

A MODIFIED NORMALIZED FXLMS ALGORITHM FOR ACTIVE CONTROL OF IMPULSIVE NOISE

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ABSTRACT

In this paper we consider active noise control (ANC) of impulsive noise having peaky distribution with heavy tail. Such impulsive noise can be modeled using non-Gaussian stable process for which second order moments do not exist. The most famous filtered-x least mean square (FxLMS) algorithm for ANC systems is based on second order moment of error signal, and hence, becomes unstable for the impulsive noise. Previously, a modified-FxLMS algorithm has been proposed, where the reference signal used in update equation is modified on the basis of the thresholding parameters. In the practical ANC systems, these thresholding parameters need to be estimated offline and cannot be updated during online operation of ANC systems. In this paper we propose an ad hoc basis normalized FxLMS algorithm for ANC of impulsive noise sources, which does not require any thresholding. The computer simulations are carried out to verify the effectiveness of the proposed algorithm.

1. INTRODUCTION

Active noise control (ANC) is based on the principle of destructive interference between acoustic waves [1, 2]. Essentially, the primary noise is cancelled around the location of the error microphone by generating and combining an antiphase canceling noise [3]. As shown in Fig. 1, a single-channel feedforward ANC system comprises one reference sensor to pick up the reference noise $x(n)$, one canceling loudspeaker to propagate the canceling signal $y(n)$ generated by an adaptive filter $W(z)$, and one error microphone to pick up the residual noise $e(n)$. The most famous adaptation algorithm for ANC systems is the filtered-x LMS (FxLMS) algorithm [4, 5], which is a modified version of the LMS algorithm [6]. The FxLMS algorithm [4] is obtained by minimizing the mean square error cost function; $J(n) = \mathbb{E}\{e^2(n)\} \approx e^2(n)$, where $\mathbb{E}\{\cdot\}$ is the expectation operator; and is given as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n) \mathbf{x}_f(n), \quad (1)$$

where μ is the step size parameter, and $\mathbf{x}_f(n)$ is the vector for the filtered-reference signal. The FxLMS algorithm is a popular ANC algorithm due to its robust performance, low computational complexity and ease of implementation [4].

Over the past few decades a great progress has been made in ANC, yet the practical applications are limited. One important challenge comes from the control

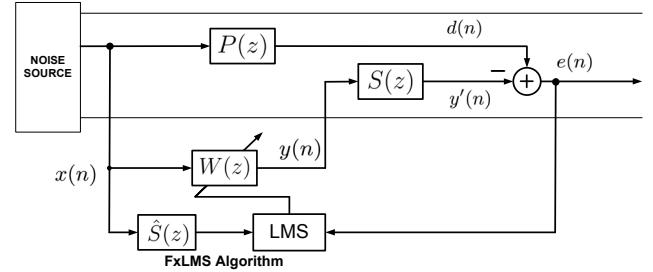


Figure 1: Block diagram of FxLMS algorithm based single-channel feedforward ANC systems.

of impulsive noise. In practice, the impulsive noises are often due to the occurrence of noise disturbance with low probability but large amplitude. An impulsive noise can be modeled by stable non-Gaussian distribution [7]. We consider impulse noise with symmetric α -stable (SaS) distribution $f(x)$ having characteristic function of the form [7]

$$\varphi(t) = e^{-\gamma|t|^\alpha} \quad (2)$$

where $0 < \alpha < 2$ is the shape parameter called as characteristics exponent, and $\gamma > 0$ is the scale parameter called as dispersion. If a stable random variable has a small value for α , then distribution has a very heavy tail, i.e., it is likely to observe values of random variable which are far from its central location. For $\alpha = 2$ it is Gaussian distribution, and for $\alpha = 1$ it is the Cauchy distribution. An SaS distribution is called standard if $\gamma = 1$. In this paper, we consider ANC of impulsive noise with standard SaS distribution, i.e., $0 < \alpha < 2$ and $\gamma = 1$. A few examples are shown in Fig. 2.

