INTERPOLATION-BASED CALIBRATION FOR NEAR-FIELD SOURCE LOCALIZATION

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ABSTRACT

In this paper, we present an interpolation-based method for near-field array calibration. Based on an analysis of the received signal model, the position-dependent errors are modeled by a correction matrix. By interpolating the correction matrix, pseudo calibration data can be obtained numerically. The proposed method allows us to calibrate the array responses on a dense grid by taking a small number of measurements. It is shown via simulations that this method has a good performance in near-filed array calibration.

Index terms- source localization, array signal processing, position measurement, calibration

1. INTRODUCTION

Source localization with an array of sensors has become a popular topic since last several decades. Most proposed methods presume the knowledge of the array response such as Minimum Variances in [1], the subspace-based methods in [5], Maximum Likelihood in [10] and approximated approaches for near-field sources localization in [3,4,13]. It is shown in [11, 12] that any inaccuracy in the presumed array response results in severe degradation of estimation performance. Array calibration via the measurement of the array response, is therefore a crucial step in the implementation of localization techniques.

Many calibration approaches have been developed for localization of far-field sources. For example, the global calibration method is proposed in [9]; the self-calibration techniques are summarized in [7] and methods in presence of multi-path are introduced in [6]. When the sources are located in near-field, the parameters to estimate are both the directions of arrival (DOAs) and the ranges. The measurement of array response must then be carried on in a two dimension (2D) environment; however, it is too expensive.

Recently, an interpolation-based calibration method for far-field source localization has been proposed in [8]. From measurements made on a sparse calibration grid, this approach allows us to obtain pseudo measurements data by interpolating the original data. The DOA-dependent errors are well corrected due to the use of local error model.

In near-field situation, the model errors (e.g. caused by multi-path propagation) may depend not only on the DOAs, but also on the ranges. In this paper, we propose an interpolation algorithm for array calibration in near-field. The position-dependent errors are modeled by a correction matrix which can be estimated from a small number of measurements. By interpolating the correction matrix, new calibration data on a dense grid can be calculated numerically. Furthermore, instead of the optimization-based interpolation algorithm in [8], spline interpolations [2] are used in this approach for sake of efficiency. This proposed method is tested by simulations and shown to have good performance on the estimation of source positions.

The rest of this paper is organized as follows: in section two, the background and signal model are introduced; section three presents the interpolating algorithm; section four elaborates on the application of the calibration data to localization algorithms; the proposed method are verified via simulations in the fifth section and the whole paper is concluded in section six.

2. BACKGROUND

Consider a near-field scenario of K uncorrelated narrow-band signals impinging to the $(2M+1)$-element array in an environment with multi-path propagation as illustrated in figure 1. Let the array center be the reference point of the coordinates system. The inter-element spacing of the array is d. The received signal at the array can be modeled as

$$\mathbf{x}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k, r_k) s_k(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{a}(\theta_k, r_k)$ denotes the $(2M+1) \times 1$ vector of the array response to the k^{th} source located at (θ_k, r_k), $s_k(t)$ is the signal emitted by the k^{th} source and $\mathbf{n}(t)$ is the vector of additive white Gaussian noise (AWGN).

To estimate the source positions from the received signal in (1), we need the knowledge of the array response $\mathbf{a}(\theta, r)$ to searching area. Most localization algorithms presume $\mathbf{a}(\theta, r)$ to be simply the phase shift vector due to the direct propagation from source location to the array. In this case, the array response $\mathbf{a}(\theta, r)$ can be calculated from Green function. In a real environment, the contributions to the array response may consists of several parts:

1. Direct propagation from source to the array
2. Multi-path propagation from source to the array
3. Errors caused by non-perfect effects (e.g. non-synchronization, channel fading, position errors of the array, etc.)

