ESTIMATION OF N-MODE RANKS OF HYPERSPECTRAL IMAGES FOR TENSOR DENOISING

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ABSTRACT

This paper deals with n-mode subspaces in tensor based denoising. Actually, the main issue of tensor signal processing is the estimation of n-mode ranks since a subspace based approach is considered. In hyperspectral images, an efficient denoising method could allow more accurate results for classification or unmixing. In this paper, we propose to extend subspace identification methods to tensors for n-mode rank estimation. The estimation of endmembers in hyperspectral images is equivalent to estimate the 3-mode rank of a tensor. HySime and Neyman-Pearson detection theory-based thresholding method (HFC) are practical benchmarks. Therefore, we adopt tensor formalism to extend reference algorithms to determine n-mode ranks of tensors. We compare different adapted criteria both on simulated and real data.

1. INTRODUCTION

Image restoration aims at estimating the original image from a noisy observation. There are many application fields for image restoration [1]. It has also been a topic of huge interest during the past few decades. As one increasingly has to work with multidimensional data, *e.g* medical or hyperspectral images, restoration filtering methods have to be modified accordingly. In this paper, we consider color or hyperspectral images as multidimensional arrays [2, 3], where there are two indexes for spatial localization and one index for spectral channel which can consist of about 200 bands.

Usually, noise removal techniques are based on the processing of each band separately. But this may lead to loss of information since correlation between bands are not considered. Hybrid filters have then be introduced [4]. Minimum noise fraction have also been proposed in the case of remotely sensed images [5], choosing a transformation of data that maximizes the signal to noise ratio (SNR) instead of the variance. But the information used is only spectral, and it does not consider spatial information.

Recent works [2] have proposed a tensor approach to noise removal in multidimensional data sets. This permits to both consider spatial and spectral information with a subspace based approach.

Therefore, an important issue in tensor filtering is the estimation of n-mode ranks, which correspond to n-mode signal subspace dimensions. For such purpose, an extension to tensor of information criteria were proposed in [2, 6]. But this extension does not take care of hyperspectral imagery specificities. Indeed, spectral bands are highly correlated since they correspond to the same observed scene.

In hyperspectral imagery, it is often interesting to determine the different materials of a scene (*i.e.* endmembers). Therefore, several works are focused on the estimation of the number of endmembers. In the state of the art, Neyman-Pearson detection theory-based thresholding method [7], namely HFC and NWHFC enable to estimate the virtual dimensionality of a hyperspectral image. An hyperspectral signal subspace identification by minimum error (HySime) has also been introduced in [8]. In tensor notations, this is equivalent to estimate the 3-mode rank. In this paper we extend these criterion to determine every n-mode rank of the data set. We also propose a comparison in terms of denoising, depending on the chose criterion.

The paper is organized as follows: section 2 introduces the tensor model for hyperspectral images. Section 3 presents some methods to reduce noise in hyperspectral images. In Section 4, we propose several methods to determine n-mode ranks of tensors. A comparison between these methods is drawn in section 5. Last section concludes the paper.

2. SOME MULTILINEAR ALGEBRA TOOLS

2.1 Multiplication of a tensor by a matrix

In this paper we are concerned with TUCKER3 decomposition, that is, a N^{th} order tensor $\mathscr{A}$ can be decomposed as several products between a core tensor $\mathscr{G}$ and a set of matrices $\mathbf{U}^{(n)}, n = 1, \ldots, N$:

$$\mathscr{A} = \mathscr{G} \times_1 \mathbf{U}^{(1)} \times_2 \ldots \times_N \mathbf{U}^{(N)}, \tag{1}$$

where $\times_n$ denotes the n-mode product operator [9], whose entries are :

$$\left(\mathscr{G} \times_n \mathbf{U}^{(n)}\right)_{i_1 \ldots i_{n-1} j_n in+1 \ldots i_N} = \sum_{i_n=1}^{I_n} g_{i_1 i_2 \ldots i_{n-1} i_n i_{n+1} \ldots i_N} u_{j_n i_n}. \tag{2}$$

