PERFORMANCE EVALUATION OF A SIGN-ERROR-BASED ITERATIVE PRECODER FOR CROSSTALK CANCELATION IN DSL SYSTEMS

J. Louveaux, L. Vandendorpe

Université catholique de Louvain, Louvain-la-Neuve, Belgium
jerome.louveaux,luc.vandendorpe@uclouvain.be

ABSTRACT

In this paper, we present some theoretical performance evaluation of the crosstalk precancelation method presented in [1] for DSL systems. This algorithm uses a limited feedback from the different receivers, returning only the (complex) sign of the error on the detected symbols. It updates the precoder iteratively based on this information and on the knowledge of the transmitted symbols. Very little simulation results had been presented in [1]. Here we first propose a slightly modified and simplified version of the algorithm which is significantly less complex. We derive approximate performance expressions for this modified version and compare them to simulation results, showing a very good match. It is then shown how these results can be used to set the different parameters of the method.

1. INTRODUCTION

Due to the use of higher bandwidths and shorter loops, the FEXT (far end crosstalk) is becoming the main degradation in some DSL systems such as VDSL2 (very high bit rate digital subscriber line). For this reason, a number of precancelation techniques have been designed to decrease the effect of FEXT [2, 3, 4] in downstream, using the coordination at the CO (central office) and assuming no coordination at the receiver side, the CPE (customer premise equipment, i.e. the user's equipment).

All these precancellation schemes rely on a good estimation of the crosstalk channels between the various lines, hence the issue of crosstalk channel estimation has to be solved to be able to use those schemes. The downstream channel estimation appears to be a much more complicate task than the upstream channel estimation, and it has received some attention in the literature recently. One straightforward way to solve the problem is to use a set of pilot symbols, sent periodically, to perform the tracking of the downstream channels at the CPE. An example of this solution, applied to the VDSL system is analyzed in [5]. In [6], it is proposed to simplify the precoder to its off-diagonal elements only, and an LMS tracking algorithm is proposed that converges to the optimal off-diagonal solution. This is also essentially a pilot-based solution. These methods use part of the useful bit rate as pilot symbols and, in addition, the information about the estimates needs to be sent back to the CO periodically to update the precoder. So this may lead to a large overhead. In order to try to limit this overhead, some methods have been proposed that only require to feedback the *sign* of the error samples (slicer errors) at the receiver [7, 1]. The entire estimation processing is transferred at the central office. Recently, it has also been proposed in [8] to use SNR measurements, introducing small perturbations on the transmitted signal, and observing their effect on the received SNR, seen from the receiver side.

In this paper, we focus on the method presented in [1] which is based on the feedback of the sign of the error signal on the detected symbols. The transmitter uses the combination of this feedback and of the knowledge of the transmitted symbol to iteratively compute the precoder (or precanceler). In this method, the precoder is computed directly without the need for the intermediate estimate of the crosstalk channel coefficients. The first contribution of this paper is to present a slightly modified version of the algorithm which is less complex and avoids any matrix inversion without significant performance degradation. Then, the performance of the method is analyzed. Very little results were presented in [1], and the ones presented were using very long acquisition times of more than 50 000 symbols, which is too long for practical purposes. The second objective of this paper is to show that (and how) the method can perform efficiently with much lower acquisition times (down to a few hundred symbols). Approximate performance expressions are derived to quantify the potential of the method, and they are compared to simulation results to be validated. Finally, these expressions can also easily be used to help setting the parameters of the method.

2. SYSTEM MODEL AND PRECODER

2.1 System model

In this section, the system model is presented, and the iterative precoder algorithm is summarized. We consider a DSL environment where DMT modulation is employed. It is also assumed that the cyclic prefix is long enough and that the individual user signals are transmitted synchronously from the CO (central office) so that after DMT demodulation the channels (including crosstalk) are free of intersymbol interference and intercarrier interference. The considered model is depicted in Figure 1. All operations can be applied tonewise so we focus on one given tone only. There are N lines in the binder. The information symbols to be transmitted by the different users are denoted by u_i, $i = 1, \ldots, N$ and are grouped into a vector $\mathbf{u} = [u_1 \ldots u_N]^T$. The variance of the symbols on line i is denoted by $\sigma^2_{u_i}$. For simplicity, we assume some normalization of the symbols so that $\sigma^2_{u_i} = 1$ on all lines. For the tone of interest, the channel model is written as

$$\mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{n}. \tag{1}$$

Here, $\mathbf{x}$ and $\mathbf{y}$ are N-dimensional vectors of transmitted and received samples with entries corresponding to the different users (or equivalently, to the different lines). $\mathbf{C}$ is the $N \times N$