The array response vector must then be modeled as

$$\mathbf{a}(\theta, r) = \rho_d \mathbf{a}_d(\theta, r) + \sum_{i=1}^{I} \rho_i \mathbf{a}_i(\theta, r) + \mathbf{b} \tag{2}$$

where $\mathbf{a}_d(\theta, r)$ denotes array response in phase of direct propagation from the source to the array, $\mathbf{a}_i(\theta, r)$ is the response in phase of the i^{th} indirect path, $\mathbf{b}$ is the vector of

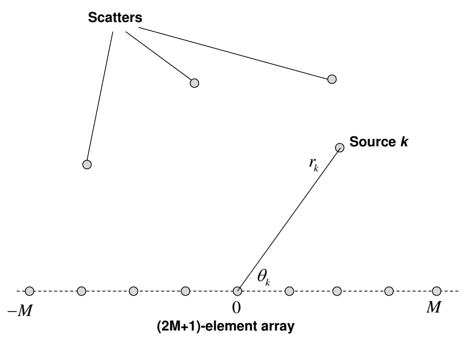

Figure 1: Array configuration: a 2M+1-element array is employed to localize sources in presence of multi-path propagation and model errors

errors caused by non-perfect effects, ρ_d is the signal attenuation factor of the direct propagation, ρ_i is the attenuation factor of the propagation and the reflection in the i^{th} indirect path, I denotes the number of indirect paths. Without loss of generality, we can normalize the propagation attenuation by taking $\rho_d = 1$.

To simplify the array response model (2), we introduce a correction matrix $\mathbf{Q}(\theta, r) \in C^{(2M+1)\times(2M+1)}$ which is dependent on (θ, r). We can rewrite (2) as

$$\mathbf{a}(\theta, r) = \mathbf{Q}(\theta, r)\,\mathbf{a}_d(\theta, r) \tag{3}$$

We note that all the contributions to the array response can be modeled by (3). If mutual coupling of the array is neglected, $\mathbf{Q}(\theta, r)$ is a diagonal matrix . Under this hypothesis (which is considered in the whole paper), the diagonal matrix $\mathbf{Q}(\theta, r)$ can be obtained by

$$q_m(\theta, r) = \frac{a(\theta, r, m)}{a_d(\theta, r, m)} \tag{4}$$

where $a(\theta, r, m)$ and $a_d(\theta, r, m)$ are the elements of $\mathbf{a}(\theta, r)$ and $\mathbf{a}_d(\theta, r)$ corresponding to the m^{th}sensor, $q_m(\theta, r)$ is the diagonal element in the $(2M+1-m)^{th}$ row of $\mathbf{Q}(\theta, r)$. Practically, $\mathbf{a}(\theta, r)$ is calibrated via measurements. $\mathbf{a}_d(\theta, r)$ is the phase shift in the direct path, which can be calculated with Green function and the positions of the array elements. We note that the array configuration can be arbitrary. To simplify the presentation, we consider uniform linear arrays (ULA) in this paper. Then, we write $\mathbf{a}_d(\theta, r)$ as

$$\mathbf{a}_d(\theta, r) = \left[e^{j\tau_{-M}}, \ldots, e^{j\tau_m}, \ldots, e^{j\tau_M}\right]^T \tag{5}$$

with

$$\tau_m = \frac{2\pi}{\lambda}\left(\sqrt{r^2 + m^2 d^2 - 2rmd\cos\theta} - r\right) \tag{6}$$

where the superscript T denotes the matrix transposition and λ is the wavelength of the source signal.

3. INTERPOLATION OF CALIBRATION

To calibrate the array response in presence of multi-path propagation, we are required to make measurements. The precision of localization is dependent on the density of the calibration grid. A large number of measurements (on a dense calibration grid) are required to achieve a good performance in localization, however, it is too expensive to make lots of measurements.

Another solution for the calibration of array response is to interpolate the calibration data measured on a sparse grid. Since the array response varies largely with respect to source position, interpolating $\mathbf{a}(\theta, r)$ may bring a lot of noise, which could degrade greatly the precision of the localization. From Green function and array response model (3), we know that the attenuation of multi-path propagation is bigger than that of direct propagation, so the most important contribution of the array response comes from the direct propagation. Consequently, we could consider that the correction matrix $\mathbf{Q}(\theta, r)$ in (3) varies near identical matrix. The interpolation algorithm is then performed to the correction matrix $\mathbf{Q}(\theta, r)$ via five steps.

