A NEW CRITERION FOR DETERMINING THE EFFICIENCY OF CDMA CODES

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ABSTRACT

In Code Division Multiple Access (CDMA) systems, the traditional method for selecting spreading codes in order to reduce multiple-access interference (MAI) is to compare the maximum absolute value of the periodic (even) Cross-Correlation Function (CCF) for different code-sets. In many cases this method fails to introduce the best code-set. In this paper we propose a new criterion which is based on the Bit Error Rate (BER) performance of one set of CDMA codes. Through the development of this method, we can also compare and select the best code-set for our application. Also, our method gives us this possibility to consider the channel noise together with the inter-user interference. Using the proposed method, we analyzed the performance of some well-known Direct Sequence (DS) CDMA spreading codes. While the traditional method to calculate the BER is based on the approximation of the MAI using central-limit theorem, our approach is applicable even when the number of codes is too small to use this theorem. Finally we have analyzed the performance of a family of ternary codes.

1. INTRODUCTION

Spreading sequence design for CDMA (Code Division Multiple Access) applications is still an open area for research [1]. The efficiency of the codes participating in a CDMA scheme depends on their distinguishability [2]. A huge amount of works have been done to introduce efficient codes with the lowest MAI [3]. The problem rises here is how to select the set (family) of codes which results in the lowest total interference and gives the smallest probability of error. The traditional method for comparison of spreading code-sets in order to reduce MAI is to use the maximum absolute value of the periodic (even) CCF. This method fails to introduce the best code-set in many cases (one such example is given in section 3), because it only considers the peak values. However, not only the peak values, but also all cross-correlation values affect the overall performance of the system. So, the maximum of Auto-Correlation Function (ACF) which occurs in delay 0, must be compared with a combination of other CCFs because the effective amount of this combination can be higher than the intended ACF. This observation is our principal motivation to look for another criterion to test the signature signals in a DS-CDMA system.

In this paper, we propose a new comparison criterion to choose the CDMA code-set which is based on the BER performance of the system. Our new method offers two significant advantages over the previous methods: 1-The accuracy of the result (we propose an exact method to calculate Probability Density Function (PDF) of MAI), 2-The traditional method to find the PDF of MAI in a CDMA network is based on an approximation using central-limit theorem, which needs a large number of codes participating in the network. Therefore the final PDF of MAI is always approximated by a Gaussian distribution, but, our method is applicable even when the central-limit theorem cannot be used.

The paper is organized as follows: In section 2, we provide the problem formulation and we review the traditional method. In section 3 we introduce our method and in the remaining sections we compute and compare the efficiency of some of well-known codes using our method, namely some families of the m-sequences and ternary codes.

2. PROBLEM FORMULATION

The selection of a family of CDMA codes is usually performed using the following criterion, namely the minimization of the absolute value of periodic correlation between signals. Consider a set of J sequences, each with period N, denoted by $\{a_n^{(j)}\}, j = 1,2,...,J$. The periodic cross correlation $C_{jk}(\tau)$ at shift τ between two sequences from this set is defined as

$$C_{jk}(\tau) = \sum_{n=1}^{N} a_{n+\tau}^{(j)}(a_n^{(k)})^* \qquad (1)$$

The maximum out of phase periodic auto-correlation magnitude C_A for this signal set is defined as

$$C_A = \max_j \max_{0<\tau<N} |C_{jj}(\tau)| \qquad (2)$$

And the maximum cross correlation magnitude C_C between signals in this set is given by

$$C_C = \max_{j\neq k} \max_{0\leq\tau<N} |C_{jk}(\tau)| \qquad (3)$$

According to the traditional criterion, the signal sets with the smallest $C_{max} = \max(C_A, C_C)$ is preferred (in the sense that it makes smaller MAI) , it is expected that a signal set which optimizes network performance probably has a small value of C_{max} and it is argued that no other design criteria have proven tractable in choosing sequences up to now [2]. But from an analytic point of view the above criterion for sequence selection does not correspond to any DS-CDMA network-performance measure, like BER.

