SELF-TUNING SUB-SAMPLE DELAY ESTIMATOR

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ABSTRACT

In this paper a novel solution of a self-tuning time delay estimator operating in the discrete time domain on a sinusoidal signal is presented. The proposed solution is based on a concept of series connection of a fractional delay filter and linear-phase Hilbert transform filter. The novelty of this solution lies in a very low numerical complexity, smaller than presented in hitherto published results. The performance of our self-tuning time delay estimator is analysed and illustrated by the results of its operation in presence of noise.

1. INTRODUCTION

Estimation of propagation delay between signals received at two separate sensors is a problem of considerable interest in areas such diverse as GPS, radar, sonar, biomedicine, ultrasonic, geophysics and others. In classical time domain estimators, the resolution is limited to the sampling period. However, now, sub-sample resolution is possible using interpolation by means of fractional delay (FD) filtering in discrete time domain [1] – [5] although other possibilities also exist, e.g., in [6], by using the FFT for this aim.

Here we achieve the delay estimation in discrete time domain via Hilbert transform filtering together with FD filtering. The performance of our self-tuning time delay estimator is analysed and illustrated in presence of noise.

Novelty of the proposed solution lies in low numerical complexity. It is economical and efficient due to reduced load of processing in comparison with, e.g., [3].

2. PROBLEM STATEMENT

We assume that the received signal has the form

$$x[n] = A\cos(\omega_0 n + \varphi) + g[n], \quad \omega_0 \in (0,\pi) \qquad (1)$$

where the amplitude A>0, ω_0 is the angular frequency in radians per sample whose value is known, φ is the unknown initial phase whose value we want to estimate/measure, $g[n]$ stands for a white Gaussian noise realisation and n stands for the time index (number) of sample; $n = 0, \pm 1, \cdots$. We estimate the value of φ by comparison of (1) with the reference cosine signal (chrono-signal otherwise called the timing waveform)

$$x_r[n] = A_1 \cos(\omega_0 n) \qquad (2)$$

whose initial phase is, by assumption, of zero value and A_1 may differ from A. For solution we apply a fractional delay FD filter with a fractional delay value d restricted to the interval $d \in [-1/2, 1/2]$ and a complex linear-phase Hilbert filter HTF.

3. THE PROPOSED SOLUTION

The block scheme of the proposed solution is shown in Figure 1.

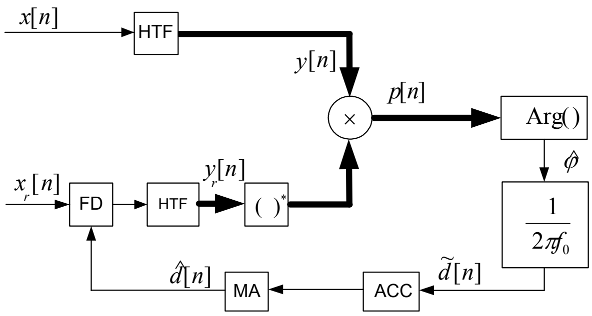


Figure 1 – Block scheme of the proposed solution of self-tuning time delay estimator. MA stands for moving averager and ACC stands for accumulator.

In Figure 1, in the upper part of the scheme, the received real-valued signal $x[n]$ passes through the linear-phase HTF giving a complex-valued output with Hilbertian noise term $g_H[n]$

$$y[n] = A\exp[j(\omega_0 n + \varphi)] + g_H[n] \qquad (3)$$

whose spectrum is periodical, Hilbertian, thus right-hand sided, in other words equal to zero for $\omega_0 \in (-\pi, 0)$. This complex-valued signal $y[n]$ is pointed out in the scheme by using a bold line. Then in Figure 1 the same signal $y[n]$ is multiplied by another complex-valued signal $y_r^*[n]$ obtained from the reference real-valued signal $x_r[n]$ by passing it through a series connection of FD with d of, generally, nonzero value, and the linear-phase HTF [7], [8], and taking the complex conjugate value (denoted by $(\ )^*$) of the result. Therefore, at the multiplier output we have

$$\begin{aligned} p[n] &= y[n]y_r^*[n] \\ &= \{A\exp[j(\omega_0 n+\varphi)]+g_H[n]\}A_1\exp(-j\omega_0 n) \\ &= AA_1\exp(j\varphi)+g_0[n] \end{aligned} \qquad (4)$$

with noise term denoted by $g_0[n]$, assuming that both filters are ideal. Thus for given n we obtain a complex number $AA_1\exp(j\varphi)$ depending on unknown $\varphi=\varphi[n]$ plus a noise component. If $\omega_0=\pi/2$ as common in telecommunications, then the value of φ has to be restricted to the interval $\varphi\in[-\pi/4,\pi/4]$ in order to have

$$d=\frac{\varphi}{\omega_0}\in[-1/2,1/2] \qquad (5)$$

as desired.