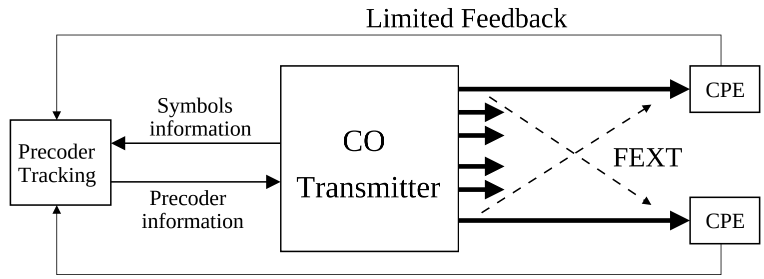

Figure 1: Principle of the structure for the iterative precoder algorithm

channel matrix for this tone, and $\mathbf{n}$ is the vector of noise samples at the different receivers. The additive noise is assumed to be Gaussian with independent elements. The noise variance for user (receiver) i is denoted by $\sigma_{n,i}^2$. In the model (1), the diagonal entries of $\mathbf{C}$ correspond to the line transmission (also called direct channel later in this paper), the off-diagonal entries correspond to crosstalk channels. The off-diagonal entries are always much smaller than the diagonal entries in the row (row-wise diagonal dominance).

Because the receivers are not collocated, the i-th receiver only has access to the i-th entry of $\mathbf{y}_k$ for detection and/or estimation purposes. To mitigate the effect of crosstalk in advance, the CO uses a precoder. We assume a linear precoder as presented in [2], and later improved in [4]. To this end, the CO designs a precoding matrix $\mathbf{F}$ and sends

$$\mathbf{x} = \mathbf{F}\mathbf{u} \tag{2}$$

on the different lines. In [2, 4], the design is such that the combined precoder-channel matrix $\mathbf{C}\mathbf{F}$ is diagonal, or

$$\mathbf{F} = \alpha \mathbf{C}^{-1}\mathbf{C}_d \tag{3}$$

where the notation $\mathbf{C}_d$ denotes the diagonal matrix formed by keeping only the diagonal entries of $\mathbf{C}$, and where α is a normalization factor that ensures that the transmitted power does not increase. The computation of the precoder requires an estimate of the crosstalk channel matrix.

When the precoder is active, a new global model can be obtained to take into account the presence of the precoder:

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{u} + \mathbf{n} && (4) \\ \mathbf{H} &= \mathbf{C}\mathbf{F}. && (5) \end{aligned}$$

2.2 Iterative precoder algorithm

The principle of the feedback used in this scheme is the following. The received symbols on line i at the tone of interest is given by

$$y_i = H_{ii}u_i + \sum_{j\neq i} H_{ij}u_j + n_i. \tag{6}$$

where H_{ij} is the corresponding entry in the matrix $\mathbf{H}$. After frequency equalization, assuming this equalization is accurate, we get the following decision variable

$$\hat{u}_i = u_i + \sum_{j\neq i} \frac{H_{ij}}{H_{ii}} u_j + \frac{n_i}{H_{ii}}. \tag{7}$$

The decision $\tilde{u}_i$ is taken as the closest symbol in the constellation. It is assumed that the decision is correct with very high probability. At the receiver, the decision noise (that is the noise that was present on the decision variable) is computed as

$$v_i = \hat{u}_i - \tilde{u}_i. \tag{8}$$

Finally, the principle used in this method is to feedback the sign of the decision noise (both real and imaginary part):

$$v_i^{\text{sgn}} = \text{sign}(v_i). \tag{9}$$

Regrouping all the users simultaneously, it can be written as

$$\begin{aligned} \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{n} && (10) \\ &= \mathbf{H}\mathbf{u} + \mathbf{n} && (11) \\ \hat{\mathbf{u}} &= \mathbf{H}_d^{-1}\mathbf{y} && (12) \\ \mathbf{v} = \hat{\mathbf{u}} - \mathbf{u} &= (\mathbf{H}_d^{-1}\mathbf{C}\mathbf{F} - \mathbf{I})\mathbf{u} + \mathbf{H}_d^{-1}\mathbf{n}. && (13) \end{aligned}$$