For stable distributions, the moments only exist for the order less than the characteristic exponent [7], and hence the mean-square-error criterion, which is bases for FxLMS algorithm, is not an adequate optimization criterion. In [8], the filtered-x least mean p -power algorithm (FxLMP) has been proposed, which is based on minimizing a fractional lower order moment (p -power of error) that does exist for stable distributions. For some $0 < p < \alpha$, minimizing the p th moment $\mathbb{E}\{|e(n)|^p\} \approx |e(n)|^p$, the stochastic gradient method to update $W(z)$ is given as [8]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu |e(n)|^{p-1} \text{sgn}(e(n)) \mathbf{x}_f(n). \quad (3)$$

It has been shown that FxLMP algorithm with $p < \alpha$

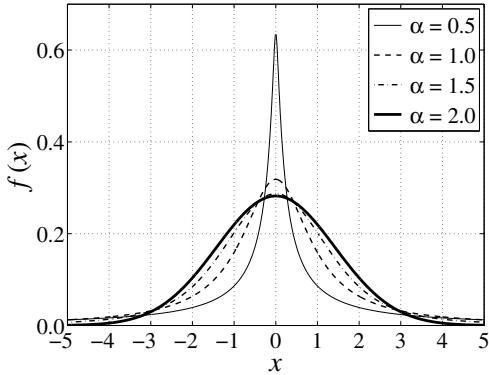


Figure 2: The PDFs of standard symmetric α -stable (SaS) process for various values of α .

shows better robustness to ANC of impulsive noise, however, due to the calculation of fractional power at each iteration, the computational complexity of FxLMP algorithm may be formidable. Furthermore, it requires prior estimation of p , which depends on α and is not an easy task.

In [9], Sun et. al. has proposed a simple variant of FxLMS algorithm for ANC of impulsive noise. The basic idea here is to ignore the sample of the reference signal $x(n)$ if its amplitude is above a certain value set by its statistics. As compared with the FxLMS algorithm, this algorithm gives stable and robust performance. However, the stability cannot be guaranteed and algorithm might become unstable particularly when α is small. In the previous work [10], we have suggested improved performance FxLMS algorithms for ANC of impulsive noise. The main problem is that these algorithms [9, 10] require estimation of appropriate thresholding parameters which is not possible during the online operation of ANC system. In this paper we attempt to solve this problem, and propose a modified normalized FxLMS (MNFXLMS) algorithm for ANC of impulsive noise. The modification is suggested on an ad hoc basis and extensive computer simulations are carried out to demonstrate the effectiveness of the proposed algorithm.

The rest of the paper is organized as follows. Section 2 gives overview of the Sun's algorithm [9]. The authors' previous work [10] in comparison with the Sun's algorithm, and the proposed algorithm are described in Section 3. The simulation results are discussed in Section 4, and the concluding remarks are given in Section 5.

2. SUN'S ALGORITHM

In Fig. 1, assuming that $W(z)$ is an FIR filter of tap-weight length L , the secondary signal $y(n)$ is expressed as

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n), \quad (4)$$

where $\mathbf{w}(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$ is the tap-weight vector for $W(z)$, and $\mathbf{x}(n)$ is the tap-input vector

being given as

$$\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-L+1)]^T. \quad (5)$$

The residual error signal $e(n)$, being picked by the error microphone is given as

$$e(n) = d(n) - y'(n), \quad (6)$$

where $d(n) = p(n) * x(n)$ is the primary disturbance signal, $y'(n) = s(n) * y(n)$ is the secondary canceling signal, $*$ denotes linear convolution and $p(n)$ and $s(n)$ are impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. This residual error $e(n)$ is used in the update of FxLMS algorithm (1). In (1) $\mathbf{x}_f(n)$ is the vector for the filtered-reference signal and is given as, $\mathbf{x}_f(n) = [x_f(n), x_f(n-1), \dots, x_f(n-L+1)]^T$, where $x_f(n) = \hat{s}(n) * x(n)$ is the reference signal $x(n)$ filtered through a model of the so-called secondary path $S(z)$, following the adaptive filter, where $\hat{s}(n)$ is impulse response of the secondary path modeling filter $\hat{S}(z)$.

