DESIGN OF BLOCK-BASED LINEAR MMSE PRECODING AND EQUALISATION FOR BROADBAND MIMO RELAY NETWORKS

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ABSTRACT

This paper addresses the design of MMSE block-based linear precoding and equalisation for relayed communications over frequency-selective MIMO channels. Using a combination of leading zero and trailing zero guard intervals to eliminate inter-block interference, jointly optimal MMSE designs for the block processors in the relay layer and the destination are derived under the assumption of channel state information availability across most of the link. This proposed technique is compared to an extension of state-of-the-art block based methods, which are operated back-to-back across the link and may contain an intermittent non-linear detection stage in the relay. Using the latter as a benchmark, simulation results highlight the benefit of the proposed design for broadband relay communications.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) relaying has recently received considerable attention by researchers due to the fact that the advantages offered through the use of multiple antennas, such as increased spectral efficiency and hence data rates, can be coupled with improved range and coverage as well as a reduction in required transmit power [1].

Relaying networks are generally either regenerative or non-regenerative, also commonly referred to as decode and forward (DF) or amplify and forward (AF). In the regenerative case the relay re-generates the original information from the previous node before transmitting to the subsequent node in the network [1]. In AF schemes the relay layer simply transmits an amplified version of the received signal. Hybrid designs, which use a combination of both methods, have also been proposed in e.g. [2]. In relay networks half duplex signalling is generally assumed, with two orthogonal channels being used for transmission and reception [3].

Relay architectures differ in the topography, and the availability of channel state information across different nodes of the system. For a specific architecture, a common optimisation criterion for deriving system parameters in various stages of the relayed link is based on the system capacity and its bounds, see e.g. [1, 3, 4, 5]. Besides the maximisation of system capacity, other design criteria such as zero forcing (ZF) and minimum mean square error (MMSE) criteria have also been considered [6, 7]. In general such cooperative methods demonstrate an improved system performance over non-cooperative approaches.

For high data rate application, the MIMO channels in a relay system generally are assumed to be broadband, i.e. frequency selective, and the above narrowband methods cannot be directly applied. One solution is orthogonal frequency division multiplexing (OFDM) , which enables the narrowband assumption for each subcarrier when fully synchronised. We here want to focus on general block-based transceiver design methods that achieve higher performance than OFDM because of the availability of channel state information (CSI). Such methods have been investigated for the SISO [8, 9] and MIMO case [10], but not for cooperative networks.

Therefore, this paper aims to explore the use of block based methods. A number of such methods have been derived in the past, which outperform OFDM and provide an excellent performance due to the joint design of precoding and equalisation [8, 9, 10]. Here we are exploiting the ideas of inter-block interference eliminiation in these designs to generalise an existing narrowband MMSE design of a cooperative relay link in [11], which combats the frequency-selectivity of the overall transmission link by a jointly optimised approach between the relay layer and the processor at the destination. The aim is to demonstrate the correctness of this generalisation and to compare it to joint optimal precoder and equaliser designs when operated back-to-back across the relay link.

The remainder of this paper is organised as follows. In Sec.2 we describe the signal model for the MIMO relaying scheme employing block transmission. Based on this analysis, we establish an MMSE cost function for the overall system in Sec. 3, from which designs for the MMSE processors in the relay layer and the destination can be derived. We demonstrate the performance of this system, benchmarked against back-to-back systems constructed according to [10] in Sec. 4. Finally, conclusions are drawn in Sec. 5.

In terms of notation, $\mathscr{E}\{\cdot\}$ is the expectation operator. Vectors and matrices are represented by lowercase and uppercase boldface variables, while $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$, and $(\cdot)^{\mathrm{H}}$ are the transpose, complex conjugate and Hermitian transpose operators. An $M \times M$ identity matrix is represented by $\mathbf{I}_M$, while $\mathbf{0}_{M\times L}$ denotes an $M \times L$ matrix with zero entries. The sets of real and complex valued numbers are $\mathbb{R}$ and $\mathbb{C}$, which in case of vector quantities indicate dimensions by means of a superscript. Finally, the tilde operator applied to vectors and matrices indicates that these quantities contain multiplexed polyphase components that arise from the block design targetted in this paper. The sampling rate changes associated with multiplexing operations are not explicitly elaborated, and the time index n is assumed to always to refer to the "local" time.

