BLIND IMAGE WATERMARKING BASED ON SAMPLE ROTATION WITH OPTIMAL DETECTOR

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ABSTRACT

This paper present a simple watermarking approach based on the rotation of low frequency components of image blocks. The rotation process is performed with less distortion by projection of the samples on specific lines according to message bit. To have optimal detection Maximum Likelihood criteria has been used. Thus, by computing the distribution of rotated noisy samples the optimum decoder is presented and its performance is analytically investigated. The privilege of this proposed algorithm is its inherent robustness against gain attack as well as its simplicity. Experimental results confirm the validity of the analytical derivations and also high robustness against common attacks.

1. INTRODUCTION

Digital watermarking is a process in which some information is embedded within a digital media so that the inserted data becomes part of the media. Several watermarking techniques have been proposed so far [1]. Among the proposed algorithms, Quantization Index Modulation (QIM) [2] has attained great popularity due to its lossless performance when lattice-based codebooks are used.

The main drawback of QIM algorithm is its vulnerability against gain attack which can easily occur through a simple channel without degrading the quality of the watermarked signal. Three types of solutions have been proposed to tackle this problem: i) Adopting auxiliary pilots through the watermarked signal known at both the encoder and decoder [3] ii) using spherical codewords [4] with correlator decoding [5], or using Angle QIM (AQIM) [6], [7], , and iii) introducing a domain in which the embedding process is invariant to gain attack called Rational Dither Modulation (RDM) [8].

The first solution decreases the security of the algorithm, since the malicious attacker can change either the watermark or the pilot signals. Besides, pilots are deterministic objects in the main signal and can be easily detected. Although the second and third approach keeps the security of the QIM algorithm, it causes high computational cost which increases the complexity of the algorithm. Besides, the low robustness of the AQIM against AWGN attack and also high Peak to Average Power Ratio (PAPR) of the RDM are the main drawbacks that should be addressed.

In this paper, we perform our embedding procedure on the slope of two points in which contains four samples. By this work the effect of noise decreases and better robustness is achieved in comparison with AQIM. Since the slope of two points varied, the PAPR does not change significantly. In order to have less distortion instead of rotating the line the projection on two specific lines determining the value of embedded bits is performed. Using analytic geometry, we embed the watermark bit by multiplication of specific matrices to the input vector. For the sake of simplicity, throughout the paper, the projection is done just on two lines. Consequently, for optimal detection, a simple hypothesis test has been used. It is straight forward but rather complicated to apply the projection on several lines and apply M-hypothesis test for detection purpose to increase the capacity . The performance of the proposed algorithm is analytically investigated and evaluated via simulations.

2. SYSTEM MODELING

In this section, we introduce the model we consider for our watermarking algorithm. We assume that we have an i.i.d Gaussian distributed random signal with four samples as the host signal. We show it as $\mathbf{u} = [u_1, u_2, u_3, u_4]$ which has the Gaussian distribution of $N(0, \sigma_u^2)$. We model these 4 sample as two points $p = [u_1, u_2]$ and $q = [u_3, u_4]$ in the 2-D space. The slope of the line which connects these two points is as follows:

$$c = \frac{u_4 - u_2}{u_3 - u_1} \tag{1}$$

If we depict the numerator and the denominator of the above equation with a and b, we have $a, b \sim N(0, 2\sigma_u^2)$. For the case that a and b are independent, c which is the ratio of zero mean independent normally distributed variates is Cauchian as $f_C(c) = \frac{1}{\pi} \frac{\frac{\sigma_a}{\sigma_b}}{c^2 + (\frac{\sigma_a}{\sigma_b})^2}$. Where, σ_a and σ_b are respectively the standard deviation of a and b.