1. Measure the array response $\mathbf{a}(\theta, r)$ on a sparse grid with
$$\theta = \theta_{meas_1}, \theta_{meas_2}, \ldots, \theta_{meas_J}$$
and
$$r = r_{meas_1}, r_{meas_2}, \ldots, r_{meas_L}.$$
2. Choose the density of the new calibration grid:
$$\theta = \theta_{new_1}, \theta_{new_2}, \ldots, \theta_{new_O}$$
and
$$r = r_{new_1}, r_{new_2}, \ldots, r_{new_P},$$
where $O > J$, $P > L$ with O and P being positive integers which indicate the density of the new grids.
3. Estimate the correction matrix $\mathbf{Q}(\theta, r)$ at all the measured positions by (4), (5) and (6), i.e. for $\theta = \theta_{meas_1}, \theta_{meas_2}, \ldots, \theta_{meas_J}$ and $r = r_{meas_1}, r_{meas_2}, \ldots, r_{meas_L}$.
4. Calculate the correction matrix $\mathbf{Q}(\theta, r)$ at all the interpolated positions by using cubic spline interpolation [2] (More details are presented in appendix).
5. Calculate the array response $\mathbf{a}(\theta, r)$ by (3) with the results obtained in the fourth step. The new calibration data are obtained with $\theta = \theta_{new_1}, \theta_{new_2}, \ldots, \theta_{new_O}$ and $r = r_{new_1}, r_{new_2}, \ldots, r_{new_P}$.

4. MUSIC METHOD FOR LOCALIZATION

With the pseudo array response obtained from the last section, the DOAs and ranges of the sources can be estimated by 2-D MUSIC method. The pseudo MUSIC spectrum is given by

$$P_{MUSIC}(\theta, r) = \frac{1}{\mathbf{a}^H(\theta, r)\,\mathbf{U}_n\mathbf{U}_n^H\,\mathbf{a}(\theta, r)} \tag{7}$$

where θ and r denote positions on the corresponding calibration grid, the superscript H indicates matrix conjugate transposition and $\mathbf{U}_n \in C^{(2M+1)\times(2M+1-K)}$ is the matrix of the eigenvectors associated with the smallest $2M+1-K$ eigenvalues of the covariance matrix of the received signal vector defined as

$$\mathbf{R} = E\left[\mathbf{x}(t)\,\mathbf{x}(t)^H\right]. \tag{8}$$

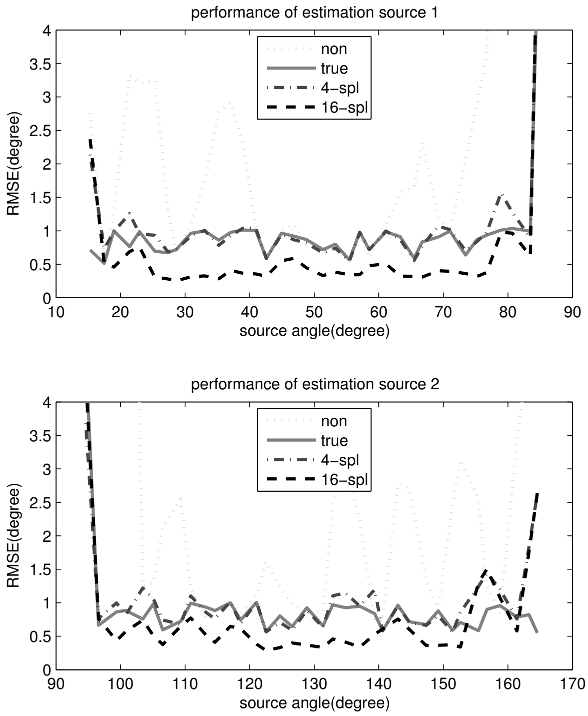

Figure 2: Four calibration strategies are compared in terms of the RMSE of the estimates for 2 moving sources: non-calibration (solid line); true calibration (dotted line); 4 spline interpolation (dashed line); 16 spline interpolation (dashdot line).

Since the array response to the source locations spans the signal subspace of the covariance matrix, the peaks of the pseudo MUSIC spectrum in (7) imply the source positions.

5. NUMERICAL RESULTS

In this section, two simulations are carried out to test the performance of the proposed calibration method.

5.1 Calibration by interpolation

First, we employ a 5-element sensor array to localize 2 moving sources: one moving from 15° to 85°; the other moving from 165° to 95°. The distance between the sources and the array is fixed at 3λ. The inter-element spacing of the array is $\lambda/4$. Two scatters are placed at $(50°, 5.5\lambda)$ and $(100°, 5.2\lambda)$ with their reflection coefficients being $0.15+0.2j$ and $0.2-0.1j$ respectively. 100 runs of independent Monte Carlo experiments are carried out for signal to noise ratios (SNRs) of the received signal fixing at 10dB. 200 snapshots are used for the estimation of the covariance matrix of the received signal. The results are obtained via MUSIC method using four different calibration strategies:

Non-calibration: No calibration is performed. The array response is considered to be the phase shift due to the direct propagation.

