Ergodic Capacity of Block-Fading Gaussian Broadcast and Multi-access Channels for Single-User-Selection and Constant-Power

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***Abstract*— We consider the ergodic capacity region of block-fading Gaussian multiuser channels with channel-state information at both the transmitters and the receivers. We assume a single constraint on the total long-term average power used for both broadcast and multi-access channels. In addition to the optimal solution known from the literature, we provide analytic expressions – some of which are novel – to characterize the boundary surface of the capacity region under *auxiliary constraints* which include single-user-selection per block, constant total transmit-power per block and the combination of both. We also provide optimal resource allocation schemes to achieve the capacity limits for each case under consideration. Moreover, we provide numerical examples to compare the cases. As an illustrative example, we analyze the two-user case, although the results carry over to the M-user case.**

I. Introduction

Fading channels (both time and frequency selective) can be modeled as a family of parallel Gaussian channels: this is called a *block-fading channel* [1]. Each of the parallel Gaussian channel blocks corresponds to a fading state. In general, the capacity of block-fading multiuser channels with channel-state-information (CSI) at both the transmitter(s) and the receiver(s) can be achieved by (i) optimal power allocation over the channel blocks and (ii) optimal resource (rate and power) allocation over the users in each of the channel blocks. This is applicable to both the broadcast channel (BC) (one-to-many multiuser channel) [2], and the multi-access channel (MAC) (many-to-one multiuser channel) [3].

From a practical communications engineering perspective, the optimal solutions are in most cases difficult if not impractical to implement. Thus, sub-optimal solutions which have close-to-optimum performance and, at the same time, lend themselves to an easy implementation are favorable.

The optimal power allocation scheme over (block-)fading Gaussian broadcast and multi-access channels is given by the water-filling approach: more power is allocated when the channel is better and, depending on the desired operating point on the capacity region's boundary surface, some users are assigned higher average power to meet their rate demands.

As a consequence of this power allocation policy, the total and individual transmission powers will vary hugely. This will cause problems when, e.g., the transmitter (i.e. the base station in the broadcast case) has maximum power constraints in order not to cause too much interference in adjacent cells. Furthermore, adaptive power control requires additional computational complexity to maintain the average power constraint, and variable transmission power is also likely to require more expensive radio-frequency circuitry.

The optimal resource allocation over a (flat-faded) channel block involves applying the optimal channel-access scheme, which is code division multiple access (in MAC) or superposition coding (in BC) with successive interference cancellation (SIC) at the receivers. Furthermore, the number of users scheduled in a channel block varies depending on the channel conditions. Superposition coding with SIC at the receivers can hardly be implemented in practice, because of (i) the complexity involved, (ii) the necessity to inform all users about the order in which successive cancellation has to be conducted including the coding schemes used (signaling overhead), and (iii) different blocksizes used for encoding of different users: cancellation of a user's signal is only possible when the whole codeword[1] for this user has been received, although the user to be detected – due to delay constraints – may well have a much shorter (although still long) channel coding blocksize. As this user would have to wait for decoding until the "interfering" user's much longer codeword has been received, delay constraints are likely to be violated.

II. Objectives

We investigate the ergodic capacity limits and the optimal solutions to achieve these limits under practically relevant restrictions that enforce the use of constant total transmission power per fading state (channel block), single-user selection per fading state or both. "Constant total power per block" is to be interpreted such that in each and every channel block the sum power for all users is constant. For a broadcast channel this means that the total power used by the base station for all users is the same in every channel block although the number of users scheduled in every block is variable and subject to

[1] In practice, coding for a user will be spread over as many blocks as possible to obtain long codewords that will allow for efficient channel coding.

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optimization. In the multiple-access case, again the sum of all powers of all users' transmitters is assumed to be constant.

In our analysis of the *"multi-access channels"* (MAC), we assume a single long-term average sum-transmit-power constraint instead of individual power constraints for the users that are assumed in previous work [3]. This case is also relevant in practice [4]. Furthermore, it gives a more general solution with an extra information (cannot be obtained from [3]) about the optimal average powers to be allocated to each user to achieve a certain operating point. In [5] the duality of the MAC and BC channels was discussed. It was shown that the capacity region of the MAC channels with sum-power constraint is identical to the capacity region of the dual[2] BC channels. Thus, in all the cases under consideration, the equations characterizing the boundary surface of the capacity region are applicable to both the BC and the dual MAC channels. Furthermore, there exists a striking similarity between the optimal resource allocation for both channels.