Thus, we have two correlated Gaussian variables (with the correlation coefficient r) and their joint distribution is given as:

$$f_{ab}(a,b) = \frac{1}{2\pi\sigma_a\sigma_b\sqrt{1-r^2}} e^{-\frac{1}{2(1-r^2)}\left(\frac{a^2}{\sigma_a^2} - \frac{2rab}{\sigma_a\sigma_b} + \frac{b^2}{\sigma_b^2}\right)} \tag{2}$$

Thus, the cumulative distribution function of c is given as:

$$\begin{aligned} F_C(c) &= P\left\{\frac{a}{b} \le c\right\} \\ &= P\left\{a \le bc, b > 0\right\} + P\left\{a \ge bc, b < 0\right\} \\ &= \int_{b=0}^{\infty}\int_{a=-\infty}^{bc} f_{ab}(a,b)\, da\, db \\ &+ \int_{b=-\infty}^{0}\int_{a=bc}^{\infty} f_{ab}(a,b)\, da\, db \end{aligned} \tag{3}$$

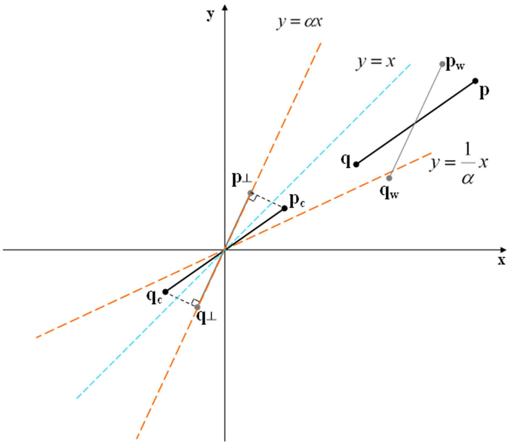


Figure 1: Steps of the proposed watermark embedding method.

Therefore, the probability density function will be:

$$f_C(c) = F'_C(c) = \int_{-\infty}^{\infty} |b| f_{ab}(bc, b)\, db \tag{4}$$

Substituting (2) in (4) and considering the fact that $f_{ab}(a,b)$ is an even function with respect to a and b we have:

$$f_C(c) = \frac{1}{2\pi\sigma_a\sigma_b\sqrt{1-r^2}} \int_0^{\infty} b e^{-\frac{b^2}{2\sigma_0^2}} = \frac{\sigma_0^2}{\pi\sigma_a\sigma_b\sqrt{1-r^2}} \tag{5}$$

where

$$\sigma_0^2 = \frac{1-r^2}{(\frac{c}{\sigma_a})^2 - \frac{2rc}{\sigma_a\sigma_b} + \frac{1}{\sigma_b^2}}$$

As a consequence,

$$f_C(c) = \frac{1}{\pi} \frac{\sigma_a\sigma_b\sqrt{1-r^2}}{\sigma_b^2(c - \frac{r\sigma_a}{\sigma_b})^2 + \sigma_a^2(1-r^2)} \tag{6}$$

In addition, $F_C(c)$ can be computed as:

$$F_C(c) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{\sigma_b c - r\sigma_a}{\sigma_a\sqrt{1-r^2}} \tag{7}$$

3. PROPOSED METHOD

In this section we introduce our blind watermarking algorithms.

3.1 Watermark embedding

To embed the watermark code in the image, we segment the host image into non-overlapping blocks and embed a single bit in each block based on the strategy given bellow.

In each block, we select four approximate level coefficients We apply 2-D Discrete Wavelet Transform (DWT) to each image block. Then we select four coefficient out of the approximate level coefficients. To provide security, the indices of these subsets are produced by a random generator which is the same for all blocks and is sent to the decoder side through a secure channel.

