# SPECTRUM SENSING IN MULTICHANNEL COMMUNICATION SYSTEMS USING RANDOMIZED SAMPLING SCHEMES

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**ABSTRACT**

*This paper studies a spectrum estimation method that utilises Digital Alias-free Signal Processing (DASP) to continuously sense the spectrum in a multichannel communication environment using relatively low sampling rates compared to the classical approaches. Various estimators that use similar concepts are considered. The effect of noise on the accuracy of the chosen detector is analysed. Most importantly, general guidelines on choosing the average sampling rate within a given scenario are provided in order to guarantee sensing reliability. The extra requirement on the needed sampling rate imposed by the presence of noise is given. The analytical results are illustrated by numerical examples.*

## 1. INTRODUCTION

Wireless sensor networks that use multichannel communication protocols [1] to detect the occurrence of an event/phenomenon e.g. boarders surveillance and fire detection are expected to have low channel occupancy due to their occasional nature of transmissions. The occupancy is defined by the ratio between the simultaneously active channels and the total number of channels in the system. If the channels occupy separate frequency bands, the signal sampling rate of the receiver monitoring their activity should normally exceed twice the total bandwidth occupied by the channels. Failing to do so could result in signal aliasing and irresolvable problems of separating their transmissions. When dealing with low channel occupancy scenario, such sampling rates would exceed many times Landau rate. Landau rate [2] is the theoretically lowest sampling rate that allows avoiding aliasing in DSP. In our case it equals twice the bandwidth of the active channels. For many bandpass and multiband signals, sampling at Landau rate requires deployment of nonuniform sampling as well as a priori knowledge of which channels are occupied. Exceeding significantly the Landau rate may indicate potential inefficiencies in using the receiver's resources such as power and/or deployment of high-cost, fast hardware capable of dealing with excessive sampling rates.

In this paper we demonstrate that even if the active channels are unknown we can still detect them by suitable use of low-rate nonuniform sampling and appropriate processing of the signal – a methodology sometimes referred to as DASP. One of the earliest DASP-type algorithms was introduced in [3] and [4]. Few monographs, such as [5] and [6], give an overview on the topic.

In this paper, we study techniques that are based on DASP approach to sense the activity of the channels in multichannel communication environment. The adopted spectrum detection approach relies on estimating the spectrum of the incoming signal and sensing its magnitude. The problem of estimating the spectrum using nonuniformly sampled data has been studied in several publications [7-13]. We note that none of the cited publications; except [13]; took into account the presence of noise. Therefore the results produced there should be treated with caution when applied to practical situations.

The paper is organised as follows: in Section 2 we formulate the studied problem. In Section 3, we consider a set of available DASP techniques that enable us to sense the spectrum and select the ones that are suitable for the task on the basis of simplicity, relevance to the considered problem and practicality. The effect of noise on the accuracy of the estimators is studied in Section 4. Primary contribution in this paper is presented in Section 5: we give general recommendations on the required average sampling rate given the actual bandwidth of the processed signal and noise level. Finally, in section 6 we give numerical examples to demonstrate the presented results.

## 2. PROBLEM FORMULATION

Let $L$ be the total number of channels over which data is transmitted in a multichannel communication system. Each channel has a bandwidth of $B_C$, hence the total bandwidth to be monitored is $B = LB_C$. The maximum number of simultaneously active channels and their joint bandwidth are given by $L_A$ and $B_A = L_A B_C$ respectively. All channels' central frequencies are known. We assume that the maximum channel occupancy is low, i.e. $L_A/L << 1$. Our task is to produce an algorithm capable of scanning the monitored bandwidth $B$ and identify which channel(s) are active. The algorithm should operate on sampling rates significantly less than $2B$. Since the use of the Landau rate requires a priori knowledge of the position(s) of the active channel(s) [11], which in our case is not available, we aim at using a sampling rate which is substantially smaller than $2B$ but still above $2B_A$. Similar task has already been solved in [11] by the use of universal sampling. However, here we search for algorithms that avoid huge computational costs inevitably accompanying solutions proposed in [11]. We note that the approach proposed in [3] is capable of detecting/estimating frequency components of present signal using arbitrary low sampling rates. Nevertheless, it entails the use of infinitely long signal observation window which has limited practical applicability.