The reference signal vector (5), used in the update equation of the FxLMS algorithm (1) and in generating the cancelling signal (4), shows that the samples of the reference signal $x(n)$ at different time are treated "equally". It may cause the FxLMS algorithm to become unstable in the presence of impulsive noise. To overcome this problem, in Sun's algorithm [9], the samples of the reference signal $x(n)$ are ignored, if their magnitude is above a certain threshold set by statistics of the signal . Thus the reference signal is modified as

$$x'(n) = \begin{cases} x(n), & \text{if } x(n) \in [c_1, c_2] \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Effectively, this algorithm assumes the same PDF for $x'(n)$ with in $[c_1, c_2]$ as that of $x(n)$, and simply neglects the tail beyond $[c_1, c_2]$. Thus Sun's algorithm [9] is given as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n) \mathbf{x}'_f(n), \quad (8)$$

where $\mathbf{x}'_f(n) = [x'_f(n), x'_f(n-1), \dots, x'_f(n-L+1)]^T$ and $x'_f(n) = \hat{s}(n) * x'(n)$ is modified-filtered-reference signal.

3. PREVIOUS WORK AND PROPOSED ALGORITHM

3.1 Modifications to Sun's Algorithm

It is evident that the residual error signal, $e(n)$, may also be peaky, and in the worst case the algorithm may become unstable. In order to improve the stability of the Sun's algorithm, the idea of (7) is extended to the error signal $e(n)$ as well, and a new error signal is obtained as [10]

$$e'(n) = \begin{cases} e(n), & \text{if } e(n) \in [c_1, c_2] \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

and modified-Sun's algorithm [10] for ANC of impulse noise is proposed as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e'(n) \mathbf{x}'_f(n). \quad (10)$$

As a one step further modification, in [10] we have proposed thresholding the peaky samples in the reference and error signals as

$$x''(n) = \begin{cases} c_1, & x(n) \leq c_1 \\ c_2, & x(n) \geq c_2 \\ x(n), & \text{otherwise} \end{cases} \quad (11)$$

$$e''(n) = \begin{cases} c_1, & e(n) \leq c_1 \\ c_2, & e(n) \geq c_2 \\ e(n), & \text{otherwise} \end{cases} \quad (12)$$

and have proposed a modified FxLMS algorithm (hereafter referred as Previous algorithm) for ANC of impulsive noise as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e''(n) \mathbf{x}_f''(n) \quad (13)$$

where samples of $\mathbf{x}_f''(n)$ are obtained as $x''_f(n) = \hat{s}(n) * x''(n)$.

3.2 Proposed Modified Normalized FxLMS (MNFxLMS) Algorithm

It is worth mentioning that all algorithms discussed so far; Sun's algorithm [9] and its variants, modified-Sun's algorithm and previous algorithm [10]; require an appropriate selection of the thresholding parameters $[c_1, c_2]$. As stated earlier, the basic idea of Sun's algorithm is to ignore the samples of the reference signal $x(n)$ beyond certain threshold $[c_1, c_2]$ set by the statistics of the signal [9]. Here the probability of the sample less than c_1 or larger than c_2 are assumed to be 0, which is consistent with the fact that the tail of PDF for practical noise always tends to 0 when the noise value is approaching $\pm\infty$. Effectively, Sun's method assumes the same PDF for $x'(n)$ (see Eq. (7)) with in $[c_1, c_2]$ as that of $x(n)$, and neglects the tail beyond $[c_1, c_2]$. The stability of Sun's algorithms depends heavily on appropriate choice of $[c_1, c_2]$. In Authors' previous work, we have extended this idea, that instead of ignoring, the peaky samples are replaced by the thresholding values c_1 and c_2 . Effectively, this algorithm adds a saturation nonlinearity in the reference and error signal paths. Thus, the performance of the previous algorithm also depends on the parameters c_1 and c_2 .

