZED EIGENVECTORS

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***Abstract* — In this paper we consider two-way relaying with a MIMO amplify and forward (AF) relay. Assuming that the terminals have perfect channel knowledge, the bidirectional two-way relaying channel is decoupled into two parallel effective single-user channels by subtracting the self-interference at the terminals.**

We derive the relay amplification matrix which maximizes the (weighted) sum rate in the case where the terminals have a single antenna. By algebraic manipulation of the rate expressions we can rewrite the optimization problem as a generalized eigenvalue expression which depends on two real-valued parameters. The optimum is then found by a 2-D exhaustive search, which can be efficiently implemented via the bisection method. The resulting method is called RAGES (RAte-maximization via Generalized Eigenvectors for Single-antenna terminals).

Moreover, both parameters have a physical interpretation which allows to find sub-optimal heuristics to reduce the complexity of the search even further. As shown in simulations, a corresponding suboptimal 1-D search is very close to the optimum sum rate.

***Index Terms*—** Two-Way Relaying, Amplify and Forward, Beamforming

1. INTRODUCTION

Future mobile communication systems target not only significantly higher data rates but should also provide a certain quality of service. Relaying is considered as a promising candidate technology to enable this vision. Among the numerous relaying schemes, two-way relaying [2] is known as a technique which uses the radio resources in a particularly efficient manner.

In two-way relaying, a bidirectional transmission between two terminals is achieved in two subsequent transmission phases: First both terminals transmit to the relay, then the relay transmits back to both terminals. This compensates the spectral efficiency loss in one-way relaying due to the half duplex constraint of the relay [3, 1]. Note that unlike in the one-way relaying case [7], the rate-optimal strategy for two-way relaying is in general not known yet.

In contrast to decode and forward (DF) relays, which decode the transmission from the terminals and reencode them in the second phase, we focus our attention on amplify and forward (AF) relays which just amplify the received signal to transmit it back to the terminals. This causes less delay in the transmissions and lowers the hardware complexity of the relay stations. Moreover, we consider the case that the terminals are equipped with a single antenna but the relay may have multiple antennas. It is desirable to find the relay transmit strategy which maximizes the (weighted) sum rate of both users. In [8], the capacity region for this scenario is discussed, and an iterative scheme to compute the relay amplification matrix is proposed. The search space is reduced to four complex numbers with further simplifications only in some special cases, e.g., parallel or orthogonal channels.

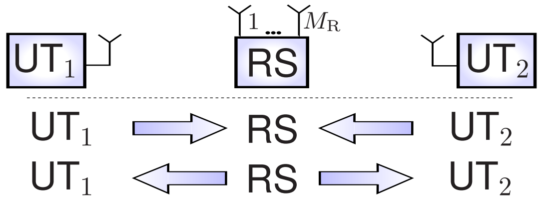

Fig. 1. Two-way relaying system model.

In this paper we reduce the complexity further by rewriting the optimization problem in terms of a generalized eigenvalue equation which depends only on two real-valued parameters. These parameters have a physical interpretation which allows to compute simple bounds and find suboptimal heuristics to reduce the complexity even more. The resulting method is called **RA**te-maximization via **G**eneralized **E**igenvectors for **S**ingle-antenna terminals (RAGES). At the end of the paper, the sum rate performance is compared to existing solutions via numerical computer simulations.

To distinguish between scalars, vectors, and matrices, the following notation is used throughout the paper: Scalars are denoted as italic letters (a, b, A, B), vectors as lower-case bold-faced letters ($\boldsymbol{a}, \boldsymbol{b}$), and matrices are represented by upper-case bold-faced letters ($\boldsymbol{A}, \boldsymbol{B}$). The superscripts $^{\mathrm{T}}$ and $^{\mathrm{H}}$ represent matrix transposition and Hermitian transposition, respectively. Moreover, * denotes complex conjugation. The Kronecker product between two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is represented by $\boldsymbol{A} \otimes \boldsymbol{B}$. Furthermore, $\mathrm{vec}\{\boldsymbol{A}\}$ aligns the elements of the matrix $\boldsymbol{A} \in \mathbb{C}^{M \times N}$ into a column vector of length $M \cdot N \times 1$. The two-norm of a vector $\boldsymbol{a}$ and the Frobenius norm of a matrix $\boldsymbol{A}$ are represented by $\|\boldsymbol{a}\|_2$ and $\|\boldsymbol{A}\|_{\mathrm{F}}$, respectively. Finally, the statistical expectation operator is symbolized by $\mathbb{E}\{\cdot\}$.

2. SYSTEM DESCRIPTION

Figure 1 shows the system we study in this paper. We consider the communication between two user terminals UT_1 and UT_2 with the help of an intermediate relay station RS. The terminals UT_1 and UT_2 are equipped with a single antenna; the number of antennas at the relay station is denoted by M_{R}.