In the algorithm proposed in [1], the CO iteratively updates the precoder $\mathbf{F}$ based on the received sign samples using the following update formula (see the reference for more detail on the derivation):

$$\mathbf{F}(k+1) = \mathbf{F}(k) - \mu \mathbf{F}^{-T}(k)\mathbf{C}_d \left[\mathbf{v}^{\text{sgn}}(k)\mathbf{u}^H(k)\right]_{nd} \tag{14}$$

where $\mathbf{X}_{nd}$ denotes the matrix $\mathbf{X}$ for which all diagonal entries have been set to 0. $\mathbf{v}^{\text{sgn}}(k)$ represents the set of sign information that has been fed back from the CPE's (for time index k), the transmitted symbols $\mathbf{u}^H(k)$ are known at the CO, and μ is the stepsize of the algorithm which needs to be chosen. In addition of the update equation (14), a normalization of the precoder matrix $\mathbf{F}$ needs to be done to ensure that the power sent on each line is not increased. The issue with this algorithm is that it requires a matrix inversion ($\mathbf{F}^{-T}(k)$) at each step and is thus quite complex. In practice, and thanks to the diagonal dominance of the system, it appears that the precoder $\mathbf{F}(k)$ is also mostly diagonal. The product $\mathbf{F}^{-T}(k)\mathbf{C}_d$ is almost diagonal and hence we propose to replace the algorithm with this simpler, but almost equivalent version

$$\mathbf{F}(k+1) = \mathbf{F}(k) - \mu_d \left[\mathbf{v}^{\text{sgn}}(k)\mathbf{u}^H(k)\right]_{nd} \tag{15}$$

where the stepsize μ_d is now a *diagonal matrix*. Each line has its own stepsize. It is even possible to consider varying stepsizes. We will restrict ourselves to fixed stepsizes in the development below, although some comments will be provided showing that varying stepsizes might be more efficient. Note that the complexity of this update formula is low since it involves one multiplication with the stepsize per line, plus one multiplication with a bit of sign per line.

3. PERFORMANCE EVALUATION

In this section, the performance of the algorithm is evaluated. Let us focus on the residual interference during the updates of the precoder. The matrix of residual interference is given by

$$\mathbf{R} = \mathbf{H}_d^{-1}\mathbf{C}\mathbf{F} - \mathbf{I}. \tag{16}$$

It measures how well the crosstalk is suppressed by the precoder. If we focus on a given line i, we can observe the evolution of the residual interference vector

$$\mathbf{r}_i = \left[\begin{array}{cccccc} \mathbf{R}_{i,0} & \ldots & \mathbf{R}_{i,i-1} & \mathbf{R}_{i,i+1} & \ldots & \mathbf{R}_{i,N-1} \end{array}\right] \tag{17}$$

which is line i of the residual interference matrix $\mathbf{R}$ from which the diagonal element has been removed[1]. Ideally, this vector should get close to zero as quickly as possible. Inserting the update equation (15) into the definition of the residual interference, it follows

$$\mathbf{R}(k+1) = \mathbf{R}(k) - \mathbf{H}_d^{-1}\mathbf{C}\mu_d\left[\mathbf{v}^{\mathrm{sgn}}(k)\mathbf{u}^H(k)\right]_{nd} \tag{18}$$

Thanks to the diagonal dominance, the behavior of each line can be approximated, by assuming that the channel is almost diagonal, as

$$\begin{aligned} \mathbf{r}_i(k+1) &\approx \mathbf{r}_i(k) - \mu_i H_{ii}^{-1} C_{ii} v_i^{\mathrm{sgn}}(k)\,\overline{\mathbf{u}}^H(k) && (19)\\ &\approx \mathbf{r}_i(k) - \mu_i v_i^{\mathrm{sgn}}(k)\,\overline{\mathbf{u}}^H(k) && (20)\end{aligned}$$

where μ_i is the stepsize for line i and where $\overline{\mathbf{u}}$ denotes the vector of symbols $\mathbf{u}$ from which the i^{th} entry was removed. The last line comes by assuming that $H_{ii} \approx C_{ii}$ which is another consequence of the diagonal dominance. By taking the expectation of the second term in (20), it is possible to compute the average effect of the update equation on the residual interference. This expectation depends on the constellations used for the symbols u_i on the different lines. In order to compute this, we approximate the probability distribution of the symbols with Gaussian distributions. In that case, and assuming a normalized symbol variance $\sigma_{u_i}^2 = 1$, it can be shown that (time index k is dropped for simplicity)