2.2 Flattening matrices of a tensor

In tensor processing, flattening is a common tool. It permits to reshape the tensor into a matrix, leading to N flattened matrices. Each flattening matrix is obtained choosing a specific direction of the multiarray. This correponds so slice the tensor and stack each piece in a specific order. The n-mode flattening matrix $\mathbf{A}_n$ of a tensor $\mathscr{A} \in \mathbb{R}^{I_1 \times \ldots \times I_N}$ is defined as a matrix [9] from $\mathbb{R}^{I_n \times M_n}$ where :

$$M_n = I_1 \ldots I_{n-1} I_{n+1} \ldots I_N. \tag{3}$$

3. NOISE REDUCTION IN HYPERSPECTRAL IMAGES

3.1 Problem formulation

A multidimensional data tensor $\mathscr{R}$ can be decomposed into an impaired signal $\mathscr{S}$ by an additive noise $\mathscr{N}$:

$$\mathscr{R} = \mathscr{S} + \mathscr{N} \tag{4}$$

Assuming that $\mathscr{S}$ and $\mathscr{N}$ are N^{th} order tensors of size $I_1 \times \ldots \times I_N$, the multidimensional filtering of the data set $\mathscr{R}$ is :

$$\hat{\mathscr{S}} = \mathscr{R} \times_1 \mathbf{H}^{(1)} \times_2 \ldots \times_N \mathbf{H}^{(N)}, \tag{5}$$

where matrix $\mathbf{H}^{(n)}$ is referred to as n-mode filter.

3.2 Noise Estimation

In hyperspectral images, high correlations are present along the third mode since the same scene has been shot at several wavelengths. In [8], a method based on these correlations is proposed. Therefore, flattening the data tensor $\mathscr{R}$ in its third mode leads to matrix $\mathbf{R}_3$ of size $I_3 \times M_3$. That is, each column represents a spectrum. We assume that each band of the image (*i.e.* each line of $\mathbf{R}_3$) can be written as a linear combination of the $I_3 - 1$ others. Denoting $\mathbf{R}_{3/i}$ the matrix $\mathbf{R}_3$ but the i^{th} band, we can write :

$$\mathbf{r}_i = \mathbf{R}_{3/i}\beta_i + \xi_i \tag{6}$$

β_i is the regression vector which least square estimate is given by $\hat{\beta}_i = \left(\mathbf{R}_{3/i}\mathbf{R}_{3/i}^T\right)^{-1}\mathbf{R}_{3/i}\mathbf{r}_i$. Then, the noise can be expressed as : $\hat{\xi}_i = \mathbf{r}_i - \mathbf{R}_{3/i}\hat{\beta}_i$.

3.3 Multidimensional Wiener Filtering

Tensorial approaches have been shown to overcome traditional channel-by-channel filtering [10, 2]. Some methods are based on the approximation of tensors, such as HOSVD-(K_1, K_2, K_3) or LRTA-(K_1, K_2, K_3). Although they aim at estimating the signal from the observation data set $\mathscr{R}$, the criterion minimized is the mean squared error $\|\mathscr{R} - \hat{\mathscr{S}}\|$ which is not optimal.

A method consists in extending Wiener filter to multidimensional data [10]. In that case, the mean squared error is given by :

$$\text{MSE} = \text{E}\left[\left\|\mathscr{S} - \hat{\mathscr{S}}\right\|^2\right] \tag{7}$$

The minimization of (7) leads to the expression of the n-mode filters : $\mathbf{H}^{(n)} = \mathbf{V}_s^{(n)}\Lambda^{(n)}\mathbf{V}_s^{(n)^T}$, where $\mathbf{V}_s^{(n)}$ are the left singular vectors of n-mode covariance matrix $\text{E}\left[\mathbf{S}_n\mathbf{S}_n^T\right]$ and $\Lambda^{(n)}$ is a diagonal weight matrix containing a combination of the eigenvalues of noise, signal and data n-mode covariance matrices. These n-mode filters are obtained through an iterative alternating least squares algorithm.