True-calibration: 81 measurements are taken from 10° to 170° with interval of 2°.

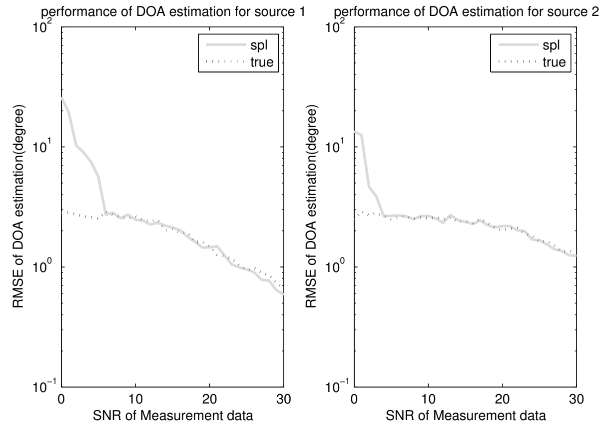

Figure 3: RMSE of DOA estimation versus SNR of measurement noise: spline interpolated calibration (solid); true calibration (dotted)

4-spline-interpolation: 21 measurements are taken from 10° to 170° with interval of 8°. With spline interpolation, 81 points are finally calibrated for the localization (with interval of 2°).

16-spline-interpolation: The same measurements are taken as 4-spline interpolation. 321 points are finally calibrated from 10° to 170°(with interval of 0.5°)

The results of this simulation are demonstrated in Fig. 2 in terms of the Root Mean Square Errors (RMSE) of the estimates. We can see that the results from non-calibrated data are largely biased due to the model errors. An accordance is shown between the results from true-calibration data and the 4-spline-interpolation calibration data, because the same density of calibration is used in these two strategies. The results from 16-spline-interpolation calibration data are better than the others because a denser calibration grid is chosen.

5.2 Interpolating-based Calibration in existence of noise

Since the calibration of the array response is performed via measurements, noise is unavoidable in the measurement data. In this simulation, we carry out 200 independents trials to test the performance of the proposed calibration method in presence of measurement noise. A 5-element sensor linear array is employed to localize 2 sources with their positions being $(70.3°, 2.93\lambda)$ and $(109.3°, 4.12\lambda)$. The inter-element spacing of the array is $\lambda/4$. Two scatters are placed at $(50°, 5.5\lambda)$ and $(100°, 5.2\lambda)$ with their reflection coefficients being $0.25+0.2j$ and $0.3-0.1j$ respectively. The SNR of the received signal is 10 dB and 200 snapshots are used for the estimation of the covariance matrix. Additive white Gaussian noise is considered in the measurement data (i.e. the vectors $\mathbf{a}(\theta, r)$ in the first step of the algorithm in section 3) with the SNR varying from 0dB to 30dB. The results from two calibration strategies are compared:

True-calibration: 81×81 measurements are taken from 50° to 130° and from 1λ to 5λ (every 1° and 0.05λ).

Spline-interpolation: 21×21 measurements are taken from 50° to 130° and from 1λ to 5λ (every 4° and 0.2λ). With spline functions, 81×81 points are totally calibrated by interpolating the noisy measurement data.(every 1° and 0.05λ).

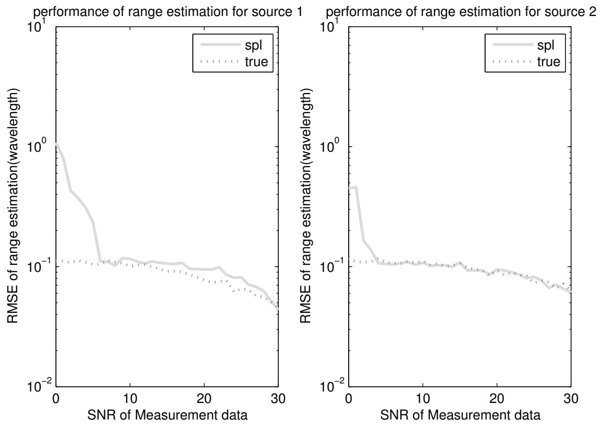

Figure 4: RMSE of range estimation versus SNR of measurement noise: spline interpolated calibration (solid); true calibration (dotted)

The results are demonstrated in Fig. 3 and Fig. 4 in terms of the RMSE of the estimates. Since the same calibration density is used, an accordance between the two calibration strategies is found from the results. The simulation results shows that the proposed interpolation-based calibration method has a good performance in presence of measurement noise.