Let's show these coefficients as $\mathbf{u} = [u_1, u_2, u_3, u_4]$. As discussed in Section 2, we model these four sample as two points $p = [u_1, u_2]$ and $q = [u_3, u_4]$ in the 2-D space. Fig. 1 shows these two points as well as the line connects them together. Let's show the slope of this line with θ. The center of this (p,q) line is in $[\frac{u_1+u_3}{2}, \frac{u_2+u_4}{2}]$. If we translate the center of this line to the origin we reach to points p_c and q_c, where

$$p_c = \begin{pmatrix} u'_1 \\ u'_2 \end{pmatrix} = \begin{pmatrix} \frac{u_1-u_3}{2} \\ \frac{u_2-u_4}{2} \end{pmatrix}, \quad q_c = \begin{pmatrix} u'_3 \\ u'_4 \end{pmatrix} = \begin{pmatrix} \frac{u_3-u_1}{2} \\ \frac{u_4-u_2}{2} \end{pmatrix} \tag{8}$$

Now, to embed the watermark code, we project this line to the line L_0 or L_1 depending on the watermarking bit. We use projection to impose less distortion which results in more invisibility of the watermark. Fig. 1 shows this step in details. We call the consequence points in the mapped line as $p_\perp$ and $q_\perp$. The slope of the L_0 line which corresponds to '0' embedding is α if θ, the slope of the primary line (p,q), is positive and $-\alpha$ otherwise. Similarly, embedding '1', the slope of L_1 line is $\frac{1}{\alpha}$ or $-\frac{1}{\alpha}$ depending on the θ.

The position of $p_\perp$ for the case that the slope of the mapped line is k, can be computed using the intersection of two following lines:

$$\begin{cases} y = kx \\ y - u'_2 = -\frac{1}{k}(x - u'_1) \end{cases} \tag{9}$$

and we can use the same approach for $q_\perp$. After some simplification, we reaches to the solution of:

$$p_\perp = \begin{pmatrix} \frac{u'_1 + ku'_2}{k^2+1} \\ \frac{ku'_1 + k^2u'_2}{k^2+1} \end{pmatrix}, \quad q_\perp = \begin{pmatrix} \frac{u'_3 + ku'_4}{k^2+1} \\ \frac{ku'_3 + k^2u'_4}{k^2+1} \end{pmatrix} \tag{10}$$

As the final step, we only need to translate back the mapped line to the center to reach points $p_w = [u''_1, u''_2]$ and $q_w = [u''_3, u''_4]$, which can be shown as:

$$p_w = \begin{pmatrix} u''_1 \\ u''_2 \end{pmatrix} = \begin{pmatrix} \frac{u'_1 + ku'_2}{k^2+1} + \frac{u_1+u_3}{2} \\ \frac{ku'_1 + k^2u'_2}{k^2+1} + \frac{u_2+u_4}{2} \end{pmatrix}$$

$$q_w = \begin{pmatrix} u''_3 \\ u''_4 \end{pmatrix} = \begin{pmatrix} \frac{u'_3 + ku'_4}{k^2+1} + \frac{u_1+u_3}{2} \\ \frac{ku'_3 + k^2u'_4}{k^2+1} + \frac{u_2+u_4}{2} \end{pmatrix} \tag{11}$$

Thus, by inserting (8) in (11) we can show the whole procedure as:

$$\begin{pmatrix} u''_1 \\ u''_2 \\ u''_3 \\ u''_4 \end{pmatrix} = \mathbf{T}(k) \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \tag{12}$$

where $\mathbf{T}(k)$ is the transfer matrix and computed as:

$$\mathbf{T}(k) = \frac{1}{2k^2+2} \begin{pmatrix} k^2+2 & k & k^2 & -k \\ k & 2k^2+1 & -k & 1 \\ k^2 & -k, & k^2+2 & k \\ -k & 1 & k & 2k^2+1 \end{pmatrix} \tag{13}$$

Therefore, the watermarking embedding process can be figured as implementation of (12) in which k is defined depending on the watermarking bit and the slope of the primary (p,q) line as:

$$k = \begin{cases} \alpha & \text{For '1' embedding when } \theta \geq 0 \\ \frac{1}{\alpha} & \text{For '0' embedding when } \theta \geq 0 \\ -\alpha & \text{For '1' embedding when } \theta < 0 \\ -\frac{1}{\alpha} & \text{For '0' embedding when } \theta < 0 \end{cases} \tag{14}$$

The watermarked signal is thus $\mathbf{u}'' = [u''_1, u''_2, u''_3, u''_4]$ which is replaced with $\mathbf{u}$ in the selected indices of the approximation scale of the block. Applying the inverse DWT we can obtain the watermarked block.

