EMPIRICAL MODE DECOMPOSITION BASED DENOISING FOR HIGH RESOLUTION DIRECTION OF ARRIVAL ESTIMATION

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ABSTRACT

In this work, Empirical Mode Decomposition (EMD) is applied to the problem of Direction of Arrival (DoA) estimation as a preprocessing method. The preprocessing stage consists of separate denoising the rows of the array data matrix where each row corresponds to the output of a particular array sensor. The chosen denoising algorithm is an iterative interval-thresholding variant of EMD. After the denoising stage, MUSIC is applied to construct the EMD-enhanced spatial spectrum. The proposed EMD-based array denoising scheme is based on the principles of wavelet-thresholding, thus it is comparable to wavelet-based denoising of array matrix. The results show that, especially in low-SNR scenarios, the estimation performance of MUSIC is significantly enhanced when denoising is applied to array data matrix prior to DoA estimation stage.

1. INTRODUCTION

Empirical Mode Decomposition, proposed by [1], is a signal-adaptive analysis technique applicable to the time-frequency analysis of nonlinear and nonstationary signals. Given an input signal, EMD decomposes the signal into a number of zero-mean functions, called Intrinsic Mode Functions (IMFs), which are sums of amplitude and frequency modulated sinusoids. As shown in [2], EMD can be implemented as an adaptive filter bank which allows a comparison basis for other subband decomposition analysis methods such as wavelets. However, as opposed to wavelet analysis, no previous information about the structure is used for the decomposition of subbands.

Traditionally, wavelet analysis has been used for array denoising purposes [3]. Denoising is accomplished using thresholding on wavelet coefficients, where certain coefficients are identified as noise bearing components and corresponding signal subbands are eliminated. In EMD, however, there are no coefficients directly related to subbands. The major difference between wavelet thresholding and EMD-thresholding is that in EMD, thresholding is directly applied to IMFs. In fact, for EMD analysis, the term 'subband' would be somewhat an abuse of the term, since there is not a prescribed filtering procedure like in wavelet analysis. When used for denoising, the problem is to properly identify which IMFs contain noise characteristics. Certain modes will consist of mainly noise, whereas other modes will contain both signal and noise characteristics.

For direction of arrival (DoA) estimation applications, subspace decomposition based algorithms (e.g. MUSIC, ESPRIT and their derivations) are widely used [4]. In order to yield their designated superior performance, the aforementioned methods need an appropriate separation of signal and noise subspaces. However, in practice, surrounding physical conditions limit severely the quality of available observation data. This, in turn, negatively affects the performance of subspace decomposition. In this work, we propose an efficient Empirical Mode Decomposition based denoising scheme for DoA estimation. The denoising process increases the performance of subspace algorithms.

2. ARRAY SIGNAL MODEL AND MUSIC ALGORITHM

In the conventional one-dimensional DoA estimation problem, K narrow-band isotropic signal sources centered about the angular frequency ω_0 impinge on a calibrated omni-directional receiving sensor array consisting of M antenna elements from the DoA angles $\theta_1, \theta_2, \cdots \theta_K$ relative to the array direction. Assuming that the sources and the array are located within the same plane, and the transmission medium is homogenous and nondispersive, the M channel array output can be modeled as:

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{s}$ is the $[K \times 1]$-length complex envelope vector of array input signal and $\mathbf{n}$ is the $[M \times 1]$-length noise vector. The steering matrix $\mathbf{A}$ of size $[M \times K]$ contains the steering vectors $\mathbf{a}(\theta_k)$:

$$\mathbf{A} = [\ \mathbf{a}(\theta_1) \quad \mathbf{a}(\theta_2) \quad \cdots \quad \mathbf{a}(\theta_K)\ ] \tag{2}$$

$$\mathbf{a}(\theta_k) = [\ a_1(\theta_1) \quad a_2(\theta_2) \quad \cdots \quad a_M(\theta_K)\ ]^T \tag{3}$$