In two-way relaying, the communication takes place in two subsequent transmission phases. In the first phase, both terminals transmit to the relay, where their transmissions interfere. Assum-

 ISSN 2076-1465

ing frequency-flat fading[1], the received signal at the relay can be expressed as

$$\boldsymbol{r} = \boldsymbol{h}_1^{(\mathrm{f})} \cdot x_1 + \boldsymbol{h}_2^{(\mathrm{f})} \cdot x_2 + \boldsymbol{n}_\mathrm{R} \in \mathbb{C}^{M_\mathrm{R} \times 1}, \quad (1)$$

where $\boldsymbol{h}_1^{(\mathrm{f})} \in \mathbb{C}^{M_\mathrm{R} \times 1}$ and $\boldsymbol{h}_2^{(\mathrm{f})} \in \mathbb{C}^{M_\mathrm{R} \times 1}$ denote the forward channel vectors between the terminals and the relay, x_1 and x_2 represent the transmitted signals from the terminals, and $\boldsymbol{n}_\mathrm{R}$ models the additive noise component at the relay with noise covariance given by $\boldsymbol{R}_{\mathrm{N,R}} = \mathbb{E}\left\{\boldsymbol{n}_\mathrm{R} \cdot \boldsymbol{n}_\mathrm{R}^\mathrm{H}\right\}$. We can define the noise power of the relay according to $P_{\mathrm{N,R}} = \mathrm{trace}\left\{\boldsymbol{R}_{\mathrm{N,R}}\right\}/M_\mathrm{R}$.

In the second time slot, the amplify and forward relay transmits an amplified version of its received signal which can be expressed as

$$\bar{\boldsymbol{r}} = \gamma \cdot \boldsymbol{G} \cdot \boldsymbol{r}. \quad (2)$$

Here, $\boldsymbol{G} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{R}}$ is the relay amplification matrix which is normalized such that $\|\boldsymbol{G}\|_\mathrm{F} = 1$ and $\gamma \in \mathbb{R}^+$ represents a scalar amplification factor that guarantees that the relay transmit power constraint of $P_{\mathrm{T,R}}$ is satisfied. For instance, to guarantee that $\mathbb{E}\left\{\|\bar{\boldsymbol{r}}\|_2^2\right\} = P_{\mathrm{T,R}}$ we choose γ according to

$$\gamma^2 = \frac{P_{\mathrm{T,R}}}{P_{\mathrm{T},1} \cdot M_\mathrm{R} \cdot \alpha_1^2 + P_{\mathrm{T},2} \cdot M_\mathrm{R} \cdot \alpha_2^2 + P_{\mathrm{N,R}} \cdot M_\mathrm{R}} \quad (3)$$

where the channel amplitudes α_1 and α_2 are defined according to

$$\alpha_n^2 = \frac{\left\|\boldsymbol{h}_n^{(\mathrm{f})}\right\|_2^2}{M_\mathrm{R}}, \; n = 1, 2 \quad (4)$$

and $P_{\mathrm{T},n} = \mathbb{E}\left\{|x_n|^2\right\}$ denotes the transmit power of terminal n.

The terminals receive the amplified signal $\bar{\boldsymbol{r}}$ from the relay via their reverse channels $\boldsymbol{h}_i^{(\mathrm{b})} \in \mathbb{C}^{M_\mathrm{R} \times 1}$. We can write the received signals as

$$y_1 = \gamma \cdot h_{1,1}^{(\mathrm{e})} \cdot x_1 + \gamma \cdot h_{1,2}^{(\mathrm{e})} \cdot x_2 + \tilde{n}_1 \quad (5)$$

$$y_2 = \gamma \cdot h_{2,2}^{(\mathrm{e})} \cdot x_2 + \gamma \cdot h_{2,1}^{(\mathrm{e})} \cdot x_1 + \tilde{n}_2, \quad (6)$$

where we have introduced the short hand notations $h_{i,j}^{(\mathrm{e})} = \boldsymbol{h}_i^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{h}_j^{(\mathrm{f})}$ for the effective channels between terminal i and j. Moreover, $\tilde{n}_i = \gamma \cdot \boldsymbol{h}_i^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{n}_\mathrm{R} + n_i$ denotes the effective noise term at terminal i consisting of the terminal's own noise and the forwarded relay noise, which are assumed to be independent. The noise powers at both terminals are defined as

$$P_{\mathrm{N},i} = \mathbb{E}\left\{|n_i|^2\right\}, \; i = 1, 2. \quad (7)$$

As it is evident from (5) and (6), each terminal receives the transmission from the other terminal via one of the effective channels $h_{i,j}^{(\mathrm{e})}$ which is superimposed by the effective noise terms $\tilde{n}_i$ as well as the self-interference from its own transmission. However, if the terminal possesses channel knowledge, the self-interference can be subtracted since its own transmitted symbols are known. We assume that the self-interference is perfectly canceled and only consider the power of the desired signal component and the effective noise terms. For a discussion on the acquisition of channel knowledge in two-way relaying scenarios with time division duplex (TDD), the reader is referred to [6], [5], and [4].

[1] In presence of frequency-selective fading, OFDM can be applied to transform the frequency-selective channel into a set of parallel flat fading channels. The RAGES scheme can then be applied on each subcarrier individually (or on a chunk of adjacent subcarriers jointly to further reduce the complexity).