3.2 Watermark decoding

To extract the hidden bit in each block, we implement an optimum decoder as follows. Suppose that $\mathbf{y} = [y_1, y_2, y_3, y_4]$ represents the approximate coefficients of the selected indices in the received block in the which the watermarked coefficients contaminated by zero mean Additive White Gaussian Noise (AWGN) $n\ N(0, \sigma_n^2)$. That is, $\mathbf{y} = \mathbf{u}'' + \mathbf{n}$.

As shown in [9], the image approximation coefficients can be well modeled by Gaussian distribution function. Therefore, we implement this distribution to develop our decoder and we assume the host signal $\mathbf{u}$ to be independently and identically distributed (iid) Gaussian coefficients. Consequently, the watermarked coefficients $\mathbf{u}''$ are also Gaussian as they are resulted by some linear transform through matrix $\mathbf{T}(\mathbf{m})$. Thus, the received coefficients $\mathbf{y}$ are Gaussian with the variance of $\sigma_y^2 = \sigma_u''^2 + \sigma_n^2$.

Now, we can use the model discussed in Section 2. To this aim we show the four coefficients in y as two points $p_r = [y_1, y_2]$ and $q_r = [y_3, y_4]$ in the 2-D space and we calculate the slope of the line connect them to each other as:

$$c = \frac{y_4 - y_2}{y_3 - y_1} = \frac{u''_4 - u''_2 + n_4 - n_2}{u''_3 - u''_1 + n_3 - n_1} \tag{15}$$

As we fixed the slope of the line which connects $p_w = [u''_1, u''_2]$ and $q_w = [u''_3, u''_4]$ to k which is defined in (14), we can say:

$$k = \frac{u''_4 - u''_2}{u''_3 - u''_1} \Rightarrow u''_4 - u''_2 = k(u''_3 - u''_1) \tag{16}$$

Therefore, if we define $v = u''_3 - u''_1$we can rewrite (15) as:

$$c = \frac{kv + n_4 - n_2}{v + n_3 - n_1} \tag{17}$$

Again, we can show the numerator and the denominator of the above equation with a and b. To compute the distribution of these two variable, first we consider the distribution of v. Using (12)and (13) we can say:

$$v = u''_3 - u''_1 = \frac{1}{k^2+1}(-u_1 - ku_2 + u_3 + ku_4)$$

As the host signal $\mathbf{u}$ is supposed to be iid Gaussian, we can say that v is a Gaussian random variable with mean $\mu_v = 0$ and variance $\sigma_v^2 = \frac{2}{1+k^2}\sigma_u^2$. Consequently, the distribution of the two random variables a and b can be given as $a \sim N(0, k^2\sigma_v^2 + 2\sigma_n^2)$ and $b \sim N(0, \sigma_v^2 + 2\sigma_n^2)$. The correlation coefficient between a and b can be computed as:

$$r = \frac{k\sigma_v^2}{\sqrt{(k^2\sigma_v^2 + 2\sigma_n^2)(\sigma_v^2 + 2\sigma_n^2)}} \tag{18}$$

Now we have $c = \frac{a}{b}$ where a and b are two correlate zero mean Gaussian random variables. Thus, we can use the discussions in Section 2 to compute the distribution function of c which will be defined as in (6).