3. PROPOSED CRITERION

The issue requiring scrutiny is the selection of the best family of the codes such that the *total* effective cross-correlation

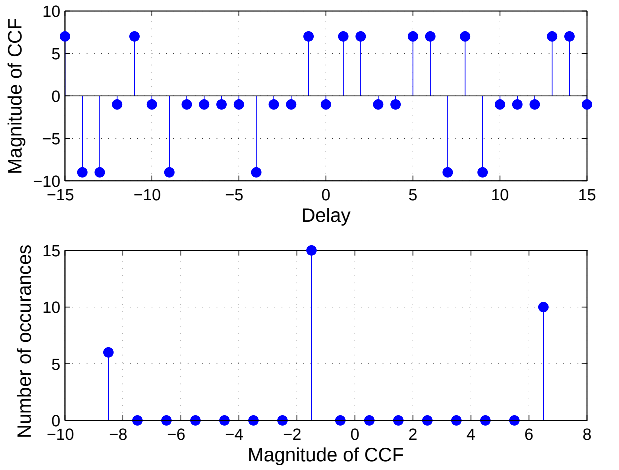

Figure 1: An example of $C_{ik}(\tau)$ and the corresponding $P(n)$ for a pair of Gold codes

between them would be minimized. However, we should exactly define the meaning of the word "total" here; it means the code-family that makes the smallest probability of error due to MAI in the receiver. The traditional method can not provide analytic answer to the problem of code selection and risks leading to a wrong conclusion. To better understand this problem, let's consider two sets of sequences $\{a_n^{(j)}\}$ and $\{b_n^{(j)}\}$, such that C_{max} for $\{a_n^{(j)}\}$ would be less than C_{max} for $\{b_n^{(j)}\}$, whereas the effective CCF of the first set $\{a_n^{(j)}\}$ would be much higher than that of the second set $\{b_n^{(j)}\}$. As one such example let's compare a set of preferred-pair of m-sequences ($\{a_n^{(j)}\}$) and a small set of Kasami sequences ($\{b_n^{(j)}\}$) proposed in [4] with the code length of 63. C_{max} for the first set is 23 and for the second set is 9. According to the traditional method the second set is preferable, however using our method, we will show that the effective MAI produced by the second family is almost twice as that of the first family (section 4.1).

To go beyond colloquial descriptions such as maximum-ACF and average-CCF, we propose a short method to calculate the PDF of MAI and hence the probability of error. We are going to use this principal fact that in a CDMA channel, the cross-correlation functions are added together, so their PDF will be convolved. As the first step to develop our model, consider the periodic cross correlation $C_{ik}(\tau)$ between sequences $\{a_n^{(i)}\}$ and $\{a_n^{(k)}\}$. If we consider the relative occurrence of various amplitudes in $C_{ik}(\tau)$, it shows the PDF of the interference between $\{a_n^{(i)}\}$ and $\{a_n^{(k)}\}$. We will show the result by $P(n)$ where n is the amplitude of CCF. $P(n)$ can be easily achieved by normalizing the histogram of $C_{ik}(\tau)$ and shows what proportion of amplitudes in $C_{ik}(\tau)$ equals n. Figure 1 shows an example of $C_{ik}(\tau)$ and the corresponding $P(n)$ for a pair of Gold codes.

However there is a problem with $C_{ik}(\tau)$ when dealing with information-modulated codes. In fact, when the spreading codes are modulated with information bits, there are different possibilities for the CCF depending on the successive information bits of each user (as shown in figure 2). In this figure the successive information bits for $\{a_n^{(k)}\}$ in the first case are $(+1,+1)$, whereas in the second case they are $(-1,+1)$, so the CCF is totally different for these cases. We assume that each information bit is modulated (multiplied) by exactly one complete period of the spreading code and also we assume equal probability of occurrence for different cases in figure 2. Lets denote the CCF for the first case in figure 2 by $C_{ik}^{++}(\tau)$. (i.e. the case with successive +1 information bits for user k)