Our objective, in this paper, is to study how much we will lose in terms of system capacity when we apply one or both of the specified auxiliary constraints. In order to answer this question, we (i) give closed-form expressions – which are novel contributions – that characterize the capacity limits and (ii) describe resource allocation schemes (for BC and MAC) – again with some novel contributions – to achieve these limits for the following four cases with different constraints:

- **OPT**: optimal solution, without any auxiliary constraints (new analytical results that complement the original work in the literature, i.e. [2] for the BC case and [3] for the MAC case, presented in this paper).
- **CP**: constant sum power of all users in every channel state (new analytical results presented in this paper).
- **SU**: selection of a single-user only in every channel state.
- **CP-SU**: constant sum power *and* single-user selection in every channel state.

We provide numerical results in which we compare the four cases. To visualize the capacity limits, we consider the two-user case, with the assumption of different long-term average channel qualities of the users. Qualitatively, the results carry over to the M-user case. We perform analysis for a higher number of users as well by selecting a specific operating point (max. sum-throughput) for symmetric channels.

III. Channel Model

The *block-fading channel* is used to model time and frequency selective fading channels. The fading channels are divided into a family of parallel Gaussian "constant" channels, each corresponds to a "flat" fading state. These constant channels are called blocks. A channel block could last for several time slots as long as the channel quality is almost constant (dependent on fading statistics).

The M-user Gaussian block-fading broadcast channel (BC) consists of a single transmitter and M receivers. In channel block k, the transmitter broadcasts a signal $x[k]$, and the received signals are

$$y_i[k] = \sqrt{h_i[k]}x[k] + n_i[k], \quad i = 1, \cdots, M$$

where $h_i[k] > 0$ is the constant channel quality (i.e. power gain)[3] between the transmitter and the i-th receiver at channel-block k, and $n_i[k]$ is Gaussian noise with zero mean of that receiver. The noises $n_i[k]$ are statistically independent, and are assumed to have a common variance σ^2 .

The M-user Gaussian block-fading multi-access channel (MAC) consists of a single receiver and M transmitters. At channel block k, each transmitter i transmits a signal $x_i[k]$, and the receiver receives the composite signal

$$y[k] = \sum_{i=1}^{M} \sqrt{h_i[k]}x_i[k] + n[k]$$

where $h_i[k] > 0$ is the constant channel quality between the i-th transmitter and the receiver at channel-block k.

The fading processes of all users are independent of each other, are stationary and have continuous probability density functions, $f_i(h)$. In the numerical examples through the paper, we assume the fading processes have the Rayleigh[4] distribution. The cumulative distribution functions of the fading processes are denoted by $F_{h_i}(x) \doteq \int_0^x f_{h_i}(h')dh'$.

We use the notation $P_i[k]$ and $R_i[k]$ to indicate the power[5] and the rate (bits/sec/Hz) respectively that are allocated to user i in channel block k. The long-term average rate that is allocated to user i is denoted as R_i. The long-term average sum-power constraint is denoted as $\bar{P}$.

IV. Problem Formulation

The ergodic capacity region is defined as the set of all achievable rate vectors $\mathbf{R}$ such that the long-term average power constraint $\bar{P}$ over all channel blocks is not exceeded. The optimum points within the capacity region are those that are located on the boundary surface. The latter can be characterized as the closure of the parametrically defined surface

$$\left\{\mathbf{R}(\boldsymbol{\mu}) : \boldsymbol{\mu} \in \Re_+^M, \sum_i \mu_i = 1\right\} \tag{1}$$

where for every weighting factor vector $\boldsymbol{\mu}$, the rate vector $\mathbf{R}(\boldsymbol{\mu})$ can be obtained by solving the optimization problem:

$$\max \frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{M} \mu_i R_i[k] \text{ subject to } \frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{M} P_i[k] = \bar{P} \tag{2}$$

where K is the total number of channel blocks, and M is the number of active users. We assume w.l.o.g. that all channel blocks have identical frequency bandwidth and time duration.

The two auxiliary constraints that are considered in this work to be added to the problem definition in (2) are:

- Constant sum power per channel block:

$$P[k] = \sum_{i=1}^{M} P_i[k] = \bar{P} \tag{3}$$

- Single-user selection per channel block: $\mathbf{R}[k]$ has a maximum number of one non-zero element.