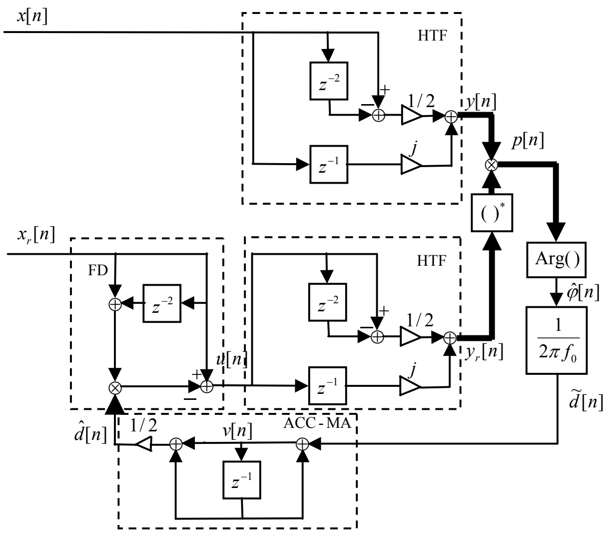


Figure 2 – Detailed block scheme of the proposed self-tuning sub-sample time delay estimator from Figure 1.

Next, in Figure 1, the estimated value $\hat{\varphi}=\hat{\varphi}[n]$ is computed at the output of the multiplier by using the block Arg( ) – the four quadrant arctangent referred to as `angle.m` in MATLAB. Further on, $\hat{\varphi}[n]$ is divided by the angular frequency $\omega_0=2\pi f_0$, $|f_0|\in(0,1/2)$ to give an estimate $\tilde{d}[n]$ of the delay between the signals: $x[n]$ (1) and $x_r[n]$ (2). This estimate is accumulated in the ACC block as indicated in Figure 1 and the result is averaged over two neighbouring samples in a MA – moving average block to give the value $\hat{d}[n]$ responsible for tuning the fractional delay in FD block operating in the self-tuning sub-sample time delay synthesis by analysis loop in Figure 1.

The general rule is that the lower the loop delay, the faster is the reaction of the loop to phase jumps in the received signal $x[n]$. That is why the MA block in Figure 1 is the simple first order finite impulse response (FIR) filter in Figure 2, with the following transfer function:

$$H_{\mathrm{MA}}(z)=(1+z^{-1})/2 \qquad (6)$$

This averaging is preceded by an accumulator

$$H_{\mathrm{ACC}}(z)=\frac{1}{1-z^{-1}} \qquad (7)$$

The transfer function of the ACC and MA cascade is

$$H_{\mathrm{ACC}}(z)H_{\mathrm{MA}}(z)=\frac{1+z^{-1}}{2(1-z^{-1})} \qquad (8)$$

and the corresponding frequency response of ACC-MA is

$$H_{\mathrm{ACC}}(e^{j\omega})H_{\mathrm{MA}}(e^{j\omega})=\frac{1+e^{-j\omega}}{2(1-e^{-j\omega})}=-j\frac{1}{2}\cot\frac{\omega}{2} \qquad (9)$$

whose values are: $\mp j/2$ for $\omega=\pm\pi/2$, 0 for $\omega=\pm\pi$ and $\mp j\infty$ for $\omega=0$.

Below we present the algorithm of the proposed self-adjusting sub-sample time delay estimator from Figure 2 based on the possibly shortest maximally flat FD and HTF and the recursive accumulator followed by a simple FIR averager in the last line of the loop.

$y[n]=(x[n]-x[n-2])/2+jx[n-1]$ HTF upper

$u[n]=x_r[n]-\hat{d}[n](x_r[n]+x_r[n-2])$ FD

$y_r[n]=\frac{1}{2}\{u[n]-u[n-2]\}+ju[n-1]$ HTF lower

$p[n]=y[n]y_r^*[n]$

$\hat{\varphi}[n]=\mathrm{Arg}(p[n])$

$\tilde{d}[n]=\hat{\varphi}[n]/\omega_0$

$v[n]=\tilde{d}[n]+v[n-1]$ ACC

$\hat{d}[n]=(v[n]+v[n-1])/2$ MA

For small $\mathrm{Arg}(p[n])$ values – in the frame shown above and also in Figures 1 and 2 – one can efficiently use an approximation

$$\mathrm{Arg}(p[n])\cong\mathrm{imag}(p[n])/|p[n]|.$$

The phase delay of the $H_{\mathrm{ACC}}(z)H_{\mathrm{MA}}(z)$ cascade (8) is $\pi/(2\omega)$ which is not astonishing. However, the group delay introduced is desirably of zero value. Thus we deal with a zero-group-delaying accumulator followed by the MA just tailored to our application in a feedback loop.