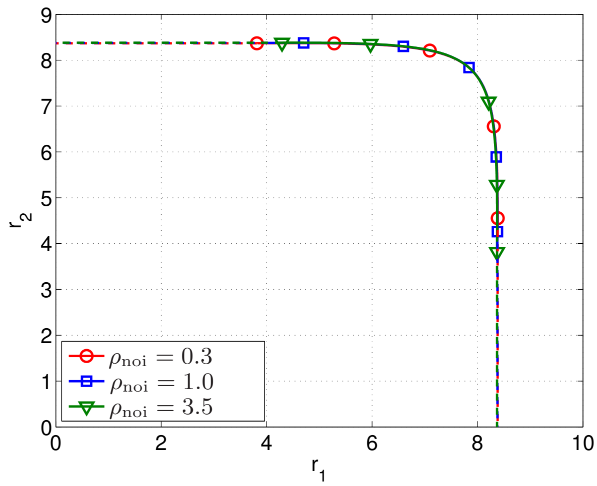

Fig. 2. Achievable rate pairs via RAGES if ρ_sig is varied from the lower bound to the upper bound (provided below (24)) for different ρ_noi (cf. eqn. (22)). Scenario: $M_\mathrm{R} = 5$, $\alpha_1 = \alpha_2 = 1$, $P_{\mathrm{N,R}} = P_{\mathrm{N},1} = P_{\mathrm{N},2} = 0.001$.

3. SUM-RATE MAXIMIZATION VIA RAGES

In this section we derive the relay amplification matrix $\boldsymbol{G}$ which maximizes the sum rate in the system. Under the conditions mentioned in the previous section, the sum rate can be expressed as

$$r = r_1 + r_2 = \frac{1}{2}\log_2\left(1 + \frac{P_{\mathrm{R},1}}{\tilde{P}_{\mathrm{N},1}}\right) + \frac{1}{2}\log_2\left(1 + \frac{P_{\mathrm{R},2}}{\tilde{P}_{\mathrm{N},2}}\right), \quad (8)$$

where $P_{\mathrm{R},1}$, $P_{\mathrm{R},2}$ denote the power of the received signals given by

$$P_{\mathrm{R},1} = \mathbb{E}\left\{\left|\gamma \cdot h_{1,2}^{(\mathrm{e})} \cdot x_2\right|^2\right\} = \gamma^2 \cdot P_{\mathrm{T},2} \cdot \left|\boldsymbol{h}_1^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{h}_2^{(\mathrm{f})}\right|^2,$$

$$P_{\mathrm{R},2} = \mathbb{E}\left\{\left|\gamma \cdot h_{2,1}^{(\mathrm{e})} \cdot x_2\right|^2\right\} = \gamma^2 \cdot P_{\mathrm{T},1} \cdot \left|\boldsymbol{h}_2^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{h}_1^{(\mathrm{f})}\right|^2,$$

and $\tilde{P}_{\mathrm{N},1}$, $\tilde{P}_{\mathrm{N},2}$ represent the effective noise powers

$$\begin{aligned}\tilde{P}_{\mathrm{N},i} &= \mathbb{E}\left\{\left|\gamma \cdot \boldsymbol{h}_i^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{n}_\mathrm{R} + n_i\right|^2\right\} \\ &= \gamma^2 \cdot \boldsymbol{h}_i^{(\mathrm{b})^\mathrm{T}} \cdot \boldsymbol{G} \cdot \boldsymbol{R}_{\mathrm{N,R}} \cdot \boldsymbol{G}^\mathrm{H} \cdot \boldsymbol{h}_i^{(\mathrm{b})^*} + P_{\mathrm{N},i}, \; i = 1, 2.\end{aligned}$$

The maximization of the sum rate is performed with respect to the normalized relay amplification matrix $\boldsymbol{G}$

$$\begin{aligned}\boldsymbol{G}_\mathrm{opt} &= \underset{\boldsymbol{G}, \|\boldsymbol{G}\|_\mathrm{F}=1}{\arg\max}\left[\frac{1}{2}\log_2\left(1 + \frac{P_{\mathrm{R},1}}{\tilde{P}_{\mathrm{N},1}}\right) + \frac{1}{2}\log_2\left(1 + \frac{P_{\mathrm{R},2}}{\tilde{P}_{\mathrm{N},2}}\right)\right] \\ &= \underset{\boldsymbol{G}, \|\boldsymbol{G}\|_\mathrm{F}=1}{\arg\max}\; \frac{1}{2}\log_2\left(\frac{(\tilde{P}_{\mathrm{N},1} + P_{\mathrm{R},1})}{\tilde{P}_{\mathrm{N},1}} \cdot \frac{(\tilde{P}_{\mathrm{N},2} + P_{\mathrm{R},2})}{\tilde{P}_{\mathrm{N},2}}\right) \\ &= \underset{\boldsymbol{G}, \|\boldsymbol{G}\|_\mathrm{F}=1}{\arg\max}\; \frac{(\tilde{P}_{\mathrm{N},1} + P_{\mathrm{R},1}) \cdot (\tilde{P}_{\mathrm{N},2} + P_{\mathrm{R},2})}{\tilde{P}_{\mathrm{N},1} \cdot \tilde{P}_{\mathrm{N},2}}, \quad (9)\end{aligned}$$

where we have dropped the logarithm in the last step since it is a monotonous function. To solve the optimization problem in (9) we first notice that the received signal power and the effective noise