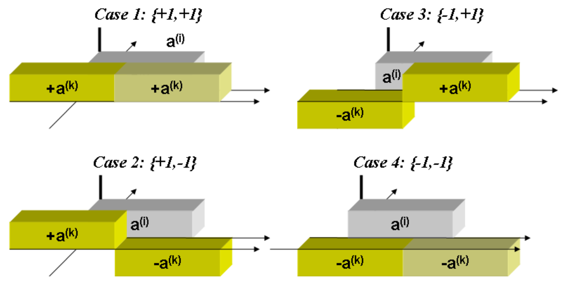

Figure 2: Successive information bits of user k

Using the same terminology we have $C_{ik}^{+-}(\tau)$, $C_{ik}^{-+}(\tau)$ and $C_{ik}^{--}(\tau)$ respectively for the information bits $(+1,-1)$, $(-1,+1)$ and $(-1,-1)$. Obviously we have

$$C_{ik}^{+-}(\tau) = -C_{ik}^{-+}(\tau) \tag{4}$$

$$C_{ik}^{--}(\tau) = -C_{ik}^{++}(\tau) \tag{5}$$

corresponding to the above notation, we have:

$$P_{ik}^{+-}(n) = P_{ik}^{-+}(-n) \tag{6}$$

$$P_{ik}^{--}(n) = P_{ik}^{++}(-n) \tag{7}$$

Since all information bits are iid (independent and equally-probable) we can average the probability functions in (6) and (7) to get the total PDF of interference between users i and k. Let us denote this average by $P_{ik}^{total}(n)$. Then by convolving the PDF of interference between user i and all of the other users, we have the PDF of total Cross-Correlation on our intended code. In other words, we have the exact PDF of MAI in the channel.

$$P_{ik}^{final}(n) = P_{i1}^{total}(n) * P_{i2}^{total}(n) * \cdots * P_{iN}^{total}(n) \tag{8}$$

Error happens when the overall effect of MAI has a bigger absolute value than A_i with opposite sign, hence, we can calculate the total probability of error by integrating (summation) $P_{ik}^{final}(n)$ from minus infinity to $-A_i$ (where $A_i = C_{ii}(0)$).

$$P_e = \sum_{-\infty}^{n=-C_{ii}(0)} P_{ik}^{final}(n) \tag{9}$$

P_e is the probability of the event that the interference exceeds the maximum ACF of the intended code ($C_{ii}(0)$). The traditional method to calculate this error is based on central limit theorem, so it's only applicable when:

1- The number of the codes in each family is large enough.

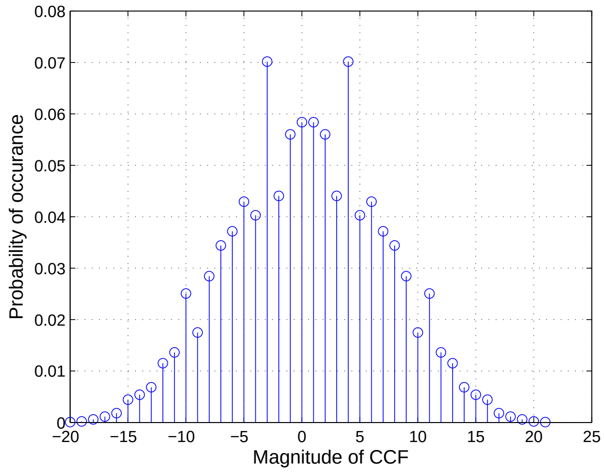

Figure 3: PDF due to MAI for a small set of Kasami sequences.

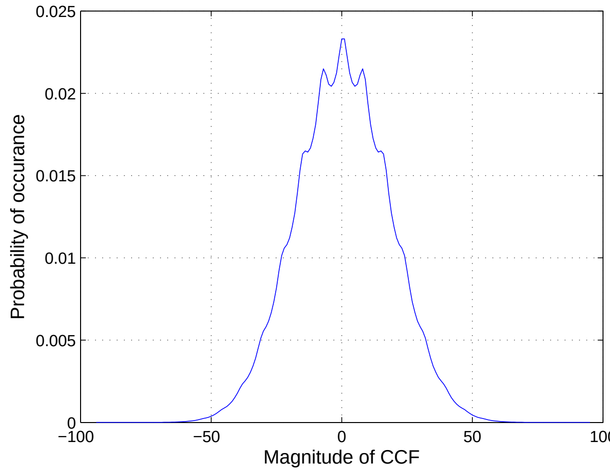

Figure 4: PDF due to MAI for six codes in a large set of Kasami sequences.