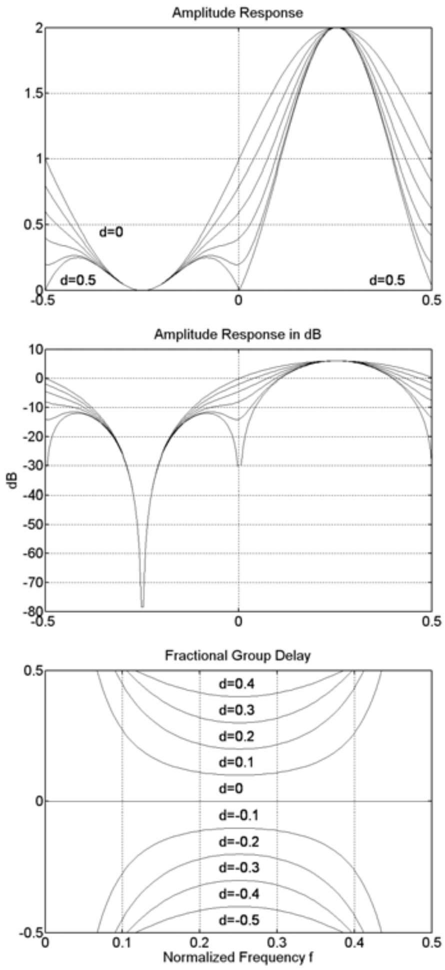


Figure 3 – The performance of a cascade of FD filter and HTF, maximally flat at $\omega_0 = \pi/2$, thus f_0=0.25. For better visibility the fractional part of the group delay response is presented only in the region of interest, i.e. for nonnegative frequency *f*.

Finally, the transfer function H_0 between $x[n]$ and $y[n]$ of the HTF of length *N*=2 implemented in the upper part of Figure 2 is

$$H_0(z) = \frac{1}{2}(1 - z^{-2}) + jz^{-1} \tag{10}$$

and the transfer function H_d in the lower part of the scheme between $x_r[n]$ and $y_r[n]$, in steady state, when we can assume that $d \cong \hat{d}[n]$

$$H_d(z) = \left[1 - d(1 + z^{-2})\right]\left[\frac{1}{2}(1 - z^{-2}) + jz^{-1}\right] \tag{11}$$

The amplitude response corresponding to (11) achieves the desired values: 2 for $\omega = \omega_0 = \pi/2$ and 0 for $\omega = \omega_0 = -\pi/2$. Of course, (11) resolves to (10) for *d*=0.

4. PERFORMANCE

The performance of the proposed self-tuning sub-sample time delay estimator is illustrated in Figures 3, 4, 5, 6 and 7.

The magnitude of the Complex Approximation Error (CAE) curves between the ideal (desired) frequency response of FD in series with HTF and that corresponding to (11) are presented in Figure 4. The best performance is observed in the vicinity of the frequency of the sine signal (1) in the upper part of Figure 1.

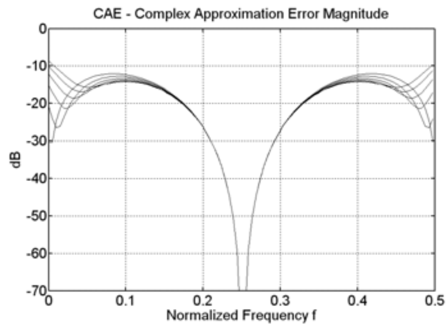


Figure 4 – The CAE for the cascade of FD filter and HTF from Figures 2 and 3.

Figure 5 shows the characteristics of the fractional delay introduced to (2) in order to compensate for the delay in the signal (1) having $\omega_0 = \pi/2$ and with $g[n] = 0$, thus without noise. We see that the convergence of the algorithm from Figure 2 applied here without averaging and accumulation is very fast. The speed of convergence does not depend on the value of *d*. Further drawings allow us to track the influence of averaging in MA – Figure 5b and accumulation in ACC – Figure 5c independently as well as applied together – Figure 5d for the input signal not corrupted by noise.