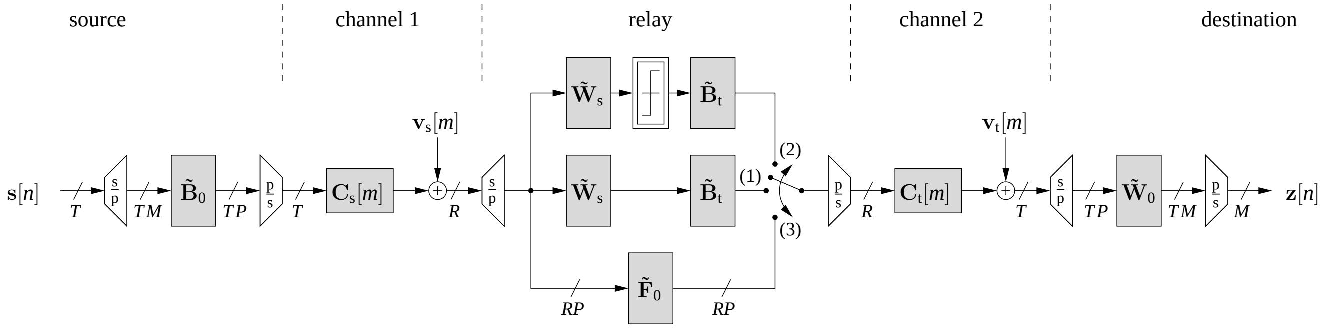

Figure 1: System setup with source and destination connected by a relayed link over two broadband MIMO channels $\mathbf{C}_\mathrm{s}[m]$ and $\mathbf{C}_\mathrm{t}[m]$; the relay layer operation can be switched between MIMO block-based designs [10] (1) without and (2) with decision prior to forwarding, and (3) the proposed joint linear design.

2. SIGNAL MODEL

The assumed communications topology comprises of a source and destination, connected by a single relay layer. The design method can accommodate different dimensions, but for the sake of simplicity it is here assumed that source and destination each comprise of T antennas, with $R \geq T$ antennas in the relay layer. This general configuration can be seen in Fig. 1, which permits three types of operation in the relay stage — back-to-back block-based filter designs according to [10], which are optimised for each channel separately and can contain a decision at the relay stage or not, and the proposed system, which is jointly optimised from source to destination. For tractability of the optimisation procedure, we will however restrict ourselves to a very simple precoder at the source in accordance with the method in e.g. [11].

In order to describe the system in Fig. 1, we first highlight the dispersive and noise corrupted channels $\mathbf{C}_\mathrm{s}[n]$ and $\mathbf{C}_\mathrm{t}[n]$, followed by the description of the individual components at the source, relay and destination of the link, that are introduced to mitigate inter-symbol interference, co-channel interference, and noise effects in the system.

2.1 MIMO Transmission Channels

We consider that the source to relay and relay to destination channels are stationary over the transmission period and frequency selective, with respective transfer functions

$$\mathbf{C}_\mathrm{s}(z) = \sum_{n=0}^{L} \mathbf{C}_\mathrm{s}[n] z^{-n} \quad (1)$$

$$\mathbf{C}_\mathrm{t}(z) = \sum_{n=0}^{L} \mathbf{C}_\mathrm{t}[n] z^{-n} \quad (2)$$

where L is the channel order and the matrices $\mathbf{C}_\mathrm{s}[n] \in \mathbb{C}^{R\times T}$ and $\mathbf{C}_\mathrm{t}[n]n \in \mathbb{C}^{T\times R}$ contain the nth time slice of the FIR MIMO channels.

The noise processes $\mathbf{v}_\mathrm{s}[n] \in \mathbb{C}^R$ and $\mathbf{v}_\mathrm{t}[n] \in \mathbb{C}^T$ are zero mean, circularly symmetric, additive white Gaussian noise (AWGN) sequences with covariance matrices $\mathscr{E}\{\mathbf{v}_\mathrm{s}[n]\mathbf{v}_\mathrm{s}^\mathrm{H}[n]\} = \sigma_{v_\mathrm{s}}^2 \mathbf{I}_R$ and $\mathscr{E}\{\mathbf{v}_\mathrm{t}[n]\mathbf{v}_\mathrm{t}^\mathrm{H}[n]\} = \sigma_{v_\mathrm{t}}^2 \mathbf{I}_T$.