Having the distribution of the slope function we can simply use the Maximum Likelihood detection as:

$$f_C(c|1) \gtrless_0^1 f_C(c|0) \tag{19}$$

where $f_C(c|1)$ and $f_C(c|1)$ are the distribution functions in condition that the embedded bit respectively is '0' or '1'. Here, without loss of generality we suppose that the received c is positive. We will see that the discussion is also hold for negative case. Thus, we can show the conditional distribution functions as:

$$f_C(c|1) = \frac{1}{\pi}\frac{\sigma_{a_{|1}}\sigma_{b_{|1}}\sqrt{1-r_{|1}^2}}{\sigma_{b_{|1}}^2(c - \frac{r_{|1}\sigma_{a_{|1}}}{\sigma_{b_{|1}}})^2 + \sigma_{a_{|1}}^2(1-r_{|1}^2)},$$
$$f_C(c|0) = \frac{1}{\pi}\frac{\sigma_{a_{|0}}\sigma_{b_{|0}}\sqrt{1-r_{|0}^2}}{\sigma_{b_{|0}}^2(c - \frac{r_{|0}\sigma_{a_{|0}}}{\sigma_{b_{|0}}})^2 + \sigma_{a_{|0}}^2(1-r_{|0}^2)} \tag{20}$$

where,

$$\sigma_{a_{|1}}^2 = \frac{2\alpha^2}{1+\alpha^2}\sigma_u^2 + 2\sigma_n^2, \quad \sigma_{b_{|1}}^2 = \frac{2}{1+\alpha^2}\sigma_u^2 + 2\sigma_n^2,$$
$$\sigma_{a_{|0}}^2 = \frac{2}{1+\alpha^2}\sigma_u^2 + 2\sigma_n^2, \quad \sigma_{b_{|0}}^2 = \frac{2\alpha^2}{1+\alpha^2}\sigma_u^2 + 2\sigma_n^2,$$
$$r_{|1} = r_{|0} = \frac{\alpha\sigma_u^2}{\sqrt{(\alpha^2\sigma_u^2 + (1+\alpha^2)\sigma_n^2)(\sigma_u^2 + (1+\alpha^2)\sigma_n^2)}} \tag{21}$$

These parameters are computed by substituting k as defined in (14) in definitions of variances of a, b and their correlation coefficient r (18).

Inserting (20) and (21) in (19) and after some simplification, we reach to the following simple decision tool:

$$c^2 \gtrless_0^1 1 \tag{22}$$

We can see that the optimum decoder is obtained independent of α. Besides, with the same discussion we can see that this decoder also hold for negative c. (In that case the only change is in the sign of r_0 and r_1).

4. PERFORMANCE EVALUATION

Here, we want to analytically study the error probability of the suggested watermarking scheme in the presence of AWGN. The error occurs whenever data bit one is embedded in an image block while zero is decoded at the receiver end and vice versa. Thus, we focus on the more common changes that occur because of crossing the decision line.

First, we compute the error probability when the slope of embedding line (p,q) is positive. Therefore, according to (22), the error probability can be written as:

$$\begin{aligned} P_e^+ &= \frac{1}{2}P(c^2 < 1|1) + \frac{1}{2}P(c^2 > 1|0) \\ &= \frac{1}{2}P(-1 < c < 1|1) + \frac{1}{2}P(c > 1 \mathrm{or} c < -1|0) \\ &= \frac{1}{2}\left[(F_{C|1}(1) - F_{C|1}(-1)) + (1 - F_{C|0}(1) + F_{C|0}(-1)))\right] \end{aligned} \tag{23}$$

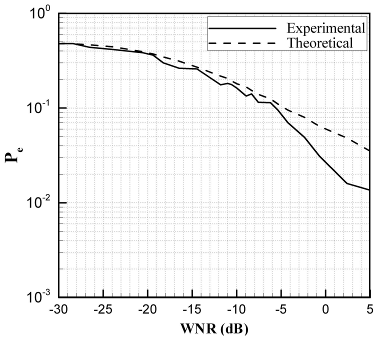


Figure 2: Comparison between the theoretical error probability and experimental one for a Gaussian random variable with $\sigma_n = 40$.