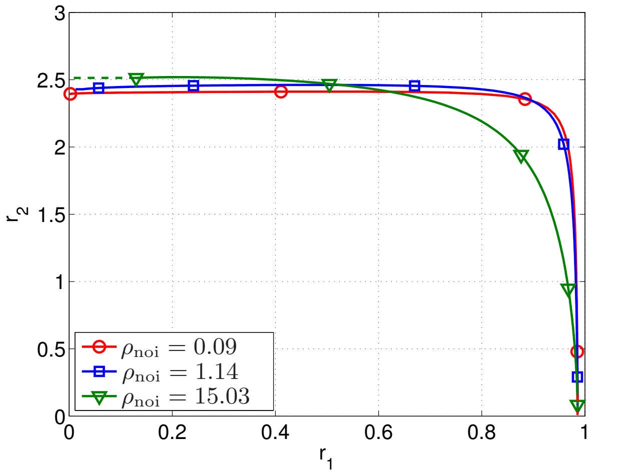

Fig. 3. Achievable rate pairs via RAGES with fixed ρ_{noi} if ρ_{sig} is varied (cf. eqn. (22)). Scenario: $M_{\mathrm{R}} = 3$, $\alpha_1 = 1$, $\alpha_2 = 0.1$, $P_{\mathrm{N,R}} = 0.01$, $P_{\mathrm{N},1} = 0.0001$, $P_{\mathrm{N},2} = 0.001$.

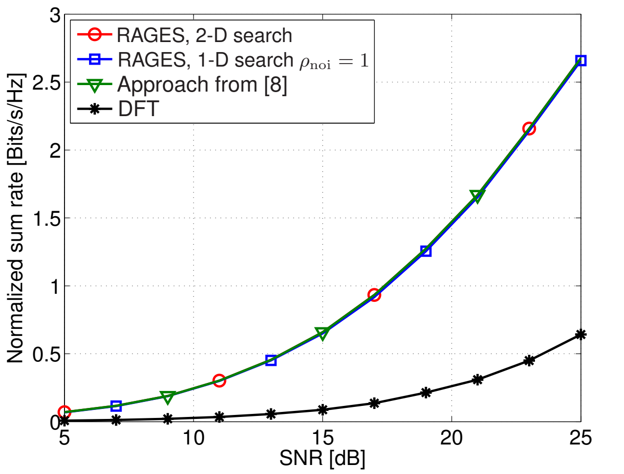

Fig. 4. Sum-rate vs. SNR for $M_{\mathrm{R}} = 4$ antennas, $\alpha_1 = 1$, and $\alpha_2 = 0.1$.

powers can conveniently be expressed in terms of the vector $\boldsymbol{g} = \mathrm{vec}\{\boldsymbol{G}\} \in \mathbb{C}^{M_{\mathrm{R}}^2}$ in the following manner

$$P_{\mathrm{R},1} = \gamma^2 \cdot P_{\mathrm{T},2} \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \boldsymbol{K}_{2,1} \cdot \boldsymbol{g} \quad (10)$$
$$P_{\mathrm{R},2} = \gamma^2 \cdot P_{\mathrm{T},1} \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \boldsymbol{K}_{1,2} \cdot \boldsymbol{g} \quad (11)$$
$$\tilde{P}_{\mathrm{N},1} = \gamma^2 \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \boldsymbol{J}_1 \cdot \boldsymbol{g} + P_{\mathrm{N},1} \quad (12)$$
$$\tilde{P}_{\mathrm{N},2} = \gamma^2 \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \boldsymbol{J}_2 \cdot \boldsymbol{g} + P_{\mathrm{N},2}, \quad (13)$$

where for $i, j = 1, 2$ the matrices $\boldsymbol{K}_{i,j}$ and $\boldsymbol{J}_i$ are given by

$$\boldsymbol{K}_{i,j} = \left(\boldsymbol{h}_i^{(\mathrm{f})} \cdot \boldsymbol{h}_i^{(\mathrm{f})^{\mathrm{H}}} \otimes \boldsymbol{h}_j^{(\mathrm{b})} \cdot \boldsymbol{h}_j^{(\mathrm{b})^{\mathrm{H}}}\right)^{\mathrm{T}} \quad (14)$$
$$\boldsymbol{J}_i = \left(\boldsymbol{R}_{\mathrm{N,R}} \otimes \boldsymbol{h}_i^{(\mathrm{b})} \cdot \boldsymbol{h}_i^{(\mathrm{b})^{\mathrm{H}}}\right)^{\mathrm{T}}. \quad (15)$$

With these definitions, the optimization problem in (9) can be rewrit-

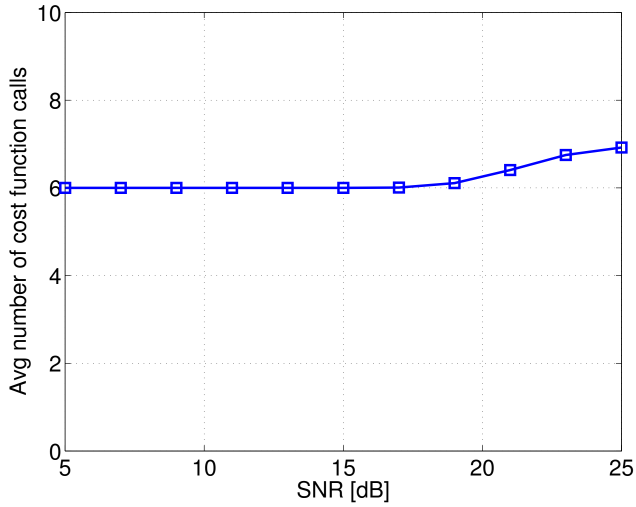

Fig. 5. Number of evaluations of the cost function (i.e., generalized eigenvalue decompositions) for RAGES via the 1-D search.