In order to overcome this difficulty, we propose a new FxLMS algorithm that does not use modified reference and/or error signals, and hence does not require selection of the thresholding parameters $[c_1, c_2]$. Following the concept of normalized LMS (NLMS) algorithm [11], the normalized FxLMS (NFxLMS) can be given as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu(n) e(n) \mathbf{x}_f(n), \quad (14)$$

where normalized time-varying step-size parameter $\mu(n)$ is computed as

$$\mu(n) = \frac{\tilde{\mu}}{\|\mathbf{x}_f(n)\|_2^2 + \delta}, \quad (15)$$

where $\tilde{\mu}$ is fixed step-size parameter, $\|\mathbf{x}_f(n)\|_2$ is l_2 -norm of the filtered-reference signal vector that can be computed from current available data, and δ is small positive

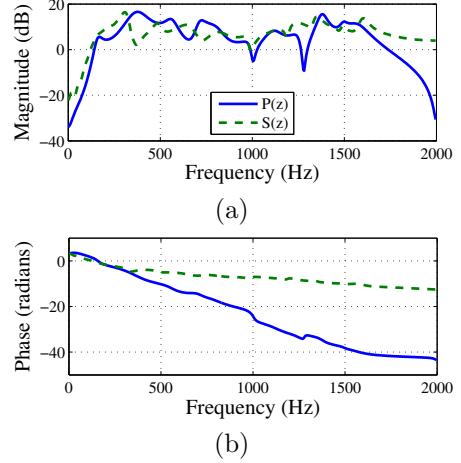


Figure 3: Frequency response of the primary path $P(z)$ and secondary path $S(z)$. (a) Magnitude response and (b) phase response.

number added to avoid division by zero. When the reference signal has a large peak, its energy would increase, and this would in turn decrease the effective step-size of NFxLMS algorithm. As stated earlier, the error signal is also peaky in nature and its effect must also be taken into account. We propose following modified normalized step-size for FxLMS algorithm of (14)

$$\mu(n) = \frac{\tilde{\mu}}{\|\mathbf{x}_f(n)\|_2^2 + E_e(n) + \delta}, \quad (16)$$

where $E_e(n)$ is energy of the residual error signal $e(n)$ that can be estimated online using a lowpass estimator as

$$E_e(n) = \lambda E_e(n-1) + (1-\lambda)e^2(n), \quad (17)$$

where λ is the forgetting factor ($0.9 < \lambda < 1$). It is worth mentioning that the proposed modified normalized FxLMS (MNFxLMS) algorithm, comprising (14), (16) and (17), does not require estimation of thresholding parameters $[c_1, c_2]$. Although the proposed MNFxLMS algorithm is based on an intuition based modification, the simulations suggest its improved performance in comparison with other algorithms discussed in this paper.

4. COMPUTER SIMULATIONS AND DISCUSSION

This section provides the simulation results to verify the effectiveness of the proposed algorithm in comparison with the FxLMP algorithm and Sun's algorithm. The acoustic paths are modeled using data provided in the disk attached with [4]. Using this data $P(z)$ and $S(z)$ are modeled as FIR filter of length 256 and 128 respectively. The frequency response characteristics of the acoustic paths are shown in Fig. 3. It is assumed that the secondary path modeling filter $\hat{S}(z)$ is exactly identified as $S(z)$. The ANC filter $W(z)$ is selected as an FIR filter of tap-weight length 192. The performance comparison is done on the basis of mean noise reduction