$$\mathbf{E}\left[v_i^{\mathrm{sgn}} u_j\right] = \frac{2}{\sqrt{\pi}} \frac{r_{ij}}{\sqrt{|\mathbf{r}_i|^2 + \sigma_{n_i}^2/|C_{ii}|^2}} \tag{21}$$

for $i \neq j$. The evolution of the residual interference vector is then given by (on average, and for normalized symbol variance $\sigma_u^2 = 1$ on all lines)

$$\mathbf{r}_i(k+1) = \mathbf{r}_i(k) - \frac{2\mu_i}{\sqrt{\pi}} \frac{\mathbf{r}_i(k)}{\sqrt{|\mathbf{r}_i(k)|^2 + \sigma_{n_i}^2/|C_{ii}|^2}}. \tag{22}$$

It is interesting to analyze this formula. It appears to cause different behaviors whether the system is in acquisition or in tracking. In acquisition, the crosstalk is dominant $|\mathbf{r}_i(k)|^2 >> \sigma_{n_i}^2/|C_{ii}|^2$, and the evolution is well approximated by

$$|\mathbf{r}_i(k+1)| = |\mathbf{r}_i(k)| - \frac{2\mu_i}{\sqrt{\pi}}. \tag{23}$$

It means that the amplitude of the residual interference coefficients decrease linearly. The resulting convergence might be somewhat slow for very high crosstalk situations, if the stepsize is not chosen to be high enough. It is thus very important to choose the stepwise wisely. Equation (23) allows us to do so if a target number of symbols is fixed and the initial level of interference is known. In tracking, the noise is dominant and the evolution is well approximated by

$$\begin{aligned} |\mathbf{r}_i(k+1)|^2 &= |\mathbf{r}_i(k)|^2 \left(1 - \frac{2\mu_i}{\sqrt{\pi}} \frac{|C_{ii}|}{\sqrt{\sigma_n^2}}\right)^2 && (24)\\ &= |\mathbf{r}_i(k)|^2 \left(1 - \frac{2\mu_i}{\sqrt{\pi}} \sqrt{\mathrm{SaNR}_i}\right)^2 && (25)\end{aligned}$$

where 'SaNR$_i$' denotes the signal to additive noise ratio for line i, that is the ratio between the power of the received signal and the noise not taking into account the influence of crosstalk (or in other terms, the SNR that would be reached if all crosstalk was removed perfectly). In our model it is given, for line i, by

$$\mathrm{SaNR}_i = \frac{\sigma_{u_i}^2 |C_{ii}|^2}{\sigma_{n_i}^2} = \frac{|C_{ii}|^2}{\sigma_{n_i}^2}. \tag{26}$$

In this tracking case, the evolution is exponential. Both behaviors are well confirmed by simulations as it is shown later on, but they provide *average* behavior only.

Asymptotically, and because of the random nature of the iterative process, the residual interference does not cancel out completely. It continues to vary over time, around zero, with some variance which characterizes the asymptotic performance of the algorithm. This asymptotic performance is evaluated below. It is obtained by taking the expectation of the square norm of (20), and then assuming that the variance of the residual interference *asymptotically* stabilizes to a constant value

$$\mathbf{E}\left[|\mathbf{r}_i(k+1)|^2\right] = \mathbf{E}\left[|\mathbf{r}_i(k)|^2\right]. \tag{27}$$

The computation of all expectations is tedious but straightforward and is skipped here. Finally, we obtain the following variance in *asymptotic* behavior:

$$\mathbf{E}\,|\mathbf{r}_i(k)|^2 = \frac{\mu_i(N-1)\sqrt{\pi}}{2\sqrt{\mathrm{SaNR}_i}}. \tag{28}$$

It can be seen that this depends on the SaNR of the line. Hence in practice, it is useful to consider different stepsizes for the different lines as suggested in (15). All the results here are given for normalized symbol variance ($\sigma_u^2 = 1$). If the variance is not unity, stepsize(s) need to be scaled accordingly.