ten in the following way

$$\boldsymbol{g}_{\mathrm{opt}} = \underset{\boldsymbol{g}, \|\boldsymbol{g}\|_2 = 1}{\arg\max} \frac{\boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{K}}_1 \cdot \boldsymbol{g} \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{K}}_2 \cdot \boldsymbol{g}}{\boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{J}}_1 \cdot \boldsymbol{g} \cdot \boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{J}}_2 \cdot \boldsymbol{g}}, \quad (16)$$

where we have introduced the new matrices $\tilde{\boldsymbol{K}}_i$ and $\tilde{\boldsymbol{J}}_i$ given by

$$\tilde{\boldsymbol{K}}_1 = \gamma^2 \cdot P_{\mathrm{T},2} \cdot \boldsymbol{K}_{2,1} + \tilde{\boldsymbol{J}}_1 \quad (17)$$
$$\tilde{\boldsymbol{K}}_2 = \gamma^2 \cdot P_{\mathrm{T},1} \cdot \boldsymbol{K}_{1,2} + \tilde{\boldsymbol{J}}_2 \quad (18)$$
$$\tilde{\boldsymbol{J}}_i = \gamma^2 \cdot \boldsymbol{J}_i + P_{\mathrm{N},i} \cdot \boldsymbol{I}_{M_{\mathrm{R}}^2} \quad (19)$$

and have exploited the fact that $\|\boldsymbol{g}\|_2 = 1$. Here, $\boldsymbol{I}_p$ represents the $p \times p$ identity matrix. Examining (16) we observe that the norm of $\boldsymbol{g}$ does not influence the cost function at all since if we replace $\boldsymbol{g}$ by $\alpha \cdot \boldsymbol{g}$ for an arbitrary scalar $\alpha \in \mathbb{C} \backslash \{0\}$, the scalar α cancels. In other words, if we computed the optimal $\boldsymbol{g}_{\mathrm{opt}}$ then $\alpha \cdot \boldsymbol{g}_{\mathrm{opt}}$ is also optimal $\forall \alpha \in \mathbb{C} \backslash \{0\}$. Consequently, we can ignore the constraint $\|\boldsymbol{g}\|_2 = 1$ for the optimization if we normalize the resulting $\boldsymbol{g}_{\mathrm{opt}}$ properly. This transforms (16) into an unconstrained optimization problem. A necessary condition for the optimum is that the gradient of the cost function becomes zero. Equating the complex gradient of (16) with respect to $\boldsymbol{g}^*$ to zero we obtain

$$\frac{\tilde{P}_{\mathrm{R},2}}{\tilde{P}_{\mathrm{N},1}\tilde{P}_{\mathrm{N},2}} \cdot \tilde{\boldsymbol{K}}_1 \cdot \boldsymbol{g} + \frac{\tilde{P}_{\mathrm{R},1}}{\tilde{P}_{\mathrm{N},1}\tilde{P}_{\mathrm{N},2}} \cdot \tilde{\boldsymbol{K}}_2 \cdot \boldsymbol{g}$$
$$= \frac{\tilde{P}_{\mathrm{R},1}\tilde{P}_{\mathrm{R},2}}{\tilde{P}_{\mathrm{N},1}^2\tilde{P}_{\mathrm{N},2}} \cdot \tilde{\boldsymbol{J}}_1 \cdot \boldsymbol{g} + \frac{\tilde{P}_{\mathrm{R},1}\tilde{P}_{\mathrm{R},2}}{\tilde{P}_{\mathrm{N},1}\tilde{P}_{\mathrm{N},2}^2} \cdot \tilde{\boldsymbol{J}}_2 \cdot \boldsymbol{g}, \quad (20)$$

where $\tilde{P}_{\mathrm{R},i} = \boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{K}}_i \cdot \boldsymbol{g}$ and $\tilde{P}_{\mathrm{N},i} = \boldsymbol{g}^{\mathrm{H}} \cdot \tilde{\boldsymbol{J}}_i \cdot \boldsymbol{g}$ for $i = 1, 2$. Rearranging the terms in (20) we obtain the equivalent condition

$$\left(\tilde{\boldsymbol{K}}_1 + \rho_{\mathrm{sig}} \cdot \tilde{\boldsymbol{K}}_2\right) \cdot \boldsymbol{g} = \frac{\tilde{P}_{\mathrm{R},1}}{\tilde{P}_{\mathrm{N},1}} \cdot \left(\tilde{\boldsymbol{J}}_1 + \rho_{\mathrm{noi}} \cdot \tilde{\boldsymbol{J}}_2\right) \cdot \boldsymbol{g}, \quad (21)$$

where we have introduced two parameters ρ_{sig} and ρ_{noi} defined via

$$\rho_{\mathrm{sig}} = \frac{\tilde{P}_{\mathrm{R},1}}{\tilde{P}_{\mathrm{R},2}} \text{ and } \rho_{\mathrm{noi}} = \frac{\tilde{P}_{\mathrm{N},1}}{\tilde{P}_{\mathrm{N},2}}. \quad (22)$$