Note that the multidimensional Wiener filter needs the n-mode ranks to determine the eigenvalues corresponding to signal or noise.

4. ESTIMATION OF N-MODE RANKS

n-mode signal subspace is an important issue. A common assumption is that signal and noise subspaces are orthogonal. The dimensions of the n-mode signal subspaces need to be estimated, since they are not known in practice. In hyperspectral images, the estimation of 3-mode signal subspace has been widely studied since it corresponds to the estimation of the endmember number in the image. It is often called virtual dimensionality (VD) [7]. But for tensor denoising methods, each n-mode rank has to be estimated.

4.1 Information Criteria

In [2, 6], an extension of Akaike criterion (AIC) or minimum description length (MDL) [11, 12] is used to obtain n-mode ranks :

$$\begin{aligned} AIC(k_n) &= -2\log\left(\frac{\prod_{j=k_n+1}^{I_n}(\beta_i^{(n)})^{\frac{1}{I_n-k_n}}}{\frac{1}{I_n-k_n}\sum_{j=k_n+1}^{I_n}\beta_i^{(n)}}\right)^{(I_n-k_n)M_n} \\ &\quad +2k_n(2I_n-k_n) \end{aligned} \tag{8}$$

where $\beta_i^{(n)}$, $i = 1$ to I_n are the I_n singular values of the covariance matrix $\text{E}[\mathbf{R}_n\mathbf{R}_n^T]$ of the n-mode flattening matrix $\mathbf{R}_n$ of data tensor $\mathscr{R}$, with $\beta_1^{(n)} \geq \ldots \geq \beta_{I_n}^{(n)}$. M_n is the number of columns of $\mathbf{R}_n$ (3). The estimated n-mode rank K_n is the value of k_n which minimizes AIC criterion.

$$\begin{aligned} MDL(k_n) &= -\log\left(\frac{\prod_{j=k_n+1}^{I_n}(\beta_i^{(n)})^{\frac{1}{I_n-k_n}}}{\frac{1}{I_n-k_n}\sum_{j=k_n+1}^{I_n}\beta_i^{(n)}}\right)^{(I_n-k_n)M_n} \\ &\quad +\tfrac{1}{2}k_n(2I_n-k_n)\log(M_n) \end{aligned} \tag{9}$$

4.2 Neyman-Pearson detectors

Some methods based on Neyman-Pearson theory are presented in [7] to estimate the number of endmembers in hyperspectral images. HFC, NWHFC and NSP are Neyman-Pearson detectors and are commonly used in hyperspectral signal subspace identification. They are shown to overcome information criteria AIC and MDL [7]. In our experiments, we propose to estimate n-mode signal subspaces of both simulated and real hyperspectral data The main overcoming of this method is the false alarm probability P_f that has to be fixed arbitrarily.

4.3 Extended HySime

In [8] propose another method called HySime to estimate the number of endmembers in hyperspectral data (*i.e.* the 3-mode rank). This method can be extended to n-mode ranks of tensors. HySime propose to minimize the mean squared error between the 3-mode signal flattening matrix $\mathbf{S}_3$ and the projection on the signal subspace of the 3-mode data flattening matrix : $\mathbf{U}_k^{(n)}\mathbf{U}_k^{(n)^T}\mathbf{R}_3$.

Let us denote the projector on the n-mode signal subspace as :

$$\mathbf{P}_{k_n}^{(n)} = \mathbf{U}_{k_n}^{(n)}\mathbf{U}_{k_n}^{(n)^T} \tag{10}$$

where subscript k_n stands for the k_n first eigenvectors kept from $\mathbf{U}^{(n)}$.