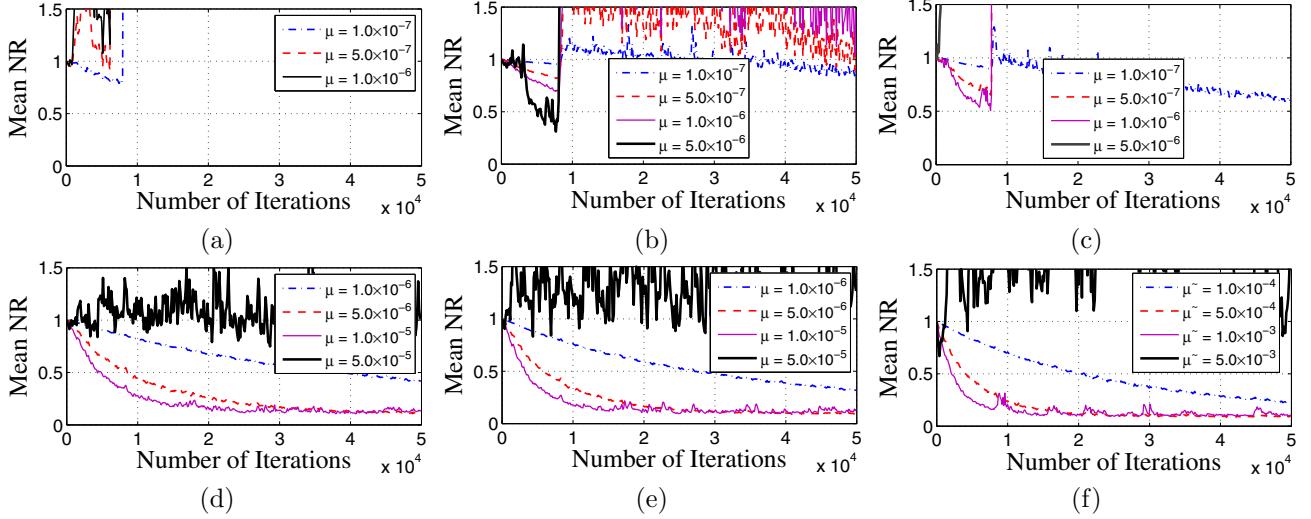


Figure 4: Mean noise reduction (MNR) curves for various algorithms for Case I ($\alpha = 1.45$). (a) FxLMS, (b) FxLMP [8], (c) Sun's algorithm [9], (d) Modified-Sun's algorithm [10], (e) Authors' previous algorithm [10], and (f) proposed MNFxLMS algorithm.