$a_m(\theta_k)$ is the complex frequency response of m-th array element relating to k-th signal source at frequency ω_0. For an uniform linear array (ULA), the steering vector is closely related to Fourier kernel. The sensor noise being assumed as uncorrelated additive white Gaussian with variance σ^2, the signal model given in has the following covariance structure:

$$\mathbf{R} = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I} \tag{4}$$

where $\mathbf{S} = E\left[\mathbf{s}\mathbf{s}^H\right]$ is the source covariance matrix of size $[K \times K]$ and $\mathbf{I}$ is the identity matrix of size $[M \times M]$. The MUSIC algorithm further exploits the covariance structure given in (4) by decomposing it into signal and noise subspaces. Eigenanalysis gives way to the following formulation:

$$\mathbf{R} = \mathbf{U}_S\mathbf{\Lambda}_S\mathbf{U}_S^H + \mathbf{U}_N\mathbf{\Lambda}_N\mathbf{U}_N^H \tag{5}$$

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where the diagonal matrix $\mathbf{\Lambda}_S$ consists of the K largest eigenvalues, the matrix $\mathbf{U}_S$ consists of corresponding signal eigenvectors, the diagonal matrix $\mathbf{\Lambda}_N$ consists of the $(M-K)$ remaining eigenvalues and the matrix $\mathbf{U}_N$ consists of corresponding noise eigenvectors. The signal subspace $\mathbf{U}_S$ lies in the column space of the steering matrix $\mathbf{A}$. The noise subspace $\mathbf{U}_N$, however, is orthogonal to the signal subspace, and therefore, to the steering matrix. Moreover, it is also orthogonal to each steeering vector $\mathbf{a}(\theta)$ which is a subspace of the steering matrix. Using these relations, the MUSIC pseudospectrum is formed as:

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{U}_N\mathbf{U}_N^H\mathbf{a}(\theta)} \tag{6}$$

The root-MUSIC version of the algorithm uses a polynomial expression for the spectrum. For the standard ULA the steering polynom is:

$$\mathbf{a}(z) = \begin{bmatrix} 1 & z & \cdots & z^{N-1} \end{bmatrix}^T \tag{7}$$

Ideally, the following relation between $\mathbf{a}(z)$ and $\mathbf{a}(\theta)$ should hold:

$$[\mathbf{a}(z)]_{z=e^{j\theta}} = e^{j\left(\frac{N-1}{2}\right)\theta}\mathbf{a}(\theta) \tag{8}$$

Using (6) the MUSIC spectrum becomes

$$P_{MUSIC}(z) = \frac{1}{\mathbf{a}^H\left(\frac{1}{z^*}\right)\mathbf{U}_N\mathbf{U}_N^H\mathbf{a}(z)} \tag{9}$$

If the eigendecomposition is performed on the actual covariance matrix $\mathbf{R}$, $[P_{MUSIC}(z)]_{z=e^{j\theta}} = P_{MUSIC}(\theta)$ must be satisfied. Therefore, the K roots of (9) should lie on the unit circle. But, in practice, we are stuck with an estimated $\hat{\mathbf{R}}$, so we focus ourselves within the unit circle and designate K roots nearest to the unit circle as the DoA parameters.

3. EMPIRICAL MODE DECOMPOSITION

The EMD technique is an algorithm defined signal analyzing method which can be summarized as follows [5]:

1. Given an input signal $x(n)$, identify its local extrema.
2. Interpolate between local maxima to construct an upper envolope $e_{max}(n)$ and between local minima to construct a lower envelope $e_{min}(n)$
3. Compute the mean of the envolopes according to $m(n) = \frac{e_{max}(n)+e_{min}(n)}{2}$
4. Extract the mean from the input signal: $d(n) = x(n) - m(n)$
5. Repeat Steps 1-4 for the detail signal $d(n)$ either until the difference between last two iterations falls below a predetermined threshold or until the detail signal can be considered as zero mean.
6. Store the detail signal as an Intrinsic Mode Function (IMF) and restart iteration from the residual signal $m(n)$.
7. Repeating this *sifting* process L-times, the input signal will be decomposed into L zero-mean IMFs and a residual signal. Ideally, the residual signal should not contain any local extrema. The original signal can be reconstructed easily by summing all IMFs $d_l(n) \quad , \quad l = 1,\dots,L$ and the last residual $m_L(n)$:

$$x(n) = m_L(n) + \sum_{l=1}^{L} d_l(n) \tag{10}$$

4. SIGNAL DENOISING

4.1 Wavelet-based Denoising

Suppose that we wish to recover the denoised sensor output $\mathbf{x}_d$ from additive noise corrupted sensor output $\mathbf{x}$. In the wavelet domain, the sensor output becomes

$$\mathbf{x}_w = \mathbf{x}_{dw} + \mathbf{n}_w \tag{11}$$

where $\mathbf{x}_w = \mathbf{Wx}$, $\mathbf{x}_{dw} = \mathbf{Wx}_d$, $\mathbf{n}_w = \mathbf{Wn}$ and $\mathbf{W}$ is the discrete wavelet transform operator. In wavelet denoising, the wavelet transform of noisy sensor output is thresholded to discard small values which are generally due to additive noise. The denoised estimate of the sensor output is obtained by the inverse DWT of the thresholded coefficients as:

$$\hat{\mathbf{x}}_d = \hat{\mathbf{W}}^{-1}\mathbf{HWx} \tag{12}$$

where $\hat{\mathbf{W}}^{-1}$ is the inverse DWT operator and $\mathbf{H}$ is the threshold operator, a diagonal matrix defined as

$$\mathbf{H} = diag\begin{bmatrix} h(1) & h(2) & \cdots & h(N) \end{bmatrix} \tag{13}$$

For hard thresholding $h(i)$'s are given by [6]

$$h(i) = \begin{cases} 1 & ,|x_w(i)| > T \\ 0 & ,|x_w(i)| \le T \end{cases} \tag{14}$$

and for soft thresholding

$$h(i) = \begin{cases} \left(1 - \frac{T}{|x_w(i)|}\right) & ,|x_w(i)| > T \\ 0 & ,|x_w(i)| \le T \end{cases} \tag{15}$$

The threshold T defined by Donoho [7] is

$$T = \hat{\sigma}\sqrt{2\ln N} \tag{16}$$

where $\hat{\sigma}^2$ is noise variance and N is the length of the input signal. For large N, $\hat{\sigma}^2$ can be estimated from the wavelet coefficients of finest scale.

4.2 EMD-based Denoising

The idea of thresholding also applies to EMD-based denoising. In the case of white Gaussian noise, the noise-only energy of the modes decreases logarithmically [8]. The first mode, carrying the highest amount of noise energy, will consist of mainly noise, the effect of noise should gradually weaken with higher modes. Therefore, the energy of a mode can be used as a measure to identify information bearing signal components. If the calculated energy of a mode is close to the noise-only energy, the mode is identified as noise-only and discarded from the partial reconstruction.

In [8], an iterative thresholding technique, named as EMD interval thresholding (EMD-IIT), is proposed for the EMD-based denoising. For convenience, we summarize EMD-IIT here as follows:

1. Perform EMD on $x(n)$
2. Leave out the first IMF: $x_p(n) = \sum_{l=2}^{L} d_l(n)$
3. Construct a new signal using the partial reconstruction $x_p(n)$ and a randomly circularly shifted version of the first IMF: $x_a(n) = x_p(n) + d_{1a}(n)$. Since the first IMF contains mainly noise-only components, $x_a(n)$ can be considered as a different noisy version of $x(n)$.

4. Perform EMD on $x_a(n)$
5. Perform denoising to the IMFs of $x_a(n)$:
   (a) Calculate the noise-only IMF energies according to

$$\hat{E}_l = \frac{\hat{\sigma}^2}{0.72} 2.01^{-l} \quad , \quad l = 2,3,4,\ldots \tag{17}$$

   where $\hat{\sigma}^2$ is noise variance which is approximately the variance of the first IMF.