The same experiment as in Figure 5, where we had $g[n] = 0$, was repeated in Figure 6 with noise, thus with $g[n] \neq 0$. The additive white Gaussian noise – AWGN was used such that the signal power to noise power ratio – SNR – was 30 dB. The presence of noise is clearly seen in Figure 6a. Simple averaging reduces in Figure 6b the influence of noise to some extend, but the main work for this aim is done by the ACC, see Figure 6c. The final characteristics shown in Figure 6d for this SNR, where both: the averaging and ACC are engaged, are very close to those from Figure 5d.

In order to better clarify the matter we present in Figures 5e and 6e the "horseshoe" of clusters of phase values corresponding to the FD values used previously in experiments from Figures 3 and 4.

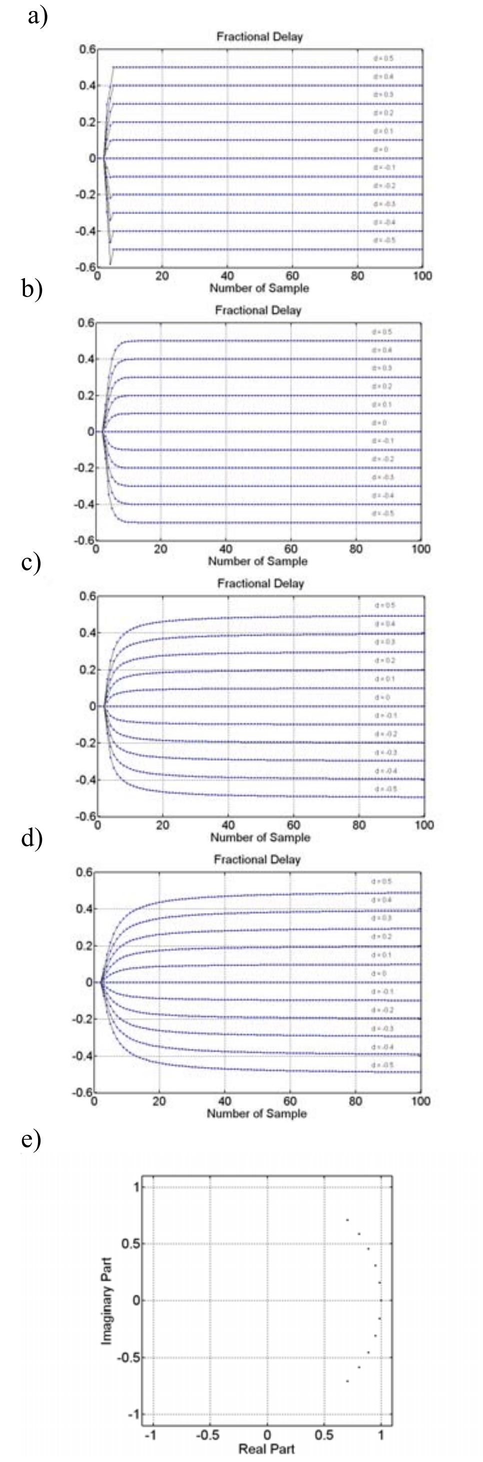


Figure 5 – The performance of self-tuning delay estimator from Figure 2 without noise and: a) without both ACC and MA, b) with MA, c) with ACC, d) complete scheme with ACC and simple MA e) the "horseshoe" of clusters of phase values for FD values as above in Figures 5 a, b, c and d.

The smaller the SNR, the dimensions of clusters increase, but they still remain distinguishable down to the SNR of approximately 15 dB, similarly as in [3], and even less. A confirmation is presented in Figure 7.