4. SIMULATION RESULTS

This paragraph presents a few simulations with the considered method. In all these simulations, we assume a system with 5 users, but only 2 lines are shown. Crosstalk channels are generated with a log-normal model. The power, channel and noise conditions are set to obtain different SNR (or more precisely SaNR) and crosstalk levels. First, It is interesting to point out that long acquisition times, as used in [1], are not necessary. In fact, if the stepsize is chosen carefully, much

[1] this element is always zero by definition of the matrix $\mathbf{R}$

quicker acquisition can be obtained. An example of simulation is shown in figure 2 for a situation where the SaNR is 40 dB and the initial interference is at 23 dB below the useful signal on all lines. The evolution of the remaining interference (crosstalk) is shown through the various iterations of the algorithm. The interference is represented as a ratio (in dB) with respect to the signal power. The stepsize has been fixed according to (23) in order to converge in roughly 300 symbols. Only 2 lines are shown. On the chosen duration, the interference is rejected to almost 10 dB below the noise.

Now we compare the theoretical performance expressions given above with several simulations in order to validate them. Figure 3 compares the theoretical acquisition performance (dash curve) with the simulation in a case with high initial crosstalk (SaNR=40 dB, and the initial crosstalk is at 14 dB below the signal). The performance prediction is quite accurate. Figure 4 compares the theoretical and simulation results for the tracking performance of the considered method. The 2 displayed lines respectively have an SaNR of 30 and 25 dB. The initial crosstalk is set below the noise to start in a tracking mode. Again, the match between theory and simulation is very good. Finally, Figure 5 provides a similar comparison for the asymptotic performance. The horizontal dash lines represent the expected remaining level of interference corresponding to the theoretical asymptotic variance of the residual interference $\mathbf{E}\left[|\mathbf{r}_i|^2\right]$. Even though there remains a lot of variation, the predictions seem reliable.

In order to evaluate the potential of the method, Figure 6 provides the performance for different crosstalk and noise situations and different stepsizes, each time averaged over 1000 simulations. The situations are indicated by their SaNR and SIR (signal to interference ratio, or in other terms, signal to initial crosstalk ratio). It illustrates the trade off between the asymptotic performance and the tracking speed that can be obtained as a function of the stepsize (when stepsize is fixed during the iterations). The tracking speed is here expressed as the number of samples needed to decrease the crosstalk interference by 10 dB. Obviously, these results confirm that the asymptotic performance does not depend on the initial crosstalk since both situation with the same SaNR and different initial crosstalk behave similarly (of course the acquisition time is different). Note that the performance decrease observed at lower tracking speed for the bottom curve is due to the limited length of the simulations. At lower tracking speed, the asymptotic region was not always reached.

All simulations have been done with fixed stepsize. In practice a fixed stepsize is not necessarily the best choice. A higher starting value will benefit a quicker acquisition, and the value can then be decreased for better asymptotic performance.

5. CONCLUSIONS

A slightly modifed version of the iterative precoder update, based on the feedback of the sign error, has been presented which avoids the need of any matrix inversion. Performance expressions have been derived and they prove to be very close to simulated performance. These expressions allow us to more easily set the parameters of the methods (mainly the stepsizes) and evaluate its potential.

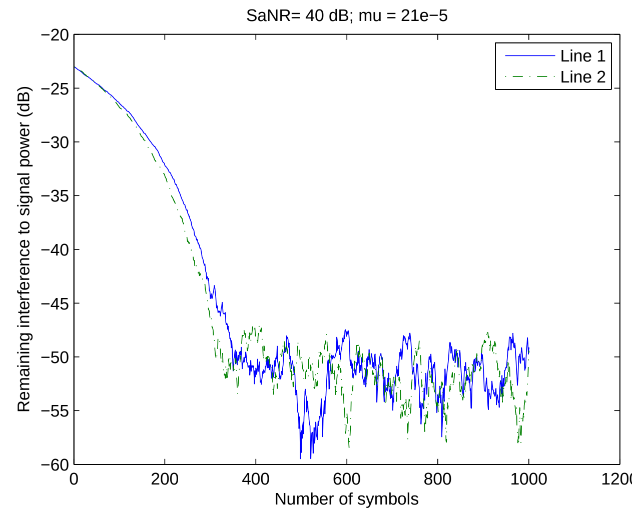

Figure 2: Example simulation (SaNR=40 dB, initial crosstalk at 23 dB)