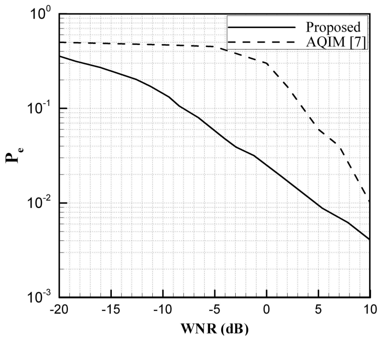


Figure 3: Comparing the probability of error with DWR=19dB between our method and the AQIM method [7]

Substituting $F_C(c)$ from (7), we have:

$$P_e^+ = \frac{1}{2} + \frac{1}{2\pi}\left[\tan^{-1}\frac{\sigma_{b|1} - r\sigma_{a|1}}{\sigma_{a|1}\sqrt{1-r^2}} - \tan^{-1}\frac{-\sigma_{b|1} - r\sigma_{a|1}}{\sigma_{a|1}\sqrt{1-r^2}} - \tan^{-1}\frac{\sigma_{b|0} + r\sigma_{a|0}}{\sigma_{a|0}\sqrt{1-r^2}} + \tan^{-1}\frac{-\sigma_{b|0} + r\sigma_{a|0}}{\sigma_{a|0}\sqrt{1-r^2}})\right] \quad (24)$$

where, $r = r_{|0} = r_{|1}$ as in (21). As we see in (21), $\sigma_{a|1} = \sigma_{b|0}$ and $\sigma_{a|0} = \sigma_{b|1}$; thus if we define $d = \frac{\sigma_{a|1}}{\sigma_{b|1}}$, we can simplify (24) as:

$$P_e^+ = \frac{1}{2} + \frac{1}{2\pi}(\tan^{-1}\frac{d^{-1} - r}{\sqrt{1-r^2}} + \tan^{-1}\frac{d^{-1} + r}{\sqrt{1-r^2}} - \tan^{-1}\frac{d + r}{\sqrt{1-r^2}} - \tan^{-1}\frac{d - r}{\sqrt{1-r^2}}) \quad (25)$$

With a similar argument we can see that the error probability when the slope of embedding is negative is the same as P_e^+; that is $P_e^- = P_e^+$. Consequently, we have $P_e = 0.5P_e^+ + 0.5P_e^- = P_e^+$. Therefore, we obtained a closed form solution for the error probability of the decoder given in (25). In Fig. 2 we compared this theoretical error probability with the experimental case of Gaussian random variable with $\sigma_u = 40$ (experimental result given for 1000 simulations). As, we can see the theoretical and experimental results match perfectly.

5. EXPERIMENTAL RESULTS

We have performed several experiments to test the proposed algorithms and evaluate its performance against various kinds of attacks.

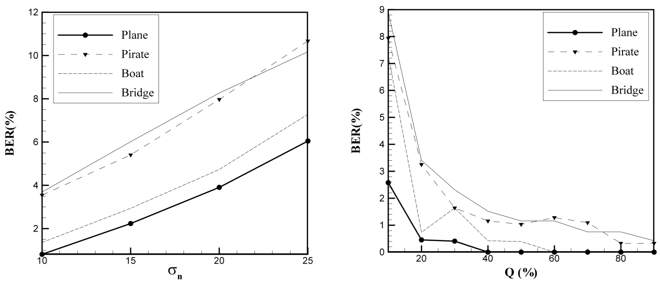
Figure 4: Original (left) and watermarked (right) test images; Top-down: *Plane*, *Pirate*, *Boat*, and *Bridge*

Table 1: BER(%) Results of extracted watermark under median and Gaussian filtering attacks

Image	Median Filtering	Gaussian Filtering		
	3×3	3×3	5×5	7×7
Plane	2.69	0.39	1.19	1.19
Pirate	5.23	1.66	2.41	2.77
Boat	15.13	1.73	2.31	2.69
Bridge	15.14	1.11	2.66	2.80