V. Characterization of the Boundary of the Capacity Region

In this section, we provide characterization of the boundary of the capacity region of block-fading BC and MAC channels for the cases

[2]BC and MAC channels are dual if they have the same channel vector $\mathbf{h}$ (i.e. h_i of receiver i in the BC equals h_i of transmitter i in the MAC).

[3]In this paper we assume, without loss of generality, that the channel gain h is real and representing the power gain of the link. h does not have imaginary part since we assume perfect phase information at the receivers.

[4]$f_{h_i}(x) = \frac{1}{\bar{h}_i}\exp\left(\frac{-x}{\bar{h}_i}\right)$, $\bar{h}_i$ is the average channel quality

[5]When we use the notation P, we mean the transmit power P_T. The received power is indicated as P_R.

under consideration[6]. This includes (i) describing resource allocation schemes to achieve the capacity boundary limits of BC and MAC channels, and (ii) giving closed-form expressions that characterize the capacity limits (the same expressions are applicable to both BC and dual MAC) for a given weighting vector $\boldsymbol{\mu}$ defining one point in the boundary surface (1).

A. ***OPT**: Optimal Case (No Auxiliary Constraints)*

As discussed in [1], problem (2) can be solved by first applying the Lagrangian characterization in order to define the problem in an unconstrained format. The resulting optimization problem is:

$$\max_{\{P[k]\}} \sum_{k=1}^{K} \left(\sum_{i=1}^{M} \mu_i R_i[k] - \lambda \sum_{i=1}^{M} P_i[k] \right) \tag{4}$$

This is equivalent to

$$\sum_{k=1}^{K} \max_{P[k]} \left(\sum_{i=1}^{M} \mu_i R_i[k] - \lambda P[k] \right) \tag{5}$$

where λ is selected such that

$$\frac{1}{K} \sum_{k=1}^{K} P[k] = \bar{P} \tag{6}$$

Thus, the main optimization problem is decomposed into (i) a family of optimization problems, one for each channel block, and (ii) an equation to control the power price λ in order to maintain the long-term average power constraint. Following the procedure described in [1] by defining marginal utility functions, and by extending these results to the MAC case, we provide a summary of the solution:

Power allocation over the channel blocks: The total power transmitted in a block k is identical in BC [1] and MAC channels (with $x^+ = \max(x, 0)$):

$$P_{sum}[k] = \max_i \left[\sigma^2 \left(\frac{\mu_i}{\lambda} - \frac{1}{h_i[k]} \right)^+ \right] . \tag{7}$$

Resource allocation in each channel block: The optimal resource allocation over a (flat-faded) channel block involves applying the optimal channel-access scheme, which is code division multiple access (in MAC) or superposition coding (in BC) with successive interference cancellation (SIC) at the receivers. The SIC at the receivers of BC channels is in order of decreasing μ. Each receiver decodes the signals sent to users of higher μ before decoding its own signal. However, in MAC channels, the receiver performs SIC in order of increasing μ [5]. We provide a summary of the greedy algorithm procedure to compute the power allocated to each user in channel block k, for both BC [1] and MAC –novel contribution by extending results of BC to MAC with sum power constraint– channels:

Marginal utilities functions ("rate revenue minus power cost") are defined for each channel block k:

$$\text{BC: } u_i(z) \equiv \frac{\mu_i}{\frac{1}{h_i[k]} + z} - \lambda, \; z \geq 0 \tag{8}$$

$$\text{MAC: } u_i(z) \equiv \frac{\mu_i}{1+z} - \frac{\lambda}{h_i[k]}, \; z \geq 0 \tag{9}$$

Then based on the marginal utilities which are dependent on the channel qualities vector $\mathbf{h}[k]$, the intervals $\mathcal{A}_i$ are obtained:

$$\mathcal{A}_i \equiv \{ z \in [0, \infty) : u_i(z) > u_j(z) \;\; \forall j \neq i \text{ and } u_i(z) > 0 \}$$

Since $u_i(z)$ is monotonically decreasing and $u_i(z)$, $u_j(z)$ ($i \neq j$) cross each other at maximum once, the interval $\mathcal{A}_i$ is continuous. The power allocation is calculated as:

$$\text{BC: } P_i[k] = \sigma^2 \int_{\mathcal{A}_i} dz \tag{10}$$

$$\text{MAC: } P_i[k] = \frac{\sigma^2}{h_i[k]} \int_{\mathcal{A}_i} dz \tag{11}$$