We assume that the information regarding the channel transfer functions and the noise covariance matrices is available across the communications link.

2.2 First Stage Transmission

The complex symbols transmitted from the source antennas are organised in a column vector $\mathbf{s}[n] \in \mathbb{C}^T$, and are uncorrelated across the antennas with zero mean and covariance $\mathscr{E}\{\mathbf{s}[n]\mathbf{s}^\mathrm{H}[n]\} = \sigma_\mathrm{s}^2 \mathbf{I}_T$. Prior to transmission, the T data streams are de-multiplexed into TM sub-streams $\tilde{\mathbf{s}}[n] \in \mathbb{C}^{TM}$ which are described by

$$\tilde{\mathbf{s}}[n] = \begin{bmatrix} \mathbf{s}[nM] \\ \mathbf{s}[nM+1] \\ \vdots \\ \mathbf{s}[nM+M-1] \end{bmatrix} \quad (3)$$

The blocks $\tilde{\mathbf{s}}[n]$ are then processed by the precoding matrix $\tilde{\mathbf{B}}_0 \in \mathbb{C}^{TP\times TM}$ producing the series of output blocks $\tilde{\mathbf{x}}[n] \in \mathbb{C}^{TP}$ defined by

$$\tilde{\mathbf{x}}[n] = \begin{bmatrix} \mathbf{x}[nP] \\ \mathbf{x}[nP+1] \\ \vdots \\ \mathbf{x}[nP+P-1] \end{bmatrix} \quad (4)$$

These blocks are multiplexed back to T data streams and transmitted across the channel $\mathbf{C}_\mathrm{s}(z)$ resulting in the received signals $\mathbf{y}[n] \in \mathbb{C}^R$ at the relays being

$$\mathbf{y}[n] = \sum_{l=0}^{L} \mathbf{C}_\mathrm{s}[l]\mathbf{x}[n-l] + \mathbf{v}_\mathrm{s}[n] \quad . \quad (5)$$

The data received data at the relays is demultiplexed into the RP sub-streams $\tilde{\mathbf{y}}[n] \in \mathbb{C}^{RP}$ prior to processing. This processing can be such that a precoder and equaliser pair $(\mathbf{B}_0, \mathbf{W}_\mathrm{s})$ is designed based on $\mathbf{C}_\mathrm{s}[m]$ [10]. A second pair $(\mathbf{B}_\mathrm{t}, \mathbf{W}_0)$ would be calculated based on $\mathbf{C}_\mathrm{t}[m]$ and operated back-to-back either with or without intermittent symbol detection, as indicated by switch positions (1) and (2) in Fig. 1. For the proposed system with switch position (3) in Fig. 1, the relay layer applies a linear processor $\tilde{\mathbf{F}}_0 \in \mathbb{C}^{RP\times RP}$.

2.3 Second Stage Transmission

The blocked data $\tilde{\mathbf{u}}[n]$ at the output of the relay layer is multiplexed into R streams and transmitted over the second stage channel $\mathbf{C}_\mathrm{t}(z)$ such that the received signals at the destination antennas $\mathbf{r}[n] \in \mathbb{C}^T$ can be described by

$$\mathbf{r}[n] = \sum_{l=0}^{L} \mathbf{C}_\mathrm{t}[l]\mathbf{u}[n-l] \ + \ \mathbf{v}_\mathrm{t}[n] \quad . \tag{6}$$

The equaliser matrix $\tilde{\mathbf{W}}_0 \in \mathbb{C}^{TM\times TP}$ processes the received blocks $\tilde{\mathbf{r}}[n] \in \mathbb{C}^{TP}$ giving output blocks $\tilde{\mathbf{z}}[n] \in \mathbb{C}^{TM}$, where $\tilde{\mathbf{r}}[n]$ and $\tilde{\mathbf{z}}[n]$ are defined similarly to (4) and (3), respectively. Finally the output of the equaliser is multiplexed into T streams reconstructing an estimate of the input signal $\mathbf{s}[n]$.