2- The PDF of cross-correlation functions are similar to each other.

Thus the proposed method has two advantages over the traditional method, because its free of the above restrictions. Our method can be summarized as follows:

1- Calculation of cross-correlation functions ($C_{ik}^{++}(\tau)$ and $C_{ik}^{+-}(\tau), k = 1, 2, \ldots, J$) between the intended code $\{a_i^n\}$ and other codes $\{a_k^n\}$.

2- Calculation of the PDFs $P_{ik}^{++}(n)$ and $P_{ik}^{+-}(n), k = 1, 2, \ldots, J$, based on the results of step one

3- Averaging $P_{ik}^{++}(n)$, $P_{ik}^{+-}(n)$, $P_{ik}^{-+}(n)$ and $P_{ik}^{--}(n)$ to get $P_{ik}^{total}(n)$ which gives the total PDF of interference between users i and k.

4- Convolving the PDFs calculated in step 3. The result is the exact PDF of MAI for user i.

5- Integrating $P_{ik}^{final}(n)$ from $C_{ii}(0)$ to infinity. Result is the total probability of error for the intended code.

6- Repeating the above steps for each code in the code-set and averaging the results to obtain the average P_e for the whole family.

In the following section, we will illustrate the performance of proposed method through some examples.

4. CASE STUDIES

4.1 m-sequences

As the first example let's consider the m-sequences. We calculate the performance of some well-known families of m-sequences in terms of P_e (probability of error due to MAI). First let's consider an example with non-Gaussian PDF for MAI. Figure 3 shows this PDF for a small set of Kasami sequences with code length of 15 and family size of 4. This non-Gaussian shape prohibits the usage of Central-Limit theorem, so the classic approach to calculate the P_e is not useful. We encounter this case in many applications. Here we consider a more common application.

It's very useful to calculate the performance of the system when just a fraction of the subscribers (codes) are active and make interference. This case is important because in practical networks, typically a fraction of the network capacity is used, so the precise performance of the channel as a function of the number of active users, is of high importance. As an example let's consider a large set of Kasami sequences with length 63 and family size of 65. Figure 4 shows the PDF of error for this family when only 6 codes are active in the network. Again a non-Gaussian PDF is achieved. Using the proposed method, we have shown the complete result in figure 5, where the probability of error P_e is drawn versus the number of the active users (i.e. the number of codes involved in the simulation).

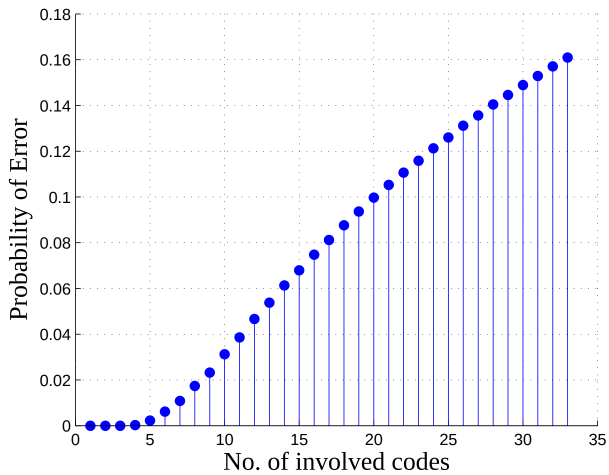

Figure 5: Probability of error due to MAI for different number of codes involved in a set of Gold sequences.