To derive equations to characterize the boundary surface of the capacity region, we complement the work in [1] to get the following equations to compute $\mathbf{R}(\boldsymbol{\mu})$ in (1). We use the assumption that the fading processes of all users are stationary with continuous probability density functions and independent of each other: for each user $i = 1, ..., M$

$$R_i^{\text{OPT}} = \frac{1}{\ln 2} \int_0^\infty \frac{1}{1+z} \int_{\frac{\lambda(1+z)}{\mu_i}}^\infty f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\alpha^*) \, dx dz \tag{12}$$

or equivalently

$$R_i^{\text{OPT}} = \frac{1}{\ln 2} \int_0^{\frac{\mu_i}{\lambda}} \int_{\frac{1}{\frac{\mu_i}{\lambda} - z}}^\infty \frac{1}{\frac{1}{x} + z} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\beta^*) \, dx dz \tag{13}$$

where λ in (12), (13) is computed based on (6) which, in our case of independent fading processes, is equivalent to:

$$\sum_i \int_{\frac{\lambda}{\mu_i}}^\infty f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\zeta^*) \left[\frac{\mu_i}{\lambda} - \frac{1}{x} \right] dx = \frac{\bar{P}}{\sigma^2} \tag{14}$$

There are two other equivalent forms to compute λ:

$$\sum_i \int_0^\infty \int_{\frac{\lambda(1+z)}{\mu_i}}^\infty \frac{1}{x} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\alpha^*) \, dx dz = \frac{\bar{P}}{\sigma^2} \tag{15}$$

$$\sum_i \int_0^{\frac{\mu_i}{\lambda}} \int_{\frac{1}{\frac{\mu_i}{\lambda} - z}}^\infty f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\beta^*) \, dx dz = \frac{\bar{P}}{\sigma^2} \tag{16}$$

α, β, ζ are given by:

$$\alpha = \frac{\lambda}{\frac{\lambda}{x} + \frac{\mu_j - \mu_i}{1+z}} \tag{17}$$

$$\beta = \frac{\mu_i x}{\mu_j + zx(\mu_j - \mu_i)} \tag{18}$$

$$\zeta = \frac{1}{\frac{1}{x} + \frac{\mu_j - \mu_i}{\lambda}} \tag{19}$$

The notation x^* in (12), (13), (14) is defined as [7]:

$$x^* \doteq \begin{cases} x & \text{if } \; x \geq 0 \\ +\infty & \text{if } \; x < 0 \end{cases} . \tag{20}$$

B. ***CP**: Constant Sum-Power per Block*

In this case, the main optimization problem (2) becomes equivalent to optimizing over each channel block k:

$$\max \sum_{i=1}^{M} \mu_i R_i[k] \text{ subject to } \sum_{i=1}^{M} P_i[k] = \bar{P} \tag{21}$$

Thus, the problem is to allocate the resources over the users in each channel block. The power allocated to each user in each channel block can be obtained using the same greedy procedure of the optimal case, but with the replacement of the global power price λ in (8), (9) by

[6]The detailed proofs are omitted due to paper length restriction. We assume here that the reader is aware of the original papers in this topic (mainly [1]) as our work complements these results. The proofs will, however, be given in a full journal paper version of this work [6].

block-dependent power price $\lambda[k]$, which is obtained as:

$$\lambda[k] = \max_i \left(\frac{\mu_i}{\frac{1}{h_i[k]} + \frac{\bar{P}}{\sigma^2}} \right) \tag{22}$$

The channel access scheme and the order of SIC is identical to the OPT case.

The boundary surface is characterized by the equation:

$$R_i^{\text{CP}} = \frac{1}{\ln 2} \int_0^{\frac{\bar{P}}{\sigma^2}} \int_0^{\infty} \frac{1}{\frac{1}{x} + z} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\beta^*)\, dx dz \tag{23}$$

where β defined in (18), and the notation $[x]^*$ in (20).