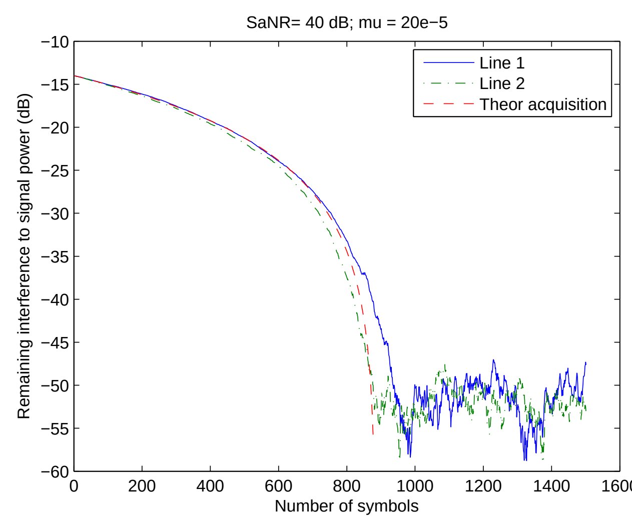

Figure 3: Comparison between predicted (dash line) and simulated acquisition speed (SaNR=40 dB, initial crosstalk at 14 dB).

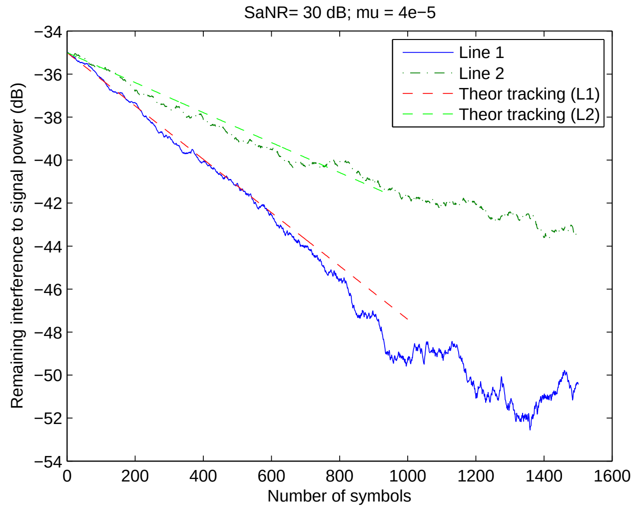

Figure 4: Comparison between predicted (dash line) and simulated tracking speed.

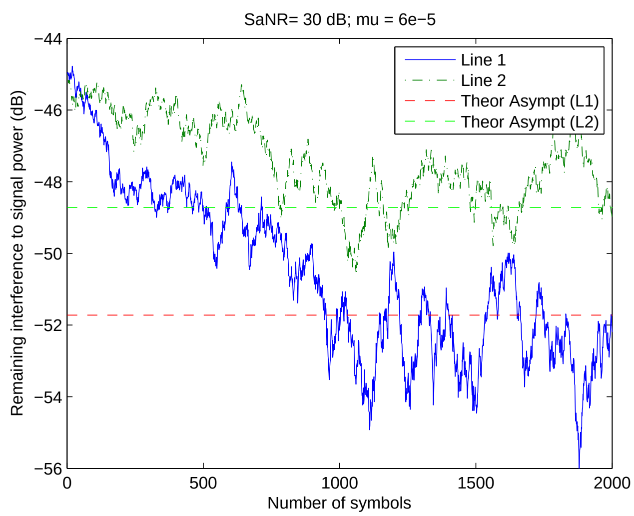

Figure 5: Comparison between predicted (horizontal dash lines) and simulated asymptotic performance.

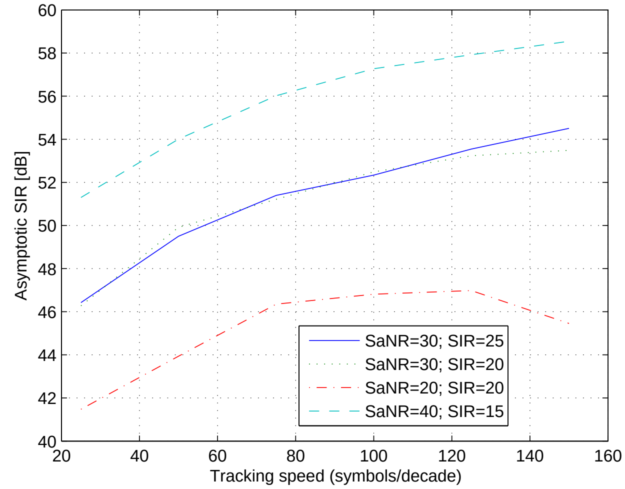

Figure 6: Asymptotic performance as a function of the tracking speed for different SNR/Crosstalk situations.

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