Random nonuniform sampling which we are going to use here eliminates signal aliasing in the form that is known in classical DSP. Instead it creates smeared aliasing – a noise-like signal present at all frequencies [5]. The sought reliable detector should be immune to the effects of smeared aliasing as well as noise. Besides, in practical systems the transmission is expected to be continuous where channels activity changes. A time moving window is typically used in such cases. A spectrum sensing procedure is carried out on each of the signal windows.

The power levels of concurrently active channels can significantly differ from each other due to the adverse practical constraints of communication systems. A practical approach to dependable sensing in such cases is to identify the active channel; if any; with the highest power level first and then extract it to reveal weaker signal components that are present. Extraction methods that are based on modelling the targeted channel are expansions of Sequential Component Extraction (SECOEX) proposed in [14]. The later targets discrete frequencies and may lead to excessive computational cost due to the high resolution spectrum of the considered communication signals given a long signal observation window as shown in the numerical examples. The details of the extraction procedure lie outside the scope of this paper.

## 3. SPECTRUM ESTIMATORS

The considered DASP techniques consist of two steps: signal sampling (nonuniformly) and calculating its spectrum with the aid of unbiased estimators. The target of such estimators is given by:

$$X_W(f) = \int_{t_0}^{t_0+T} x(t)w(t)e^{-j2\pi ft}dt \qquad (1)$$

$t_0$ is the initial time instant of the analysed signal window whilst $T$ is its width. The windowing function given by $w(t)$ is commonly used to suppress the known Gibbs phenomenon.

In this section we review a set of available spectrum estimators and exclude the ones that are less suitable for our purpose. We start first with periodograms which were introduced in [13] for arbitrarily collected data. They are commonly used to detect discrete spectral components. Their accuracy is poor compared to the other studied methods. As a result the option of using periodograms is excluded.

Blind Spectrum Sampling (BSS) [11,12] is a sensing technique that allows the use of sampling rates which are arbitrary close to the minimum limit i.e. Landau rate to sense the spectrum and reconstruct signals. It deploys periodic nonuniform sampling and blindly scans a predefined range of frequencies searching for present spectral components. BBS uses complex algorithm that makes it computationally expensive and consequently it will be abandoned.

[7] and [8] introduced estimators that are based on a total random sampling scheme where sampling instants are identically distributed, independent random variables whose Probability Distribution Functions ( PDF's) cover the whole signal observation window $[t_0, t_0+T]$. In [7] two estimators were proposed: Weighted Sampled (WS) and Weighted Probability (WP). In the latter the sampling instants PDF's are dependent on the used windowing function $w(t)$. However, the average density of sampling instants inside $[t_0, t_0+T]$ is desired to be constant especially when the window moves i.e. $t_0$ changes due to the continuous nature of transmission. The optimal estimator (in term of accuracy) proposed in [8] has sampling instants whose PDF's are dependent on the incoming signal which is assumed to be unknown. Thus, both WP and the optimal estimators are eliminated from the list of candidates.

Estimators that use stratified and antithetical stratified sampling schemes were introduced in [9] and [10]. The two methods divide the signal observation window into subintervals within which sample(s) with uniform PDF's are taken. The quality of the estimators can be improved by designing the optimal densities of these subintervals. The optimal cases are dependent on the processed signal and hence they will not be considered. In the non-optimal cases time subintervals are equal, hence the sampling schemes become a kind of jitter sampling. We will refer to those two schemes by jitter sampling thereafter.

Therefore the remaining candidates are: the WS and the two estimators that utilise jitter sampling (stratified-antithetical sampling with equal time partitions).