C. *SU*: Maximum of Single-User Selection per Block

In this case, the solution of the optimization problems over each channel block becomes a user selection strategy, where the user to be scheduled is the one who maximizes the selection argument (policy). The only user m scheduled to transmit (MAC) or receive (BC) in block k, and the power allocated to this user are calculated according to:

$$m = \arg\max_i \left(\mu_i R_i[k] - \frac{\lambda P_i[k]}{\sigma^2} \right) \tag{24}$$

where $P_i[k]$ is calculated according to:

$$P_i[k] = \sigma^2 \left[\frac{\mu_i}{\lambda} - \frac{1}{h_i[k]} \right]^+ \tag{25}$$

The boundary surface is characterized by the equation [8]:

$$R_i^{\text{SU}} = \int_{\frac{\lambda}{\mu_i}}^{\infty} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\gamma) \log\left(\frac{\mu_i x}{\lambda} \right) dx \tag{26}$$

where λ in (26) is computed according to

$$\sum_i \int_{\frac{\lambda}{\mu_i}}^{\infty} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\gamma) \left[\frac{\mu_i}{\lambda} - \frac{1}{x} \right] dx = \frac{\bar{P}}{\sigma^2} \tag{27}$$

γ in (26), (27) is given by:

$$\gamma = \frac{-\lambda}{\mu_j W\left[-\left(\frac{\lambda}{\mu_i x} \right)^{\frac{\mu_i}{\mu_j}} \exp\left(\frac{\mu_i}{\mu_j} - \frac{\lambda}{\mu_j x} - 1 \right) \right]} \tag{28}$$

with $W()$ the Lambert function [9] (inverse of $f(x) = xe^x$).

D. *CP-SU*: Both Constraints

In this case, the only user m scheduled to transmit (MAC) or receive (BC) in block k, and the power allocated to this user are calculated according to:

$$m = \arg\max_i \mu_i R_i[k] \quad \text{and} \quad P_i[k] = \bar{P} \tag{29}$$

$R_i[k]$ in (24) and (29) is the Shannon capacity of AWGN channel:

$$R_i[k] = \log\left(1 + \frac{h_i[k] P_i[k]}{\sigma^2} \right)$$

The boundary surface is characterized by [8]:

$$R_i^{\text{CP-SU}} = \int_0^{\infty} f_{h_i}(x) \prod_{j \neq i} F_{h_j}(\eta) \log\left(1 + \frac{x\bar{P}}{\sigma^2} \right) dx \tag{30}$$

where η is defined as:

$$\eta = \frac{\left(1 + x\frac{\bar{P}}{\sigma^2} \right)^{\frac{\mu_i}{\mu_j}} - 1}{\frac{\bar{P}}{\sigma^2}} \tag{31}$$

VI. Numerical Examples

A. Comparison of Two-User Case

We provide a numerical example of applying the equations to characterize the boundary surface of the capacity region for the four cases under consideration in a scenario of two users. The two-user case is selected because it is possible to visualize the capacity regions and compare the different cases. Qualitatively, the results carry over the general M-user case.

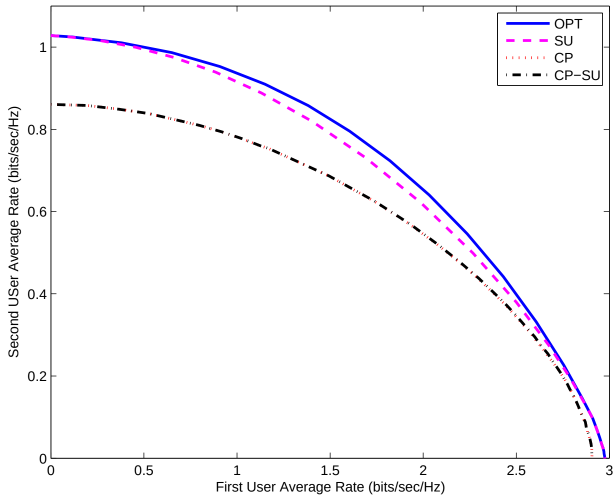

Fig. 1. Boundaries of the ergodic capacity regions for the two-user case. The users are Rayleigh-faded with 10dB difference in average channel qualities.

In Fig. 1 we show the capacity regions with the assumption that the users channels are fading independently and with Rayleigh distribution. The first user channel has 10dB better long-term average channel quality over the second user channel. Any specific point in the capacity boundary can be achieved by adjusting the weighting factors $\boldsymbol{\mu}$. We selected a relevant case in which the network average spectral efficiency can range between 1 and 3 bits/sec/Hz.