In the first experiment, to validate our method, we present the performance results for an artificial Gaussian signal under AWGN attack. The strength of the watermarking in terms of the Document-to-Watermark Ratio (DWR) is 22dB. The result of the proposed method for various Watermark-to-Noise Ratios (WNR) are given in Fig. 3. Here, we also compare the robustness with an AQIM based method [7] in the same situation. As we can see, we have lower error probability values even for very low WNRs. (Both the results for our and AQIM are for artificial signals (not the DWT of the image data)).

Then, we conduct several experiments to verify the performance of the proposed technique in real application of image watermarking. Throughout our experiments, we use the Daubechies length-8 Symslet filters with two levels of decomposition to compute the 2-D DWT. The watermark data is embedded in the second level approximation coefficients of each block. The results are obtained by averaging over 20



(a) (b)

Figure 5: (a) AWGN attack for various noise variances. (b) JPEG compression attack for various quality factors.

Table 2: Comparison between our watermarking method and [6] : BER (%) under AWGN attack.

Method	σ_n						
	1	2	3	4	5	6	7
[6]	1.00	2.00	3.00	4.00	15.00	25.00	44.00
Proposed	0.00	0.00	0.12	0.12	0.43	1.25	1.05

Table 3: Comparison between our watermarking method and Wang's method [10] : BER (%) under Median Filtering attack with window size 3×3.

Method	Image			
	Barbara	Baboon	Peppers	Goldhill
Wang [10]	24.95	31.65	29.35	25.60
Proposed	9.00	11.88	4.25	1.95

runs with 20 different pseudo random binary sequences as the watermarking signal.

For this study, we use various natural images of size 512×512. These images consist of *Plane*, *Pirate*, *Boat*, and *Bridge*. The original test images and their watermarked version using the proposed method with 16×16 block size and 128 bits message length are shown in Fig. 4. Also, the embedding slope α used in (14) is set to $\alpha = \tan^{-1}(65^o)$. This value is hand optimized to achieve the most robustness while keep the watermark almost imperceptible. As we can see, the watermark invisibility is satisfied. The mean Peak-Signal-to-Noise-Ratio (PSNR) of the watermarked images are 40.39dB, 40.44dB, 39.89, and 40.85 respectively.

As the first attack, we investigate the effect of AWGN to the proposed watermarking scheme. Fig. 5(a) shows the Bit Error Rate of the proposed method for various images versus different noise power. As we see, the method has a great resistance against noise attack.

Secondly, the proposed technique is tested against JPEG compression with different quality factor. As demonstrated in Fig 5(b), the proposed method is highly robust against JPEG with different quality factor up to 10%.

Table 1 show the BER results for median filtering, and Gaussian low-pass filtering attack with different test images. It can be seen that the proposed scheme is highly robust against various attacks.

Finally, we compare our watermarking algorithm with two of the recent blind watermarking techniques, [6] and [10] for AWGN and median filtering attacks. The simulation results are shown in Tables 2, 3. We see that the robustness of our method is considerably better than these two techniques.

6. CONCLUSION

A rotation based watermarking approach with optimum decoder is presented. The embedding is performed by multiplication of two specific matrices to vector of samples. By assuming the host samples to be iid Gaussian, the distribution of noisy watermarked samples is calculated. Then the ML detector is presented which decides on the embedded bit by a simple thresholding. The error probability of the decoder is analytically investigated. The proposed algorithm is applied to image signals by using the four approximation coefficients of image blocks. Simulation results show that the proposed algorithm is highly robust against common attacks such AWGN, filtering, etc. The future work may be performed by generalizing the algorithm for embedding more bits. In this manner the M-hypothesis test should be used. Besides, using Human Visual System (HVS) model, we can optimized the rotation angle to increase the robustness and invisibility simultaneously.

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