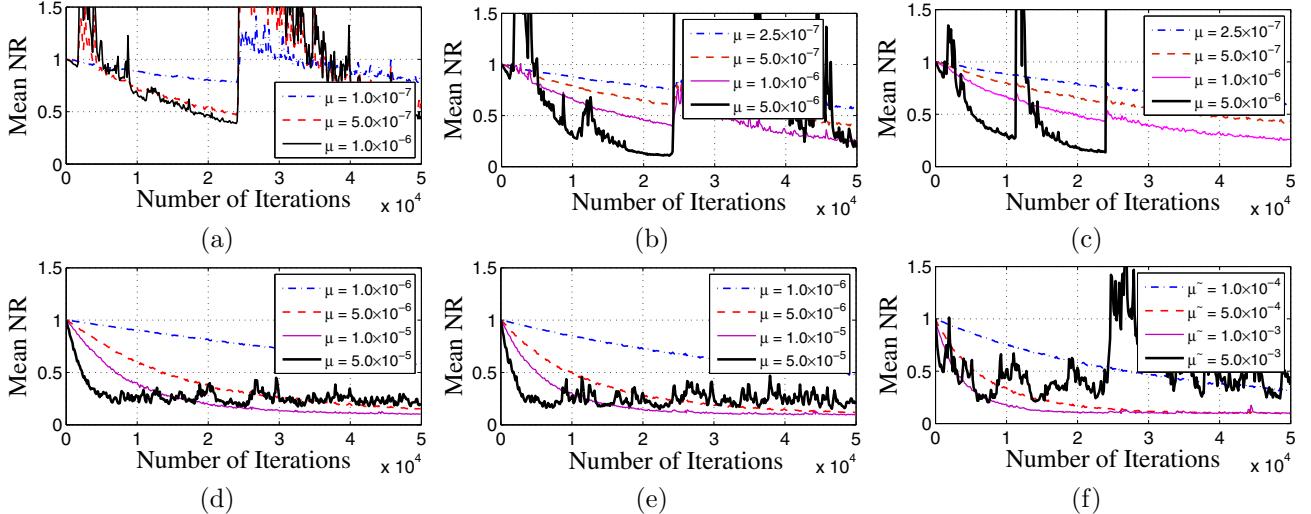


Figure 5: Mean noise reduction (MNR) curves for various algorithms for Case II ($\alpha = 1.65$). (a) FxLMS, (b) FxLMP [8], (c) Sun's algorithm [9], (d) Modified-Sun's algorithm [10], (e) Authors' previous algorithm [10], and (f) proposed MNFxLMS algorithm.

(MNR), being defined as

$$MNR(n) = \mathbb{E} \left\{ \frac{A_e(n)}{A_d(n)} \right\}, \quad (18)$$

where $\mathbb{E}\{\cdot\}$ denotes expectation or ensemble averaging of quantity inside, and $A_e(n)$ and $A_d(n)$ are estimates of absolute values of residual error signal $e(n)$ and disturbance signal $d(n)$, respectively, at the location of error microphone. These estimates are obtained using low-pass estimators as

$$A_r(n) = \lambda A_r(n-1) + (1-\lambda)|r(n)| \quad (19)$$

where $|\cdot|$ is the absolute value of quantity, and λ is same as defined in (17).

The reference noise signal $x(n)$ is modeled by standard S α S process with $\alpha = 1.45$ (Case I), and $\alpha = 1.65$

(Case II). Here Case I is more towards Cauchy distribution and Case II is more towards Gaussian distribution. All simulation results presented below are averaged over 25 realization of the process. Extensive simulations are carried to find appropriate values for the thresholding parameters $[c_1, c_2]$, and are selected as: [0.01, 99.99] in Sun's algorithm, [0.5 99.5] in modified-Sun's algorithm, and [1,99] in previous algorithm. The detailed simulation results for Case I and Case II are given in Figs. 4 and 5, respectively, where the objective is to study the effect of step-size parameter. It is seen that, the FxLMS algorithm is not able to provide ANC of impulsive noise, even for a very small step-size. Furthermore, in comparison with the Authors' algorithms, the performance of Sun's algorithm and FxLMP algorithm is very poor. On the basis of best results for the respective algorithms, the performance comparison for Case I and

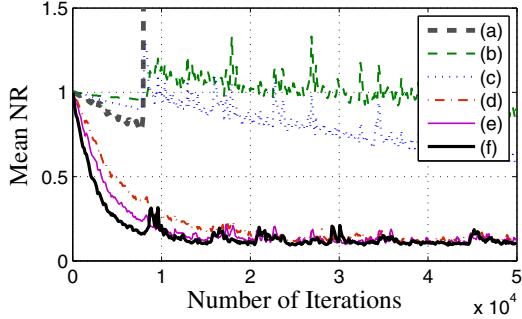


Figure 6: Performance comparison between various algorithms for Case I ($\alpha = 1.45$). (a) FxLMS ($\mu = 1 \times 10^{-7}$), (b) FxLMP ($\mu = 1 \times 10^{-7}$), (c) Sun's algorithm ($\mu = 1 \times 10^{-7}$), (d) Modified-Sun's algorithm ($\mu = 1 \times 10^{-5}$), (e) Authors' previous algorithm ($\mu = 1 \times 10^{-5}$) and (f) proposed MNFxLMS algorithm ($\tilde{\mu} = 1 \times 10^{-3}$).

II is shown in Fig. 6 and 7, respectively. These results show that the proposed algorithm outperforms the existing algorithms and, among the algorithms considered in this paper, appears as a best choice for ANC of SaS impulsive noise.