The main conclusions we obtain from the results are:

- Power control is more important when the operating point of the system has overall low spectral efficiency (to serve weak-channel users). For high spectral efficiencies, using constant power per block is justified and has minor detrimental effects to the capacity of the system.
- For constant transmit power systems, applying superposition coding provides negligible improvements to the achievable rates. Thus, using single-user selection scheme in such systems is justified. On the other hand, for systems applying optimal power control, superposition coding is useful for a range of operating points.

B. Sum-Throughput Comparison for Symmetric Channels

In this example, we compare the capacity difference between systems applying optimal power control and constant power per block systems for various number of users. Since it is not possible to visualize the capacity regions for systems with more than 3 users, we use instead a specific operating point within the capacity boundary surface. We select the maximum sum-throughput capacity and make the analysis with assumption of symmetric users channels.

Furthermore, with the assumption of Rayleigh fading channels, we can derive close-form expressions for the capacities as a function of the number of users M.

For the constant power system, we obtain:

$$R_{\text{sum}} = \frac{1}{\ln 2}\sum_{i=1}^{M}(-1)^{(i-1)}\binom{M}{i}\exp\left(\frac{i\sigma^2}{\bar{h}\bar{P}}\right)E_1\left(\frac{i\sigma^2}{\bar{h}\bar{P}}\right) \quad (32)$$

where E_1 is the exponential integral function

$$E_1(x) \equiv \int_x^{\infty}\frac{\exp(-u)}{u}du$$

For the system applying optimal power control, we obtain:

$$R_{\text{sum}} = \frac{1}{\ln 2}\sum_{i=1}^{M}(-1)^{(i-1)}\binom{M}{i}E_1\left(i\lambda\right) \quad (33)$$

where λ is adjusted so that the power constraint is achieved:

$$\frac{\bar{h}\bar{P}}{\sigma^2} = \sum_{i=1}^{M}(-1)^{(i-1)}\binom{M}{i}\left[\frac{\exp(-i\lambda)}{\lambda} - i\,E_1\left(i\lambda\right)\right] \quad (34)$$

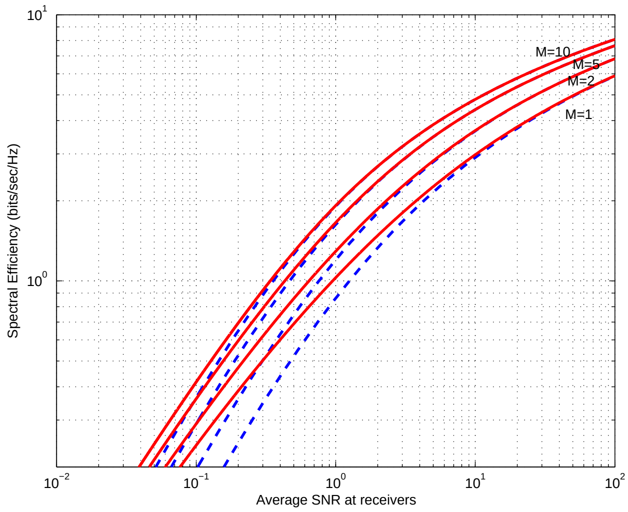

Fig. 2. Differences in spectral efficiency between optimal power control (solid lines) and constant power per block (dashed lines) for different number of users M. Rayleigh fading channels with identical long-term average channel qualities. Maximum sum-throughput is considered.

From the results in Fig. 2, we can find rough estimates of system spectral efficiencies over which the application of the constant power constraint is justified. As the number of users in the system increases, the rate level, over which the constant power system approaches the optimal power control system, decreases. For example, in a single user system, using constant power while operating above 4 bits/sec/Hz is very close to the optimal case. While a value of 3 bits/sec/Hz is applicable in two users system, and approximately 1.5 bits/sec/Hz for $M = 10$.

VII. Conclusions

We have derived novel closed-form equations to characterize the boundary surface of the ergodic capacity region of BC channels and MAC channels with sum-power constraints under practical auxiliary constraints on power per block and user selection per block. The characterization of the capacity region in these cases is important in order to be able to compare the performance of the system under the practical constraints. We have provided numerical examples to compare the cases under consideration. Additionally, we have described the optimal resource allocation schemes to operate at the capacity limits. This topic was studied in the literature for the optimal case. However, we have extended the results in order to include the cases of the auxiliary constraints.

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