## 4. SPECTRUM ESTIMATORS PERFORMANCE IN PRECENSE OF NOISE

Spectrum estimators studied in [7-10] were all evaluated in noise free environments. However, most data transmission systems are subject to noise which affects/limits the system performance. Noise is commonly modelled as zero mean Added White Gaussian Noise (AWGN). Hence, the processed signal which is composed of data $x(t)$ and noise $n(t)$ is represented by $y(t) = x(t) + n(t)$. In this section we assess the effect of noise on the unbiased nature of the WS estimator as well as on its accuracy. The presented results are linked to the estimators that use jitter (stratified-antithetical) sampling.

The sampling instants used in WS approaches are independent from each other and have identical PDF's which are given by:

$$p_{WS}(t) = \begin{cases} \frac{1}{T} & t \in \left[t_0, t_0+T\right] \\ 0 & elsewhere \end{cases} \qquad (2)$$

WS estimator is defined by:

$$X_{WS}(f) = \frac{T}{N}\sum_{n=1}^{N} y(t_n)w(t_n)e^{-j2\pi f t_n} \quad (3)$$

where $N$ is the number of the captured samples.

### 4.1 Unbiased Spectrum Estimator

In this subsection we show that the WS estimator remains unbiased despite the presence of noise. The expected value of the estimators is evaluated with respect to the sample points as well as the added noise. We note that all the components of the summation in (3) are independent and identically distributed random variables with identical PDF's, hence:

$$E\left[X_{WS}(f)\right] = TE\left[\left\{x(t)+n(t)\right\}w(t)e^{-j2\pi ft}\right] \quad (4)$$

where $t$ is a random variable defined by $p_{WS}(t)$. Given that $E\left[n(t)w(t)e^{-j2\pi ft}\right]=0$:

$$E\left[X_{WS}(f)\right] = TE\left[x(t)w(t)e^{-j2\pi ft}\right] = X_W(f) \quad (5)$$

Therefore the estimator is unbiased.

### 4.2 Accuracy of the Estimator in Noisy Environment

Although the lack of bias is a sought property for any statistical estimator, it does not indicate the estimator's accuracy. On the other hand, the standard deviation is directly related to the accuracy according to Chebychev's inequality which states that: $\Pr\left\{\left|X - E\left[X\right]\right| \geq \varepsilon\right\} \leq \frac{\sigma_X^{\,2}}{\varepsilon^2}$ where $X$ is a random variable and $\varepsilon > 0$. In this section we evaluate the effect of the noise factor on the standard deviations of the WS estimator. The variance is defined by:

$$\sigma^2_{X_{WS}}(f) = E\left[\left|X_{WS}(f)\right|^2\right] - \left|X_W(f)\right|^2 \quad (6)$$

Now:

$$\left|X_{WS}(f)\right|^2 = \frac{T^2}{N^2}\sum_{n=1}^{N}\sum_{m=1}^{N} y(t_n)y(t_m)w(t_n)w(t_m)e^{-j2\pi f(t_n-t_m)} \quad (7)$$

The expected value of (7) in terms of the sampling instants and the added noise is calculated over two stages: when indices are identical i.e. $n=m$ and when they are distinct, hence:

$$\left|X_{WS}(f)\right|^2 = \frac{T^2}{N^2}\sum_{n=1}^{N}\sum_{\substack{m=1\\ m\neq n}}^{N} y(t_n)y(t_m)w(t_n)w(t_m)e^{-j2\pi f(t_n-t_m)} + \frac{T^2}{N^2}\sum_{n=1}^{N} y^2(t_n)w^2(t_n) \quad (8)$$