5. CONCLUDING REMARKS

In this paper a modified normalized FxLMS algorithm has been proposed. The main advantage of the proposed algorithm is its simplicity and robust performance, which makes it a good candidate for practical applications. It is demonstrated by computer simulations, that the proposed algorithms have very fast convergence, good stability and robustness for ANC of α stable impulsive noises. In future, it would be interesting to investigate the adaptive filtering algorithms for ANC of impulsive noise that are not α stable, for example of transient sinusoids type as considered in [12].

REFERENCES

- [1] S. J. Elliot, *Signal Processing for Active Control*, London, U.K.: Academic Press, 2001.
- [2] S. J. Elliot, and P. A. Nelson, "Active noise control," *IEEE Signal Process. Mag.*, vol. 10, pp. 12–35, Oct., 1993.
- [3] P. Lueg, Process of silencing sound oscillations, *U.S. Patent*, 2043416, June 9, 1936.
- [4] S. M. Kuo, and D. R. Morgan, *Active Noise Control Systems-Algorithms and DSP Implementations*, New York: Wiley, 1996.
- [5] ———, "Active Noise Control: A tutorial review," *Proc. IEEE*, vol. 87, pp. 943–973, Jun., 1999.
- [6] B. Widrow, and S.D. Stearns, *Adaptive Signal Processing*, Prentice Hall, New Jersey, 1985.
- [7] M. Shao, and C. L. Nikias, "Signal Processing with Fractional Lower Order Moments: Stable Processes and Their Applications," *Proc. IEEE*, vo. 81, no. 7, pp. 986–1010, Jul. 1993.
- [8] R. Leahy, Z. Zhou, and Y. C. Hsu, "Adaptive Filtering of Stable Processes for Active Attenuation of Impulsive Noise," in *Proc. IEEE ICASSP 1995*, vol. 5, May 1995, pp. 2983–2986.
- [9] X. Sun, S. M. Kuo, and G. Meng, "Adaptive Algorithm for Active Control of Impulsive Noise," *Jr. Sound Vibr.*, vol. 291, no. 1-2, pp. 516–522, Mar. 2006.
- [10] M. T. Akhtar, and W. Mitsuhashi, "Improving Performance of FxLMS Algorithm for Active Noise Control of Impulsive Noise," *Jr. Sound Vibr.*, vol. 327, no. 3-5, pp. 647–656, Nov. 2009.
- [11] S. C. Douglas, A family of normalized LMS algorithms, *IEEE Signal Processing Letters*, vol. 1, no. 3, pp. 49–51, 1994.
- [12] G. Pinte, W. Desmet, and P. Sas, "Active Control of Repetitive Transient Noise," *Jr. Sound Vibr.*, vol. 307, pp. 513–526, 2007.

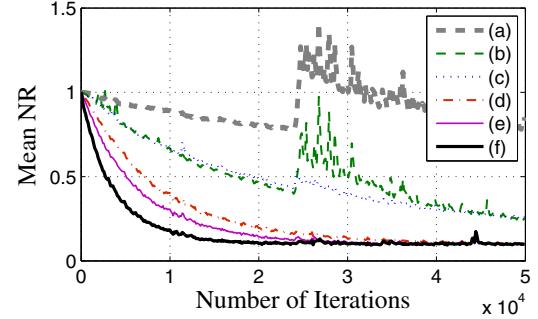


Figure 7: Performance comparison between various algorithms for Case II ($\alpha = 1.65$). (a) FxLMS ($\mu = 1 \times 10^{-7}$), (b) FxLMP ($\mu = 1 \times 10^{-6}$), (c) Sun's algorithm ($\mu = 1 \times 10^{-6}$), (d) Modified-Sun's algorithm ($\mu = 1 \times 10^{-5}$), (e) Authors' previous algorithm ($\mu = 1 \times 10^{-5}$) and (f) proposed MNFxLMS algorithm ($\tilde{\mu} = 1 \times 10^{-3}$).

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