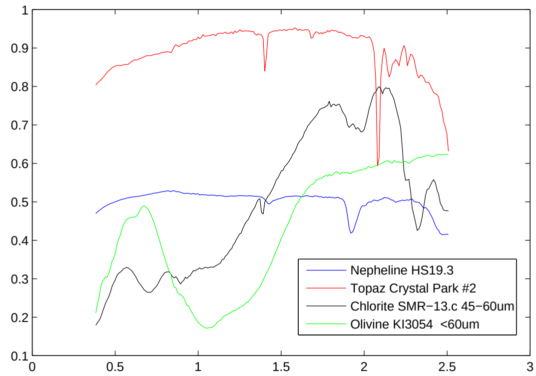

Figure 1: Spectra of five materials of the USGS digital spectral library.

We can extend HySime method in each mode, minimizing :

$$mse(k_n) = \mathrm{E}\left[\|\mathbf{S}_n - \mathbf{P}^{(n)}_{k_n}\mathbf{R}_n\|^2\right] \tag{11}$$

It can be shown, following [8], that it is equivalent to find all the negative component of the vector defined by:

$$\delta_i = \sum_{j=1}^{k_n} -\mathbf{u}^{(n)}_{i_j}\frac{1}{M_n}\mathbf{R}_n\mathbf{R}_n^T\mathbf{u}^{(n)}_{i_j} + 2\mathbf{u}^{(n)}_{i_j}\frac{1}{M_n}\mathbf{N}_n\mathbf{N}_n^T\mathbf{u}^{(n)}_{i_j} \tag{12}$$

with $i = 1, \ldots, I_n$.

5. EXPERIMENTAL RESULTS

In this section, we propose to compare the interest of above methods to determine n-mode ranks. These n-mode ranks are involved in multidimensional Wiener filtering. We show that the extention of HySime algorithm for n-mode ranks estimation leads to improve tensor filtering.

In the following experiments, the signal-to-noise ratio is defined as:

$$SNR = 10 \cdot \log \frac{\|\mathscr{S}\|^2}{\|\mathscr{N}\|^2}. \tag{13}$$

Before presenting some results, we introduce a quality criterion to quantify *a posteriori* the quality of the estimation :

$$QC(\hat{\mathscr{S}}) = 10 \cdot \log\left(\frac{\|\mathscr{S}\|^2}{\|\hat{\mathscr{S}} - \mathscr{S}\|^2}\right). \tag{14}$$

5.1 Results on simulated data

We generated a hyperspectral image from the USGS digital spectral library [13]. We selected 4 spectra from the database (fig 1) and generated a background with the first spectrum and 3 targets with the others (fig 2).

We add a Gaussian white noise to this synthetic image. Table 1 gives the estimation of n-mode ranks for several values of SNR. This table emphasizes that HFC, NWHFC and HySime outperforms the information criteria AIC and MDL since they take into account specificities of hyperspectral imagery. Notice that the improvement concerns the 3-mode rank which corresponds to virtual dimensionality in other studies.

Including the n-mode ranks estimation of table 1 in tensor filtering, we show that the use of HySime leads to a significant improvement of the reconstruction (see table 2).

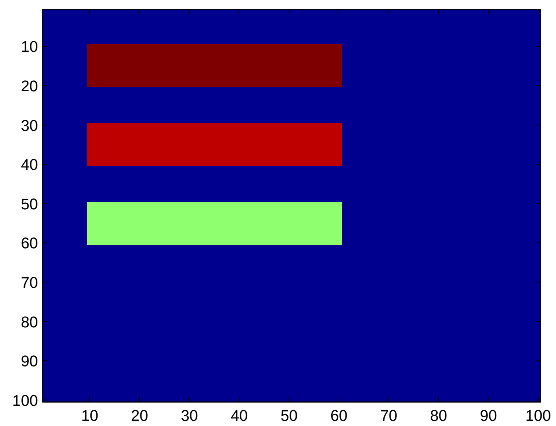

Figure 2: Synthetic images generated from the spectra given in Fig. 1.