2.4 Inter-Block Interference

With the multiplexing of the transmit and receive signals, and the selection of $P = M + L$, the effective channel between the demultiplexed blocks $\tilde{\mathbf{x}}[n]$ and $\tilde{\mathbf{y}}[n]$ results in a first order MIMO channel with transfer function [10]

$$\tilde{\mathbf{C}}_\mathrm{s}(z) = \tilde{\mathbf{C}}_{\mathrm{s},0} + \tilde{\mathbf{C}}_{\mathrm{s},1}z^{-1} \tag{7}$$

with matrix coefficients

$$\tilde{\mathbf{C}}_{\mathrm{s},0} = \begin{bmatrix} \mathbf{C}_\mathrm{s}[0] & & & \mathbf{0} \\ \vdots & \ddots & & \\ \mathbf{C}_\mathrm{s}[L] & & \mathbf{C}_\mathrm{s}[0] & \\ & \ddots & & \ddots \\ \mathbf{0} & & \mathbf{C}_\mathrm{s}[L] \ \ \dots & \mathbf{C}_\mathrm{s}[0] \end{bmatrix} \tag{8}$$

$$\tilde{\mathbf{C}}_{\mathrm{s},1} = \begin{bmatrix} & \mathbf{C}_\mathrm{s}[L] & \dots & \mathbf{C}_\mathrm{s}[1] \\ & & \ddots & \vdots \\ & & & \mathbf{C}_\mathrm{s}[L] \\ & & & \\ \mathbf{0} & & & \end{bmatrix} \quad . \tag{9}$$

This leads to the following input/output descriptions for the channels in the two transmission stages,

$$\tilde{\mathbf{y}}[n] = \tilde{\mathbf{C}}_{\mathrm{s},0}\tilde{\mathbf{x}}[n] + \tilde{\mathbf{C}}_{\mathrm{s},1}\tilde{\mathbf{x}}[n-1] + \tilde{\mathbf{v}}_\mathrm{s}[n] \tag{10}$$

$$\tilde{\mathbf{r}}[n] = \tilde{\mathbf{C}}_{\mathrm{t},0}\tilde{\mathbf{u}}[n] + \tilde{\mathbf{C}}_{\mathrm{t},1}\tilde{\mathbf{u}}[n-1] + \tilde{\mathbf{v}}_\mathrm{t}[n] \quad . \tag{11}$$

The interference by previously transmitted data blocks $\tilde{\mathbf{x}}[n-1]$ and $\tilde{\mathbf{u}}[n-1]$ in (10) and (11) is referred to inter-block interference (IBI), which needs to be suppressed [10].

Due to the different dimensions of the channels in the two transmit stages, the first RL samples of $\tilde{\mathbf{y}}[n]$ and TL samples of $\tilde{\mathbf{r}}[n]$ will suffer from IBI.

2.5 IBI-Free System Structure

In [8, 10] the authors propose two methods for the elimination of IBI. The first approach is to append the transmit blocks with sufficient zeros such that the upper right triangle in (9), and hence IBI is suppressed. This is termed the trailing zero (TZ) method. The second, known as the leading zero (LZ) technique, is to simply discard the symbols at the receiver that suffer the interference. The structure of the pre-coders and equaliser are dependant on what approach is used for each channel. For brevity we shall only consider one example, specifically, the TZ method for the first channel and the LZ for the second channel. The structure of our pre-coders and equaliser are then of the following forms

$$\tilde{\mathbf{B}}_0 = \begin{bmatrix} \tilde{\mathbf{B}} \\ \mathbf{0}_{TL\times TM} \end{bmatrix} \quad ,\tilde{\mathbf{B}} \in \mathbb{C}^{TM\times TM} \tag{12}$$

$$\tilde{\mathbf{F}}_0 = \tilde{\mathbf{F}} \quad \in \mathbb{C}^{RP\times RP} \tag{13}$$

$$\tilde{\mathbf{W}}_0 = \begin{bmatrix} \mathbf{0}_{TM\times TL} & \tilde{\mathbf{W}} \end{bmatrix} \qquad \tilde{\mathbf{W}} \in \mathbb{C}^{TM\times TM} \tag{14}$$