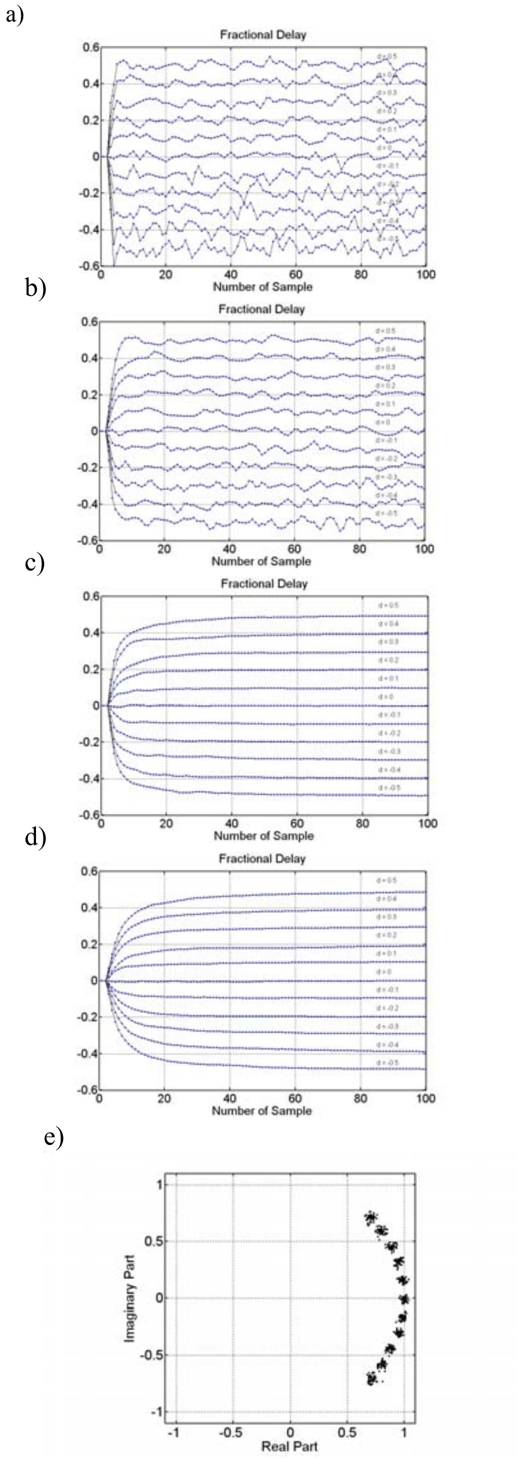
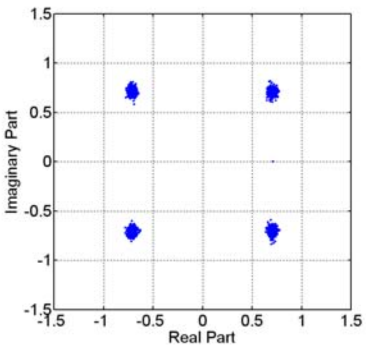


Figure 6 – The performance of self-tuning sub-sample time delay estimator from Figure 2 with additive white Gaussian noise of SNR= 30 dB; a, b, c, d, e as in Figure 5.

There, assuming the initial phase of $x[n]$ in (1) corresponding to the fractional delay d=1/2 (the worst case for processing) the clusters of the signal values after the both HTF are shown for SNR equal to 30 dB as in Figure 6 and also for SNR=15 dB. These clusters illustrate not only the phase $\hat{\varphi}[n]$ discrepancy around φ but also independence of this discrepancy on φ.

Nevertheless, our result obtained here is a solution whose numerical load of processing is significantly reduced in comparison with that needed in [3].

a)



b)

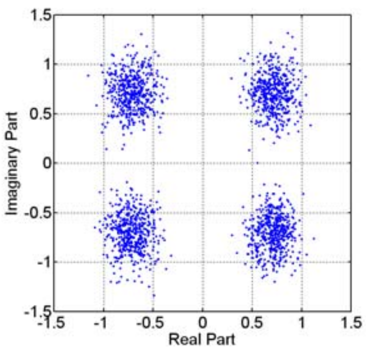


Figure 7 – Illustration of clusters of signals in Figure 2 of length 2000 samples: a) for $y[n]$ when $x[n]$ is corrupted by AWGN of SNR = 30 dB and b) of SNR=15 dB.

The FD filter and both HTF filters in the solution engineered in Figure 1, and implemented in Figure 2 need altogether only 2 multiplications (1 real-valued multiplication for the fractional delay d update and one complex-valued multiplication for $p[n]$ computation) versus 13 multiplications utilized in [3]. Moreover, they need only 6 adders of real-valued samples instead of 25 adders required in [3]. Also the number of integer delay (storage) elements in our design is smaller.

Summing up: the proposed solution achieves similar performance as that published in [3] but with much lower numerical complexity. That was accomplished by taking advantage of the extremely narrow bandwidth of the signal under processing.

5. CONCLUDING REMARKS

In the paper a novel solution of a self-tuning time delay estimator operating in the discrete time domain on a sinusoidal signal has been successfully developed. The proposed solution is made up of a series connection of fractional delay filter and Hilbert transform filter.

The performance of our self-tuning time delay estimator was illustrated by the results of its operation in presence of noise. The performance of the proposed technique is comparable to the technique presented in [3] with respect to much less computational cost.

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