   (b) Calculate the threshold

$$T_l = c\sqrt{E_l 2 \ln N} \tag{18}$$

   where N is the length of the input signal, and c is a arbitrary constant.

   (c) Perform interval-based thresholding for all extrema, that is, if the absolute value of i-th extremum falls below the threshold T_l, set all samples in the interval $\mathbf{z}_i = [z_i : z_{i+1}]$, which contains the i-th extremum, and defined between the previous and the next zero crossings, to zero:

$$d_l(\mathbf{z}_i) = \begin{cases} d_l(\mathbf{z}_i) & , |d_l(ex_i)| > T_l \\ 0 & , |d_l(ex_i)| \leq T_l \end{cases} \tag{19}$$

6. Repeat Steps 3-5 P times to obtain the denoised versions $\tilde{x}_p(n)$ of the input signal $x(n)$.
7. Averaging P denoised signal estimates, obtain the smoothed denoised signal $x_d(n) = \frac{1}{P}\sum_{p=1}^{P} \tilde{x}_p(n)$

5. THE PROPOSED DENOISING METHOD

For the problem of DoA estimation, it is possible to improve the performance of the MUSIC algorithm in a low SNR environment by applying the denoising methods given in Section 3 to the noisy sensor outputs as a preprocessing step. However, if the sensor snapshots are uncorrelated, the denoising schemes will fail, since both the noise and the information signal will be spectrally white, therefore it is necessary to implement a colored signal model which will produce a signal form with autocorrelation form such as

$$r_k(\tau) = \sigma_k^2 e^{-\alpha_k|\tau|} \tag{20}$$

α_k being the decay rate of the autocorrelation sequence of k-th source signal. Since the standard MUSIC algorithm fails in the case of full coherence, the source signals may be modeled as mutually uncorrelated or partly correlated.

Although there has been some work on wavelet-based array denoising [3, 6], no applications of EMD-based denoising to DoA estimation has been reported so far. In this work, we propose an efficient EMD-denoising based DoA estimation method similar to wavelet-based denoising approaches, which can be summarized as follows:

- Step 1: By taking N temporal snapshots from the array sensors, construct the M channel array observation matrix $\mathbf{X}$ of size $[M \times N]$.
- Step 2: For each sensor output, which corresponds to rows of $\mathbf{X}$, perform iterative EMD interval-thresholding (EMD-IIT) denoising to obtain the denoised matrix $\mathbf{X}_d$
- Step 3: Apply either the MUSIC or root-MUSIC algorithm on the denoised data matrix $\mathbf{X}_d$ and estimate the spatial spectrum and/or the related DoAs of the signal sources.

It is worthwhile to note that the proposed method performs an adaptive basis function extraction individually for each array channel. Thus, the coherent information between channels (i.e. the DoA estimate itself) cannot be guaranteed to preserved. Nevertheless, as the simulations in the next section also demonstrate, no significant information loss concerning the source localization occurs whenever the source signals are uncorrelated.

6. COMPUTER SIMULATIONS

In the simulations, two signal sources from 10^o and 15^o relative to array direction impinge on a uniform linear array. The source signals are mutually uncorrelated ($\mathbf{S} = \mathbf{I}$), the autocorrelation of each source is exponentially decaying according to (21), all decay rates being $\alpha_k = \frac{1}{200}$. The ULA consists of $M = 8$ receiving antennas. The distance between two neighboring array antennas is $d = 0.5\lambda$. $N = 1024$ temporal snapshots are available from each sensor.

In the case of wavelet-based denoising, the analysis was performed on a 4-level decomposition using db22 wavelet from the Daubechies family. The resulting wavelet coefficients were hard thresholded using the threshold given in (14). In the case of EMD-based denoising, the constant c in (18) is chosen as $c = 0.7$, partial signal reconstruction starts from 6th IMF and we average $P = 5$ iterations of EMD-denoised versions of sensor outputs.