Table 1 shows the amount of error due to MAI for some well-known families of m-sequences: Preferred pair of m-sequences, Gold sequences, small and large set of Kasami sequences. From this table it can be seen that the maximum-length and small set of Kasami sequences have the lowest MAI which is reasonable because the family size for those codes is small compared with the code length. The performance of Gold and large set of Kasami sequences is almost

Code name	*Code length*	*Family size*	*Pe*
Maximum-length	15	2	0
Maximum-length	31	6	0.0079
Maximum-length	63	6	0.00028563
Gold	31	33	0.1609
Gold	63	65	0.1593
Small Kasami	15	4	0.0069
Small Kasami	63	8	0.00056658
Large Kasami	15	15	0.1530
Large Kasami	63	65	0.1596

Table 1: Probability of error due to MAI for some well-known families of m-sequences.

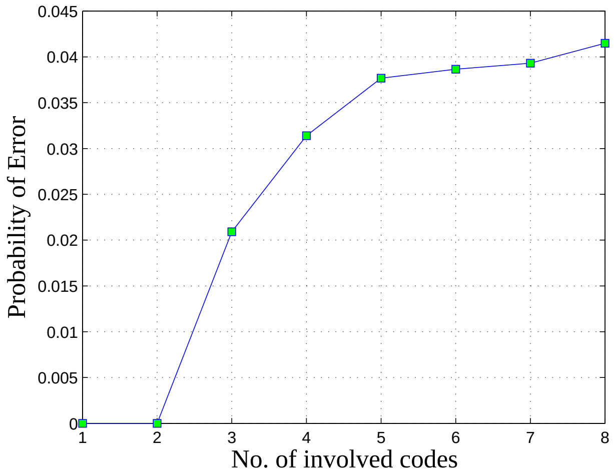

Figure 6: Probability of error due to MAI for different number of codes involved in a set ternary codes.

comparable. As another advantage of this method; we can consider the effect of the additive noise by simply convolving its PDF with the PDF of MAI. Also the reader can easily extend this method to calculate the performance of optical spreading codes (where the amplitudes are $\{0, +1\}$) through modifying steps 2 and 3 in our algorithm.

4.2 Ternary codes

In this section, we show the usage of the proposed method to evaluate the performance of ternary codes. Ternary sequences are a class of spreading sequences with better ACF and CCF properties compared to binary sequences. The elements in a ternary sequence are chosen from the alphabet $\{-1, 0, +1\}$. Here we have selected a set of ternary sequences introduced in [5], with code length of 16 and family size of 8. The codes are shown in the columns of matrix H given in equation (10). Using the proposed method, the probability of error due to MAI as a function of the number of active users for this set of ternary codes is calculated and shown in figure 6.

$$H = \begin{bmatrix} 0,0,0,+1,0,0,0,+1,0,0,0,+1,0,0,0,-1 \\ 0,0,0,+1,0,0,0,+1,0,0,0,-1,0,0,0,+1 \\ 0,0,0,+1,0,0,0,-1,0,0,0,+1,0,0,0,+1 \\ 0,0,0,+1,0,0,0,-1,0,0,0,-1,0,0,0,-1 \\ 0,+1,0,0,0,+1,0,0,0,+1,0,0,0,+1,0,0 \\ 0,+1,0,0,0,-1,0,0,0,+1,0,0,0,-1,0,0 \\ 0,+1,0,0,0,+1,0,0,0,-1,0,0,0,-1,0,0 \\ 0,+1,0,0,0,-1,0,0,0,-1,0,0,0,+1,0,0 \end{bmatrix} \quad (10)$$

5. CONCLUSIONS

In this paper we proposed a new simple method to calculate the PDF of MAI in a set of CDMA codes. Through the development of this method, we found the exact and correct answer to the problem of code selection. The proposed method gives us the possibility to consider the channel noise together with the inter-user interference. Using the proposed method, we have analyzed the performance of some well-known DS-CDMA systems. As an important advantage, our approach is applicable even when the number of the codes is not sufficiently large to get the advantage of central-limit theorem. For the large number of the codes in the code-set, this approach will lead to the traditional method of performance measurement for CDMA systems (which uses the Central-Limit Theorem). Finally we have analyzed the performance of some well-known families of m-sequences like sets of Kasami sequences and ternary codes.

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