The block FIR response of the complete system can now be written as

$$\tilde{\mathbf{z}}[n] = \tilde{\mathbf{W}}\tilde{\mathbf{H}}_\mathrm{t}\tilde{\mathbf{F}}\tilde{\mathbf{H}}_\mathrm{s}\tilde{\mathbf{B}}\tilde{\mathbf{s}}[n] + \tilde{\mathbf{W}}\tilde{\mathbf{H}}_\mathrm{t}\tilde{\mathbf{F}}\tilde{\mathbf{v}}_\mathrm{s}[n] + \tilde{\mathbf{W}}\tilde{\mathbf{v}}_\mathrm{t}[n] \tag{15}$$

with $\tilde{\mathbf{H}}_\mathrm{s} \in \mathbb{C}^{RP\times TM}$ containing the first TM columns of $\tilde{\mathbf{C}}_{\mathrm{s},0}$, and $\tilde{\mathbf{H}}_\mathrm{t} \in \mathbb{C}^{TM\times RP}$ the last TM rows of $\tilde{\mathbf{C}}_{\mathrm{t},0}$. Note that (15) is free of IBI, which permits the use of linear algebraic tools to optimise the overall system with respect of $\tilde{\mathbf{B}}$, $\tilde{\mathbf{F}}$, and $\tilde{\mathbf{W}}$ in a suitable sense.

3. PRECODER AND EQUALISER DESIGN

Having established the signal model for the system, the goal is now to design the precoders $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{F}}$ as well as the equaliser $\tilde{\mathbf{W}}$ according to a suitable design criterion. For simplicity we assume $\tilde{\mathbf{B}} = \mathbf{I}_{TM}$ such that the operation of $\tilde{\mathbf{B}}_0$ is simply to append the required zeros to the tails of the transmit blocks $\tilde{\mathbf{s}}[n]$. We note that this assumption also eliminates the need for the source antennas to possess any CSI other than knowledge of the channel order L.

3.1 MSE Formulation

In [11] the authors present a design for a narrowband MIMO relaying system where the relaying pre-coder and equaliser are jointly optimised to minimise the trace of the mean square error (MSE) matrix i.e. minimising the sum of the MSE of each data stream. Due to our utilisation of block transmission we can follow the steps in [11] to design our precoder $\tilde{\mathbf{F}}$ and equaliser $\tilde{\mathbf{W}}$.

The error signal between the transmitted and received blocks,

$$\tilde{\mathbf{e}}[n] = \tilde{\mathbf{s}}[n] - \tilde{\mathbf{z}}[n] \tag{16}$$

is used to construct the MSE cost function

$$\xi(\tilde{\mathbf{F}}, \tilde{\mathbf{W}}) = \mathrm{tr}\{\mathscr{E}\{\tilde{\mathbf{e}}[n]\tilde{\mathbf{e}}^\mathrm{H}[n]\}\} \tag{17}$$

resulting in

$$\xi(\tilde{\mathbf{F}}, \tilde{\mathbf{W}}) = \mathrm{tr}\{(\tilde{\mathbf{W}}\tilde{\mathbf{H}} - \mathbf{I}_{TM})\tilde{\mathbf{R}}_{ss}(\tilde{\mathbf{W}}\tilde{\mathbf{H}} - \mathbf{I}_{TM})^\mathrm{H}\} + \\ + \mathrm{tr}\{\tilde{\mathbf{W}}\tilde{\mathbf{R}}_{vv}\tilde{\mathbf{W}}^\mathrm{H}\} \tag{18}$$

where

$$\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_\mathrm{t}\tilde{\mathbf{F}}\tilde{\mathbf{H}}_\mathrm{s} \tag{19}$$

can be viewed as the compound MIMO channel, $\tilde{\mathbf{R}}_{\mathrm{ss}} = \sigma_s^2\mathbf{I}_{TM}$ is the covariance of the transmitted data blocks, and

$$\tilde{\mathbf{R}}_{vv} = \tilde{\mathbf{H}}_\mathrm{t}\tilde{\mathbf{F}}\tilde{\mathbf{R}}_{v_\mathrm{s}v_\mathrm{s}}\tilde{\mathbf{F}}^\mathrm{H}\tilde{\mathbf{H}}_\mathrm{t}^\mathrm{H} + \tilde{\mathbf{R}}_{v_\mathrm{t}v_\mathrm{t}} \tag{20}$$