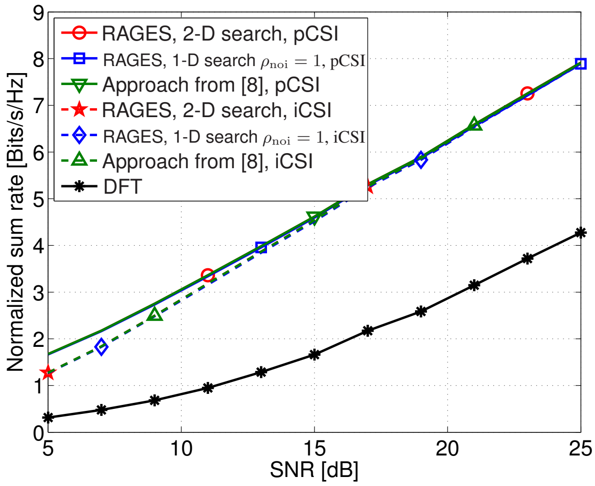

Fig. 6. Sum-rate vs. SNR for $M_{\mathrm{R}} = 4$ antennas, $\alpha_1 = \alpha_2 = 1$, comparing the case of perfectly known channels (pCSI) and channels estimated via two pilot symbols (iCSI).

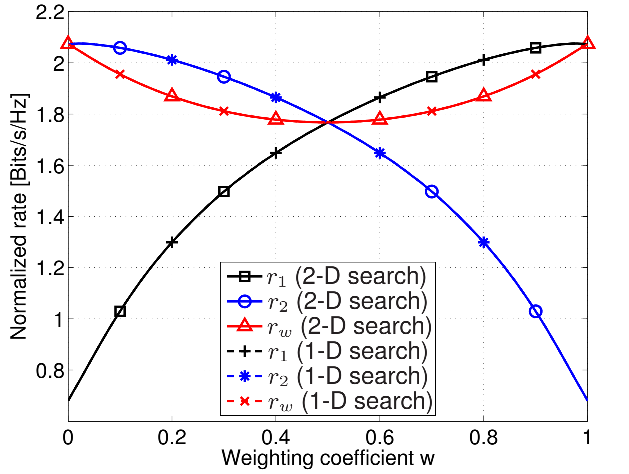

Fig. 7. User rates and weighted sum rate vs. weighting coefficient w comparing the weighted sum rate maximization via RAGES using the full 2-D search and the suboptimal 1-D search ($\rho_{\text{noi}} = 1$). The parameters are $M_{\mathrm{R}} = 4$, SNR = 10 dB, $\alpha_1 = \alpha_2 = 1$.

Equation (21) shows that the optimal $\boldsymbol{g}$ must be a generalized eigenvector of the two matrices $\tilde{\boldsymbol{K}}_1 + \rho_{\text{sig}} \cdot \tilde{\boldsymbol{K}}_2$ and $\tilde{\boldsymbol{J}}_1 + \rho_{\text{noi}} \cdot \tilde{\boldsymbol{J}}_2$, which for a given ρ_{sig} and ρ_{noi} is very simple to compute. However, the parameters ρ_{sig} and ρ_{noi} depend on $\boldsymbol{g}$ as well. Therefore, to find the optimum, a two-dimensional search over ρ_{sig} and ρ_{noi} must be performed. Compared to the approach from [8] this is a significant advantage since there, a matrix containing four complex parameters has to be found via numerical optimization schemes. Only in special cases or via suboptimal approximations, the number of parameters could be reduced. For RAGES, we have two real-valued parameters with a physical interpretation: ρ_{noi} represents the ratio of the effective noise powers and ρ_{sig} is equal to the ratio of the effective signal plus effective noise powers (which for high SNRs converges to the ratio of the effective signal powers) experienced at both terminals. Both parameters can be bounded in the following way

$$\rho_{\text{noi}} \leq \frac{P_{\mathrm{N},1}}{P_{\mathrm{N},2}} + \frac{P_{\mathrm{N,R}}}{P_{\mathrm{N},2}} \cdot \gamma^2 \cdot M_{\mathrm{R}} \cdot \alpha_1^2 \tag{23}$$

$$\rho_{\text{noi}} \geq \left(\frac{P_{\mathrm{N},2}}{P_{\mathrm{N},1}} + \frac{P_{\mathrm{N,R}}}{P_{\mathrm{N},1}} \cdot \gamma^2 \cdot M_{\mathrm{R}} \cdot \alpha_2^2 \right)^{-1} \tag{24}$$

$$\rho_{\text{sig}} \leq \frac{P_{\mathrm{N},1}}{P_{\mathrm{N},2}} + \frac{P_{\mathrm{T},2}}{P_{\mathrm{N},2}} \cdot \gamma^2 \cdot \alpha_1^2 \cdot \alpha_2^2 + \frac{P_{\mathrm{N,R}}}{P_{\mathrm{N},2}} \cdot \gamma^2 \cdot M_{\mathrm{R}} \cdot \alpha_1^2$$

$$\rho_{\text{sig}} \geq \left(\frac{P_{\mathrm{N},2}}{P_{\mathrm{N},1}} + \frac{P_{\mathrm{T},1}}{P_{\mathrm{N},1}} \cdot \gamma^2 \cdot \alpha_1^2 \cdot \alpha_2^2 + \frac{P_{\mathrm{N,R}}}{P_{\mathrm{N},1}} \cdot \gamma^2 \cdot M_{\mathrm{R}} \cdot \alpha_2^2 \right)^{-1},$$

Within this region we compute $\boldsymbol{G}$ for every combination of $(\rho_{\text{sig}}, \rho_{\text{noi}})$ by determining the dominant generalized eigenvector $\boldsymbol{g}$ corresponding to the largest generalized eigenvalue according to (21) and then reshaping it back into a $M_{\mathrm{R}} \times M_{\mathrm{R}}$ matrix. The final RAGES solution is the $\boldsymbol{G}$ for which the sum rate is maximized.