Since the sampling instants are independent from each other and with identical distributions we can make use of (4) and (5). For $n=m$, expected value of (8) reduces to:

$$\frac{T^2}{N}\left\{E\left[x^2(t)w^2(t)\right] + E\left[n^2(t)w^2(t)\right]\right\} = \frac{T}{N}\left\{E_{WS} + \sigma_0^2 E_W\right\},$$

where $E_{WS}$ and $E_W$ represent the energy of the windowed signal and the area of the used window respectively. They are defined by:

$$E\left[x^2(t)w^2(t)\right] = \frac{1}{T}\int_{t_0}^{t_0+T}\left[x(t)w(t)\right]^2 dt = \frac{E_{WS}}{T} \quad (9)$$

$$E\left[n^2(t)w^2(t)\right] = \frac{\sigma_0^2}{T}\int_{t_0}^{t_0+T} w^2(t)dt = \frac{\sigma_0^2 E_W}{T} \quad (10)$$

Now when $n \neq m$, the expected value of (8) yields: $\frac{(N-1)}{N}\left|X_W(f)\right|^2$. Hence,

$$E\left[\left|X_{WS}(f)\right|^2\right] = \frac{T\left\{E_{WS} + \sigma_0^2 E_W\right\}}{N} + \frac{(N-1)}{N}\left|X_W(f)\right|^2 \quad (11)$$

By inserting (11) back into (6) we obtain:

$$\sigma^2_{WS}(f) = \frac{E_{WS} + \sigma_0^2 E_W}{\alpha} - \frac{\left|X_W(f)\right|^2}{N} \quad (12)$$

where $\alpha = N/T$ is the average sampling rate. Hence,

$$\sigma_{WS}(f) = \sqrt{\frac{E_{WS} + \sigma_0^2 E_W}{\alpha} - \frac{\left|X_W(f)\right|^2}{N}} \quad (13)$$

The estimators that use stratified and antithetical stratified sampling schemes reserve their unbiased nature despite the presence of noise. Besides, the impact of noise on the accuracy of the two methods is identical to that of the WS estimator i.e. $\sqrt{\sigma_0^2 E_W / \alpha}$ for the equal time partitions case i.e. jitter sampling. Those two facts can be proven by following similar procedures to the ones shown above.

## 5. SPECTRUM DETECTION

The procedure of detecting active channel(s) proposed in this paper relies on creating unbiased estimates of the received signal spectrum and scanning its magnitude. In order for the detection to be reliable, we request that the spectral peak(s) associated with the targeted active channel(s) are visibly higher than the level of the smeared aliasing and noise that are present in the signal spectrum. This request will impose a lower limit on the used average sampling frequency $\alpha$.

Let $H$ be the average peak of the magnitude spectrum of the targeted channel(s) i.e. with the highest power level. We recall that $B_A$ is the bandwidth of the processed signal i.e. joint active channels bandwidth. We note that $\sigma_{WS}(f)$ is constant at frequencies where the signal is not present. This represents the maximum error of the estimator given by $\sigma_{WS,\max} = (E_{WS} + \sigma_0^2 E_W)/\alpha$. $\sigma_{WS,\max}$ features a white-noise-like error that is inversely proportional to $\alpha$. This error represents the effect of noise and smeared aliasing on the estimator's accuracy and its relation to the spectrum of the signal $X_W(f)$ is described by Chebychev's inequality. Hence in general for the spectral peak(s) of the targeted channel(s) to be visible i.e. notably above the error/smeared-aliasing level:

$$H - \sqrt{\sigma^2_{WS,\max} - \frac{\left|X_W(f_0)\right|^2}{N}} >> \sigma_{WS,\max} \quad (14)$$

where $f_0$ is the frequency where the highest peak in the magnitude spectrum of the signal exists. Experimental results; including numerical examples in [7] for noise-free case; showed that $\left|X_W(f)\right|^2/N$ contribution in term of reducing the estimator's error level according to (13) is insignificant. Thus, for a conservative approach to reliable detection (14) can be written as:

$$H > \eta\sigma_{WS,\max} \tag{15}$$

where $\eta \geq 2$. Hence: $H^2 > \eta^2(E_{WS} + \sigma_0^2 E_W)/\alpha$ and as a result the average sampling rate is decided by:

$$\alpha > \eta^2 \frac{E_{WS} + \sigma_0^2 E_W}{H^2} \tag{16}$$

If the rectangular windowing function is used, $E_{WS}$ reduces to the signal energy given by $S = E_{WS} = \int_{t_0}^{t_0+T} \left|x(t)\right|^2 dt$ while the window energy becomes $E_W = T$. According to Parseval's theorem: $S = \int_{-\infty}^{+\infty} \left|X_W(f)\right|^2 df$. Now if the used signal observation window is long enough, the signal energy can be calculated by estimating the area under the squared magnitude spectrum of the present signal i.e. :

$$S \leq 2B_A H^2 \tag{17}$$

We note that $N = \sigma_0^2 T$ embodies the noise energy. As a result (16) can be written as:

$$\alpha > \eta^2 \frac{S+N}{S/2B_A} \tag{18}$$

Hence for reliable detection of the channel(s) with the highest power level, the average sampling rate should comply with the following conservative guideline:

$$\alpha > 2B_A\eta^2\left(1+\left(\frac{S}{N}\right)^{-1}\right) \tag{19}$$

Formula (19) gives a clear lower bound on the needed average sampling rate which is a function of the channel occupancy and Signal to Noise Ratio (SNR). In practical applications such parameters are usually known.

We note that according to (19), the lowest recommended sampling rate for a noise free environment is $8B_A$ which exceeded Landau rate by a factor of 4. On the other hand for uniform sampling the required sampling rate is $2B >> 2B_A$, hence substantial savings on sampling rates can be obtained by using the adopted approach. The presence of noise imposes an additional $2B_A\eta^2(SNR)^{-1}$ factor on the required average sampling rate.

The standard deviations of the stratified-antithetical sampling estimators are shown in [9] and [10] to be less than or at most equal to that of the WS. Hence, they are theoretically more accurate. However, such an advantage over WS is noticeable only if the used number of samples is large i.e. high $\alpha$ possibly higher than the uniform sampling case. The numerical examples given in [9] and [10] also indicate this fact. We recall that the reason behind using DASP is maintaining the sampling rate lower than that of the uniform case. Therefore, the improvement gained by using stratified-antithetical sampling estimators is minor for the considered problem as will be shown in the numerical example in the next section. From this perspective, the lower bound on the needed average sampling rate for WS estimator given by (18) applies to jitter-stratified-antithetical sampling estimators. In this case the bound is expected to be even more conservative.

The used estimator is expected to process data on continuous basis. A moving time window is typically used in such cases. From this point of view, estimators that use jitter sampling schemes can be handled easier compared to the ones that use total random sampling particularly once the window shifts. Besides, implementing jitter sampling imposes fewer constraints on the used Analogue to Digital Converter (ADC). In the case of total random sampling two sampling instants can be arbitrary close to each other whereas such phenomenon can be better controlled with the use of jitter sampling.

## 6. NUMERICAL EXAMPLES

In this section we present numerical results that illustrate the analysis performed in the previous sections. Our goal is to sense the spectrum of a multichannel communication system that consists of 50 channels ($L = 50$) that are 2 MHz each ($B_C$ = 2 MHz). QPSK modulated signals with maximum bandwidths are transmitted over the channels. The monitored range of frequencies i.e. system bandwidth stretches from $f_{\text{int}} = 800$ MHz to $f_{\text{int}} + LB_C = 900$ MHz. Channel occupancy of 10% is assumed i.e. $L_A = 5$. Channels with central frequencies of [815, 841, 845, 875, 895] MHz are expected to be active and with similar power levels. Rectangular window of width $T$ = 200 μs is used.

In the first example we consider the noise free case and choose $\alpha = 125$ MHz i.e. $\eta = 2.5$. Figure 1 shows the signal's normalised magnitude spectrum versus $f - f_{\text{int}}$ for WS and jitter-stratified sampling estimators. As shown in Figure 1, the five active channels can be identified clearly. We note that the minimum uniform sampling rate in this case is approximately 400 MHz (bandpass sampling). The detection task was accomplished with a saving of 68.75% on the sampling rate with the use of the proposed method. We note that higher savings can be achieved by using lower $\alpha$. The two plots in Figure 1 show that WS estimator has an error level slightly higher than the jitter sampling one. However, the former features spectrum peaks at certain frequencies higher than those for the WS case which emphasises the statistical nature of the study. This confirms the minor differences between WS and jitter sampling estimators for the studied scenario in term of performance.