is the effective covariance of all noise terms at the equaliser input with $\tilde{\mathbf{R}}_{v_\mathrm{s}v_\mathrm{s}} = \sigma_{v_\mathrm{s}}^2\mathbf{I}_{RP}$ and $\tilde{\mathbf{R}}_{v_\mathrm{t}v_\mathrm{t}} = \sigma_{v_\mathrm{t}}^2\mathbf{I}_{TM}$ the covariance matrices of the demultiplexed noise vectors in the relay stage and the receiver, respectively.

3.2 Constrained Optimsation Problem

As well as aiming to minimise (18), limited power requires to introduce constraints on the transmit power of the relays, which can be expressed as

$$P(\tilde{\mathbf{F}}) = \mathrm{tr}\left\{\tilde{\mathbf{F}}\left(\tilde{\mathbf{H}}_{\mathrm{t}}\tilde{\mathbf{R}}_{\mathrm{ss}}\tilde{\mathbf{H}}_{\mathrm{t}}^{\mathrm{H}} + \tilde{\mathbf{R}}_{v_{\mathrm{s}}v_{\mathrm{s}}}\right)\tilde{\mathbf{F}}^{\mathrm{H}}\right\} \leq P_0 \tag{21}$$

where P_0 is the available power budget.

With the limited transmit power of the relays the constrained design problem can be formulated as

$$\arg\min_{\tilde{\mathbf{F}},\tilde{\mathbf{W}}} \xi(\tilde{\mathbf{F}},\tilde{\mathbf{W}}) \qquad \text{subject to} \quad P(\tilde{\mathbf{F}}) \leq P_0 \,. \tag{22}$$

3.3 MMSE Equaliser

For any given relay processor matrix $\tilde{\mathbf{F}}$, the optimum MSE equaliser $\tilde{\mathbf{W}}_{\mathrm{opt}}$ is provided by the Wiener solution [13, 11]. The Wiener solution can be obtained by differentiating (18) w.r.t. $\tilde{\mathbf{W}}$, and solving for $\tilde{\mathbf{W}}$ by setting the gradient to zero, leading to

$$\tilde{\mathbf{W}}_{\mathrm{opt}} = \tilde{\mathbf{R}}_{\mathrm{ss}}\tilde{\mathbf{H}}^{\mathrm{H}}\left(\tilde{\mathbf{H}}\tilde{\mathbf{R}}_{\mathrm{ss}}\tilde{\mathbf{H}}^{\mathrm{H}} + \tilde{\mathbf{R}}_{vv}\right)^{-1} \quad . \tag{23}$$

Subsituting (23) into (18) yields

$$\xi(\tilde{\mathbf{F}},\tilde{\mathbf{W}}_{\mathrm{opt}}) = \mathrm{tr}\left\{\tilde{\mathbf{R}}_{\mathrm{ss}}\left(\mathbf{I}_{TM} - \tilde{\mathbf{H}}^{\mathrm{H}}\left(\tilde{\mathbf{H}}\tilde{\mathbf{R}}_{\mathrm{ss}}\tilde{\mathbf{H}}^{\mathrm{H}} + \tilde{\mathbf{R}}_{\mathrm{vv}}\right)^{-1} \cdot \cdot \tilde{\mathbf{H}}\tilde{\mathbf{R}}_{\mathrm{ss}}\right)\right\} \quad , \tag{24}$$

which can be simplified to

$$\xi(\tilde{\mathbf{F}},\tilde{\mathbf{W}}_{\mathrm{opt}}) = \mathrm{tr}\left\{\tilde{\mathbf{R}}_{\mathrm{ss}}\left(\mathbf{I}_{TM} + \tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{R}}_{\mathrm{vv}}^{-1}\tilde{\mathbf{H}}\tilde{\mathbf{R}}_{\mathrm{ss}}\right)^{-1}\right\} \tag{25}$$

using the matrix inversion lemma [14].