The derivation can be extended to the optimization of a weighted sum rate given by

$$r_w = w \cdot r_1 + (1 - w) \cdot r_2, \tag{25}$$

where $w \in \mathbb{R}_{[0,1]}$ is the weighting parameter. Following the same steps of the derivation we obtain the following criterion for the optimal $\boldsymbol{g}$

$$\left(\tilde{\boldsymbol{K}}_1 + \rho_{\text{sig}} \cdot \bar{w} \cdot \tilde{\boldsymbol{K}}_2 \right) \cdot \boldsymbol{g} = w^2 \frac{\tilde{P}_{\mathrm{R},1}}{\tilde{P}_{\mathrm{N},1}} \left(\tilde{\boldsymbol{J}}_1 + \frac{\rho_{\text{noi}}}{\bar{w}} \cdot \tilde{\boldsymbol{J}}_2 \right) \cdot \boldsymbol{g}, \tag{26}$$

where $\bar{w} = \frac{w}{1-w}$. Consequently, the maximization of the weighted sum rate can be performed in a similar manner by optimizing over two real-valued parameters.

4. BISECTION SEARCH

So far, we have reduced the search space for the optimal relay amplification matrix $\boldsymbol{G} \in \mathbb{C}^{M_{\mathrm{R}} \times M_{\mathrm{R}}}$ from four complex parameters [8] to two real-valued and bounded parameters. However, the optimization problem in [8] is convex which allows for an efficient implementation. Unfortunately, the optimization problem for the two real-valued parameters ρ_{sig} and ρ_{noi} is not convex so that in general, a 2-D exhaustive search is required.

As we show in this section, the nature of the problem allows to reduce the complexity of the 2-D exhaustive search further since the optimum can easily be found via the bisection method.

To this end, let $\hat{\rho}_{\text{sig}}$ and $\hat{\rho}_{\text{noi}}$ be our current estimates of the actual ratios ρ_{sig} and ρ_{noi} defined in (22). Then we can define the following auxiliary functions

$$A_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}) = \hat{\rho}_{\text{sig}} - \rho_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}) \tag{27}$$

$$A_{\text{noi}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}) = \hat{\rho}_{\text{noi}} - \rho_{\text{noi}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}), \tag{28}$$

where $\rho_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}})$ and $\rho_{\text{noi}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}})$ are computed by inserting $\hat{\rho}_{\text{sig}}$ and $\hat{\rho}_{\text{noi}}$ into (21), computing the dominant generalized eigenvector and using it in (22) to compute the actual ratios ρ_{sig} and ρ_{noi}. Then, obviously, $A_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}) = A_{\text{noi}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}}) = 0$ for the values of $\hat{\rho}_{\text{sig}}$ and $\hat{\rho}_{\text{noi}}$ which maximize the sum rate. Moreover, we have observed that $A_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}})$ is a monotonous function in $\hat{\rho}_{\text{sig}}$ for every value of $\hat{\rho}_{\text{noi}}$. Similarly, $A_{\text{noi}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}})$ is a monotonous function in $\hat{\rho}_{\text{noi}}$ for every value of $\hat{\rho}_{\text{sig}}$. Therefore, the search for the optimum $\hat{\rho}_{\text{sig}}$ and $\hat{\rho}_{\text{noi}}$ can be implemented efficiently via a 2-D bisection search.

As we show in the simulations, in many cases the parameter ρ_{noi} can be ignored by simply setting it to the geometric mean of the lower and upper bounds provided in (24) and (23). In this case, the optimization we have to perform reduces to a 1-D search over the parameter ρ_{sig}. This search can be implemented via a 1-D bisection search on the function $A_{\text{sig}}(\hat{\rho}_{\text{sig}}, \hat{\rho}_{\text{noi}})$ for fixed $\hat{\rho}_{\text{noi}}$.

5. SIMULATION RESULTS

In this section we present results of numerical computer simulations to evaluate the achievable rate using RAGES to compute the relay amplification matrix. If not stated otherwise, we set $P_{\text{T},1} = P_{\text{T},2} = P_{\text{T,R}} = 1$ and $P_{\text{N,R}} = P_{\text{N},1} = P_{\text{N},2} = \sigma^2$ so that the SNR can be defined as σ^{-2}. Moreover, we consider uncorrelated Rayleigh fading channels and assume that reciprocity is valid so that $\boldsymbol{h}_i^{(\text{f})} = \boldsymbol{h}_i^{(\text{b})}$ for $i = 1, 2$.