SNR	AIC	MDL	HFC	NWHFC	HySime
40 dB	(2,2,3)	(2,2,2)	(3,2,4)	(3,2,4)	(4,2,4)
30 dB	(2,2,3)	(2,2,2)	(3,2,4)	(3,2,4)	(4,2,4)
20 dB	(2,2,3)	(2,2,2)	(3,2,3)	(3,2,3)	(4,2,4)

Table 1: n-mode rank estimation using several methods. For methods HFC and NWHFC, we use the parameter $P_{fa} = 10^{-3}$.

SNR	20 db	30 dB	40 dB
QC (AIC)	30 dB	30 dB	30 dB
QC (MDL)	30 dB	30 dB	30 dB
QC (HFC)	37 dB	37 dB	37 dB
QC (NWHFC)	37 dB	37 dB	37 dB
QC (HySime)	50 dB	61 dB	71 dB

Table 2: Denoising results with multidimensional Wiener filter given the criterion used for the n-mode ranks estimation.

5.2 Real data

The following data have been obtained by HYDICE. HYDICE is an airborne sensor which collects post-processed data in 210 wavelengths: 0.4 - 2.5 μm.

Considering the scene depicted in Fig. 3, we propose to estimate its n-mode ranks.

First, we add a Gaussian noise to obtain varying SNR. We show that HySime fails to estimate 1-mode and 2-mode ranks but is good for 3-mode rank detection. Actually, the model used to identify subspaces in HySime does not fit with 1-mode and 2-mode flattened data since the spatial bands cannot be written as a linear combination of others. But for the 3-mode rank determination, the model fits. The results

SNR		15 dB	20 dB	25 dB	30 dB	35 dB
AIC	Estimated ranks	(46,36,10)	(66,65,12	(98,94,18)	(121,121,33)	(126,128,63)
	QC	23.5 dB	26 dB	29 dB	32 dB	36 dB
HySime	Estimated ranks	(19,19,6)	(37,33,8)	(60,57,11)	(88,84,13)	(112,108,15)
	QC	22 dB	25 dB	28 dB	32 dB	35 dB
AIC+HySime	Estimated ranks	(41,36,6)	(66,65,8)	(98,94,11)	(121,121,13)	(126,128,15)
	QC	25 dB	28.5 dB	32 dB	35 dB	37 dB

Table 3: Influence of *n*-mode ranks estimation on multidimensional Wiener filtering. Results were obtained on real data.

Figure 3: HYDICE image.

obtained are in agreement with the study [6, 2] which have shown that the spatial ranks have to contain most of information. Therefore, either 1-mode and 2-mode ranks have large values [6] or a rearrangement of data have to be done to reduce these ranks [2]. In this paper we keep large values given by AIC or MDL criterion for the spatial mode ranks.

In table 3 we don't give the results obtained with HFC/NWHFC since it requires a threshold that has to be arbitrarily fixed. However, we give the estimated *n*-mode ranks and the quality of the restoration (after multidimensional Wiener filtering). The table illustrate that the combination of extended HySime (for spectral rank) and extended AIC (for spatial ranks) permits to get a better reconstruction of the signal tensor.

6. CONCLUSION

In this paper, we have proposed to extend virtual dimension to determine *n*-mode ranks. Starting from hyperspectral data specificities, that is, the high correlation in 3-mode, we developed HySime expression in tensor notation. We have shown on simulated results that the extension of HySime to tensors permits to determine efficiently the 3-mode rank. However, the spatial ranks (1-mode and 2-mode) are not very well found. Actually, the property of high correlation is not true in spatial modes. Therefore, we have proposed to use the extended information criterion AIC for 1-mode and 2-mode ranks and HySime for 3-mode rank.

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