3.4 Optimum Relay Processor

The optimisation problem in (22) has now been reduced to finding the matrix $\tilde{\mathbf{F}}$ that minimises (25) whilst abiding by the constraint in (21). This can be achieved by calculating the singular value decomposition (SVD) of the channel matrices $\tilde{\mathbf{H}}_{\mathrm{s}}$ and $\tilde{\mathbf{H}}_{\mathrm{t}}$,

$$\tilde{\mathbf{H}}_{\mathrm{s}} = \left[\mathbf{U}_{\mathrm{s}} \;\; \mathbf{U}_{\mathrm{s}}^{\perp}\right] \cdot \begin{bmatrix} \Lambda_{\mathrm{s}} \\ \mathbf{0} \end{bmatrix} \cdot \mathbf{V}_{\mathrm{s}} \tag{26}$$

$$\tilde{\mathbf{H}}_{\mathrm{t}} = \mathbf{U}_{\mathrm{t}} \cdot [\Lambda_{\mathrm{t}} \;\; \mathbf{0}] \cdot \left[\mathbf{V}_{\mathrm{t}} \;\; \mathbf{V}_{\mathrm{t}}^{\perp}\right]^{\mathrm{H}} \quad . \tag{27}$$

The relay precoding matrix is constructed as $\tilde{\mathbf{F}} = \mathbf{V}_{\mathrm{t}}\Psi\mathbf{U}_{\mathrm{s}}^{\mathrm{H}}$ with $\Psi \in \mathbb{R}^{TM\times TM}$ a diagonal matrix that loads power across the channels' singular vectors in $\mathbf{V}_{\mathrm{t}}$ and $\mathbf{U}_{\mathrm{s}}$. This results in the MIMO system being decoupled into a set of parallel single-input single-output (SISO) subchannels.

Substituting $\tilde{\mathbf{F}} = \mathbf{V}_{\mathrm{t}}\Psi\mathbf{U}_{\mathrm{s}}^{\mathrm{H}}$ into (25) and (21) results in (22) being simplified to

$$\arg\min_{\Psi}\left(\sigma_{\mathrm{s}}^2 \sum_{i=1}^{TM} \frac{|\Psi_i|^2\Lambda_{\mathrm{t},i}^2\sigma_{v_{\mathrm{s}}}^2 + \sigma_{v_{\mathrm{t}}}^2}{|\Psi_i|^2\Lambda_{\mathrm{t},i}^2\left(\Lambda_{\mathrm{s},i}^2\sigma_{\mathrm{s}}^2 + \sigma_{v_{\mathrm{s}}}^2\right) + \sigma_{v_{\mathrm{t}}}^2}\right) \tag{28}$$

subject to

$$\sum_{i=1}^{TM} |\Psi_i|^2\left(\Lambda_{\mathrm{s},i}^2\sigma_{\mathrm{s}}^2 + \sigma_{v_{\mathrm{s}}}^2\right) = P_0 \quad . \tag{29}$$

The solution to this optimisation problem can be obtained through the use of Karush-Kuhn-Tucker conditions of optimality [12], resulting in

$$|\Psi_i|^2 = \frac{1}{\Lambda_{\mathrm{t},i}^2\left(\Lambda_{\mathrm{s},i}^2\sigma_{\mathrm{s}}^2 + \sigma_{v_{\mathrm{s}}}^2\right)} \cdot \cdot \left(\sqrt{\frac{\Lambda_{\mathrm{s},i}^2\Lambda_{\mathrm{t},i}^2\sigma_{\mathrm{s}}^2\sigma_{v_{\mathrm{t}}}^2}{\mu\left(\Lambda_{\mathrm{s},i}^2\sigma_{\mathrm{s}}^2 + \sigma_{v_{\mathrm{s}}}^2\right)}} - \sigma_{v_{\mathrm{t}}}^2\right)^+ \tag{30}$$

whereby

$$(x)^+ = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases} \tag{31}$$

The optimisation of the precoder is therefore closely related to the waterfilling solution, whereby power levels $|\Psi_i|^2$ are determined to satisfy the constraint imposed by the power budget P_0. The variable μ needs to be identified, akin to the water level in waterfilling problems [15].