In the second experiment we consider a noisy case with SNR of 2 dB. The WS estimator with minimum $\alpha$ = 130.5 MHz according to (19) is depicted in Figure 2. The figure shows that with relatively high level of noise active channels can be detected with the minimum $\alpha$ which is well below its uniform sampling counterpart. This confirms the conservative nature of (19). The effect of noise can be viewed as a rise in the estimator's error floor. Experimental results (not shown due to space restrictions) showed that distribution of simultaneously active channels across the scanned bandwidth does not hinder the performance of the detection procedure.

## 7. CONCLUSION

A method that utilises DASP techniques to detect the channel(s) with the highest power in a multichannel communication environment is proposed. The method uses estimators that deploy different nonuniform sampling schemes. Their differences are insignificant for the detection problem. The estimators exploit the low channel occupancy to use sampling rates that are well below the ones used with classical approaches. Lower bounds on the required average sampling rate for noise-free and noisy cases are provided given the SNR and the channel occupancy values. The provided lower bound indicates that 25% is the maximum channel occupancy percentage for which the proposed method is advantageous over classical DSP in a noise free environment.

The used average sampling rates support channel modelling i.e. reconstruction either for effective sequential extraction or data recovery purposes. This prompts researching into effective reconstruction algorithms of noisy nonuniformly sampled data (little work on the topic is available in open literature for the best knowledge of the author). Besides, the proposed method demands high number of frequency points per channel as a result of the unsmoothness of the signal spectrum due to the relatively long signal observation window. Hence the need for a spectrum smoothing technique that would minimise the required frequency points per channel to detect its activity.

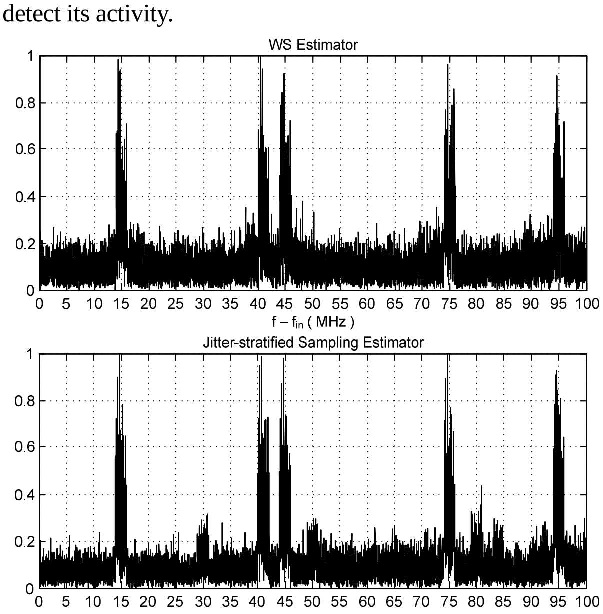

Figure 1, Normalised magnitude spectrum of the received signal.

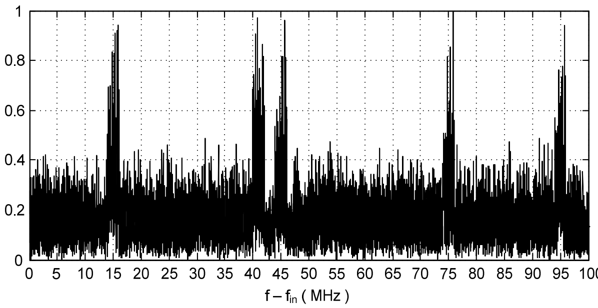

Figure 2, Normalised magnitude spectrum of the processed signal with SNR = 2 dB using WS estimator.

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