Figure 2 shows the rate pairs (r_1, r_2) we obtain by varying ρ_{sig} for three different choices of ρ_{noi}, corresponding the the lower bound from (24), the upper bound from (23), and the mid point (i.e., the geometric mean of lower and upper bounds). We set the effective channel norms α_1 and α_2 to 1, the SNR to 30 dB, and M_{R} to 5. It can be seen that in this scenario, the parameter ρ_{noi} has almost no impact on the rates and the parameter ρ_{sig} can be used to achieve different rate pairs. Therefore, in this scenario, the 2-D search for the sum rate maximum can be replaced by a 1-D search over ρ_{sig}, leaving ρ_{noi} constant. The heuristic behind this approximation is that adjusting the ratio of signal plus noise powers via ρ_{sig} offers enough degrees of freedom for sum rate optimization and therefore it is not required to additionally adjust the noise ratios via ρ_{noi}.

While a similar picture is obtained in many scenarios, for Figure 3 we choose the parameters in such a way that the impact of ρ_{noi} becomes visible. Here we consider $M_{\text{R}} = 3$, $\alpha_1 = 1$, $\alpha_2 = 0.1$, $P_{\text{N,R}} = 0.01$, $P_{\text{N},1} = 0.0001$, and $P_{\text{N},2} = 0.001$. Obviously, none of the ρ_{noi} values provides a rate curve that is always on the boundary of the rate region. Still, if we compute the maximum sum rate over ρ_{sig} for different values of ρ_{noi}, we obtain 3.24 Bits/s/Hz for $\rho_{\text{noi}} = 0.09$, 3.25 Bits/s/Hz for $\rho_{\text{noi}} = 1.14$, and 3.04 Bits/s/Hz for $\rho_{\text{noi}} = 15.03$, which is only a difference of 7 %. Consequently, if our goal is to maximize the sum rate, the loss incurred by ignoring the impact of ρ_{noi} is not very severe.

In Figure 4 we display the sum rate r (which is normalized to the bandwidth) achieved via different choices of $\boldsymbol{G}$. We choose $M_{\text{R}} = 4$ and set the channel norms to $\alpha_1 = 1$ and $\alpha_2 = 0.1$. We compare RAGES via a full 2-D search over ρ_{sig} and ρ_{noi} with RAGES via a suboptimal approximation where ρ_{noi} is fixed to 1 and a 1-D search over ρ_{sig} is used to optimize $\boldsymbol{G}$. According to Figure 4, the 1-D search and the full 2-D search perform identically well. If we compute the loss in rate we find that it is around 1 % for low SNRs and it converges to zero for high SNRs (for 25 dB it has already declined below 0.1 %). As a comparison, we also depict the sum-rate optimal approach presented in [8]. As it is evident from the simulation result, RAGES achieves the same sum rate at a significantly reduced complexity. Finally, the curve labeled "DFT" represents the case where we choose the relay amplification matrix $\boldsymbol{G}$ as a DFT matrix to show the loss incurred by the absence of channel state information at the relay. For the same scenario, Figure 5 depicts the average number of times the cost function had to be evaluated for the low-complexity 1-D search. Since we use a bisection approach this number is only a function of the initial size of the search interval as well as the desired accuracy. We observe a slight increase of this number with the SNR which is due to the fact that the lower bound on ρ_{sig} decreases and the upper bound on ρ_{sig} increases with the SNR.

In Figure 6 we investigate the robustness with respect to channel estimation errors. Since the robustness of the different relaying schemes should be compared we depict the maximum mutual information over the two-way relaying channel for the case where the relay has to estimate the channel vectors $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ and the terminals know the effective channel taps $h_{i,j}^{(\text{e})}$ perfectly. Estimation errors in $h_{i,j}^{(\text{e})}$ would affect all schemes in a similar fashion. The curves labeled "pCSI" represent the case of perfectly known channels, whereas "iCSI" represents the case where the channel vectors are estimated at the relay from the transmission of two known pilot symbols. We observe that the channel estimation error affects all schemes in a similar fashion, slightly reducing the sum rate for low SNRs.

The result of weighted sum rate maximization is displayed in Figure 7. Here we set $M_{\text{R}} = 4$, $\alpha_1 = \alpha_2 = 1$ and the SNR to 10 dB. The weighted sum rate is maximized via RAGES for different choices of the weighting coefficient w. We display the rates for each user r_1, r_2 as well as the weighted sum rate r_w (cf. equation (25)). Along with the full 2-D search we also depict the results of using the suboptimal 1-D search for $\rho_{\text{noi}} = 1$ (the range for ρ_{noi} in this example is $[0.34, 2.90]$ according to (24) and (23)). As before, the loss in rate is negligibly small (around 0.2 %).

6. CONCLUSIONS

In this paper we derive a sum-rate optimal relaying strategy for two-way relaying with a MIMO AF relay in the case where the terminals are equipped with a single antenna called RAGES (**RA**te-maximization via **G**eneralized **E**igenvectors for **S**ingle-antenna terminals). We transform the optimization problem into a generalized eigenvalue equation which depends only on two parameters. Therefore, to maximize the sum rate, a 2-D exhaustive search is required. Another advantage of RAGES compared to previous approaches is that both parameters have a physical interpretation which allows to state their bounds and develop heuristics to lower the complexity even further.

As we show in simulations, in many cases the impact of one of the two parameters can be ignored and the other parameter can be used to achieve different rate pairs (r_1, r_2) in the rate region. RAGES achieves the same sum rate as a previous proposal at a significantly lower complexity. Even with the suboptimal 1-D search the sum rate is very close to the optimum.

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