4. SIMULATIONS AND RESULTS

The performance of the proposed system design as shown in Fig. 1 with switch in position (3) is evaluated in simulations over a number channels. The simulated channel and system parameters are discussed prior to presenting a number of simulation results.

4.1 Channel Model and System Parameters

The simulations assumed a configuration with $T = R = 4$ antennas at the transmitter, the relay layer, and the destination. The two resulting 4×4 MIMO channels are of order $L = 4$, containing independently identically distributed coefficients drawn from complex Gaussian distributions with zero mean and unit variance. Any results show and ensemble average over 100 such channel realisations.

For IBI eliminations, a relatively small block size of $M = 5$ for the sake of simulation time results $P = M + L = 9$ for the block-based transceiver systems. This introduces $1 - \frac{M}{P} = 44\%$ redundancy into the transmission. With a transmit power of $TM\sigma_{\mathrm{s}}^2 = 20$ per block at the source, the power constraint at the relay layer has also been set to $P_0 = 20$. This results in the signal to noise ratios (SNRs) of the source-to-relay and relay-to-destination channels

$$\mathrm{SNR}_1 = \frac{T\sigma_{\mathrm{s}}^2}{R\sigma_{v_{\mathrm{s}}}^2} \quad , \tag{32}$$

$$\mathrm{SNR}_2 = \frac{P_0}{T\sigma_{v_{\mathrm{t}}}^2} \quad . \tag{33}$$

When assessing BER over SNR in simulations, we set $\mathrm{SNR} = \mathrm{SNR}_1 = \mathrm{SNR}_2$.

Both the proposed method and the benchmark design [10] are operating with identical power budgets and redundancies for IBI cancellation.

4.2 Results

Fig. 2 shows the BER results when transmitting QPSK symbols across the relayed communications link, using the proposed method as well as the benckmark back-to-back linear

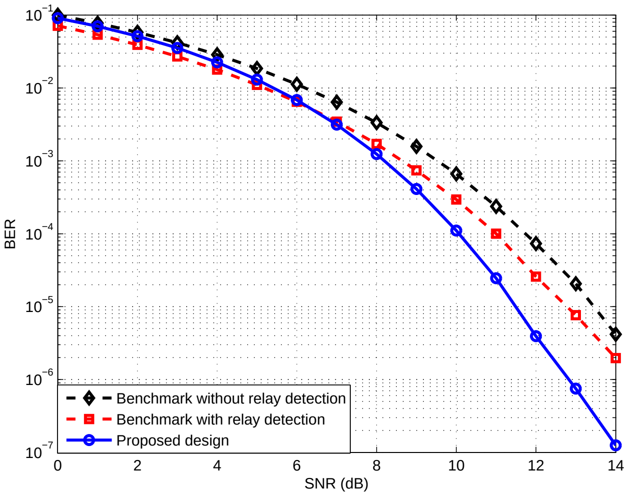

Figure 2: BER performance of the proposed designed compared to benchmark systems with and without detection in the relay layer.

MMSE system with and without symbol detection in the relay layer.

For the benchmark design, the results in Fig. 2 clearly indicate that detection in the relay layer provides superior performance. This is intuitively clear, as the noise is distributed across both relay layers, and an intermediate decision can substantially reduce the noise power in the receiver provided that decisions are correct.

From an SNR of about 6dB upwards, the proposed design outperforms the benchmark system with decisions in the relay layer. This comes despite the fact that the benchmark design performs precoding at the source, while the proposed system only introduces redundancy at the transmitter in order to mitigate IBI, but performs no beamsteering. At SNR values of about 13dB, the proposed system is an order of magnitude below the BER of the benchmark design.

5. CONCLUSIONS

In this paper we have presented a method for the design of jointly MMSE-optimal block transmission for a relayed communications link over two frequency-selective MIMO channels. The method combines system design ideas for narrowband MIMO systems with a block-processing approach that first eliminates inter-block interference by guard intervals, and then relies in linear algebraic techniques to optimise the remaining system.

At present, the processing effort of the proposed methods at the source is minimal and restricted to cater for the elimination of inter-block interference, while linear processing is admitted in the relay layer and at the destination. This system outperforms a back-to-back MMSE-optimal system with beamsteering, and still outperforms for most SNR values the same system when a non-linear for detection is employed in the relay layer.

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