In the first set of simulations, temporal and spatial white Gaussian noise is added to array sensor outputs, so the noisy signal has a signal to noise ratio of 0dB. Spectral MUSIC has been performed on 50 independent noisy datasets. In Figure 1, the mean of the MUSIC spectra is shown. As we can deduce from the figure, without denoising, the MUSIC algorithm is not able to resolve the sources, whereas wavelet-denoising slightly increases the resolution. The EMD-denoising, however, clearly increases the resolution of spectral MUSIC.

In the second set of simulations, Root-MUSIC has been performed on different noise levels. For each noise level, the algorithm has been executed on 50 independent noisy datasets and their denoised versions. For each of the simulations, the mean squared errors related to the estimated DoA angles are calculated using

$$MSE = \frac{1}{K}\sum_{k=1}^{K}\left(\hat{\theta}_k - \theta_k\right)^2 \tag{21}$$

In Figure 2 the mean of the MSE for each noise level is shown. As it is seen from the figure, the denoising stage reduces the MSE of the estimates allowing more realistic estimates in noisy environments. For SNR levels lower than 0dB, EMD-based denoising yields better DoA estimates than wavelet-based denoising. The performance improvement detoriates with increasing SNR levels. This is partly due to the fact that a fixed number of IMFs were used for reconstruction for all noise levels. To increase the estimation performance, one can search for the optimal number of IMFs and threshold values at each noise level, though at the expense of increased computational complexity. The profound reason however, is that the EMD denoising technique summarized in Section 4.2 identifies the first IMF as noise-bearing signal component and alters it for the purpose of denoising. When the SNR level is relatively high,

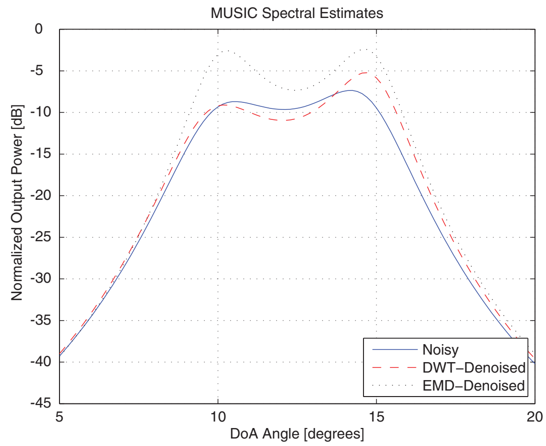

Figure 1: Spectral-MUSIC Estimates (SNR: 0dB)

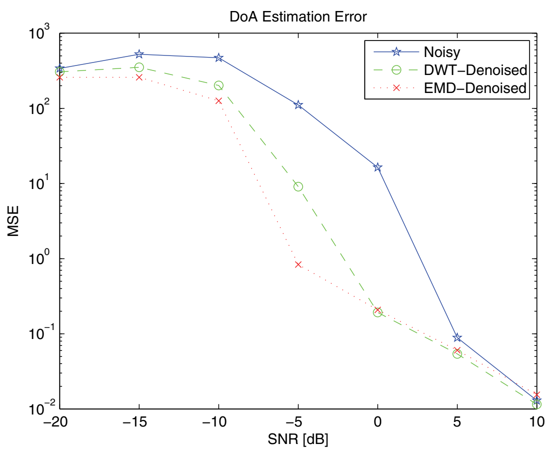

Figure 2: Root-MUSIC Mean Squared Estimation Errors

this assumption does not hold true, thus altering the IMF also alters the inherent information component. A recent work [9] addresses this problem and proposes a denoising technique called Clear Iterative EMD Interval-Thresholding (EMD-CIIT) to enhance the denoising performance in high SNR levels.

7. SUMMARY

In this work, we have inspected the one-dimensional direction of arrival (DoA) estimation of a preprocessed array. We have utilized spectral MUSIC and root-MUSIC as DoA estimators. To increase the estimation performance we have proposed an EMD-based denoising scheme which is similar to wavelet-based denoising approaches. We have shown that, especially in low-SNR scenarios, EMD-based denoising increases the resolution and reduces the mean squared error of the DoA estimates compared to undenoised case and wavelet-based denoising.

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