6. CONCLUSION

We present an interpolation-based method to calibrate the array response in near-field source localization. From an analysis on the signal model, new pseudo calibration data are obtained by interpolating the correction matrix, which allows us to calibrate the array response on a dense grid with a small number of measurements. The proposed approach is shown to have good performance.

REFERENCES

[1] J. Capon, "High resolution frequency-wavenumber spectrum analysis", *Proceeding of IEEE*, pp. 1408-1418, 1969.

[2] C. De Boor, *A Practical Guide to Splines,* New York, Springer-Verlag, 2001

[3] H. He, Y. Wang, J. Saillard, "A high resolution method of source localization in near-field by using focusing technique", *European signal processing conference*, Lausanne, Switzerland, Aug. 2008.

[4] H. He, Y. Wang, J. Saillard, "Near-field Source localization by Using Focusing Technique", *EURASIP Journal of Advances in Signal Processing*, Vol. 2008, Article ID 461517, 2008.

[5] H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach", *IEEE Signal Processing magazine*, vol. 13, Issue 4, pp. 67-94, Jul. 1996.

[6] A. Leshem and M. Wax, "Array calibration in the presence of multipath", IEEE transaction on Signal Processing, Vol. 48, Issue 1, pp. 53-59, Jan. 2000.

[7] Q. Li, L. Gan and Z. Ye, "An overview of self-calibration in sensor array processing", *IEEE 6th International symposium on antennas, propagation and EM Theory*, pp.279-282, Beijing, China, Oct. 2003.

[8] A. Lundgren, M. Lanne and M. Viberg, "Two step ESPRIT with compensation for modeling errors using a sparse calibration grid", *IEEE 2007 International conference on acoustics, speech and signal processing*, Honolulu, HI, USA, vol. 2, pp. II-1045-II-1048, Apr. 2007.

[9] J. Pierre and M. Kaveh, "Experimental performance of calibration and direction-finding algorithms", *IEEE 1991 International conference on acoustics, speech and signal processing*, vol. 2, pp. 1365-1368, Apr. 1991.

[10] P. Stoica and A. Nehorai, "MUSIC Maximum likelihood and Cramer-Rao bound," IEEE Transaction on Acoustics, Speech and Signal Processing, vol. 37, no. 5, pp. 720-741, May 1989.

[11] A.L. Swindlehurst, T. Kailath, "A performance analysis of subspace-based methods in the presence of model errors. I. The MUSIC algorithm", *IEEE Transaction on Signal Processing*, vol. 40, Issue 7, pp. 1758-1774, Jul. 1992.

[12] A.L. Swindlehurst, T. Kailath, "A performance analysis of subspace-based methods in the presence of model error. II. Multidimensional algorithms", *IEEE Transaction on Signal Processing*, vol. 41, Issue 9, pp. 2882-2890, Sep. 1993.

[13] W. Zhi and M. Y. Chia, "Near-field source localization via symmetric subarrays", *IEEE Signal Processing Letters*, vol. 14. no. 6, pp. 409-412, Jun. 2007.

APPENDIX

A. CUBIC SPLINE INTERPOLATION

A.1 Definition

Given $n+1$ distinct knots x_i such that

$$x_0 < x_1 < \cdots < x_{n-1} < x_n, \quad \text{(A-1)}$$

with $n+1$ knot values y_i, a cubic spline function $S(x)$ is a piecewise-defined function

$$S(x) = \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases} \quad \text{(A-2)}$$

which satisfies the following conditions:

1. The interpolating property, $S(x_i) = y_i$
2. The splines to join up, $S_{i-1}(x_i) = S_i(x_i) = y_i, i = 1, ..., n-1$

The n cubic polynomial pieces are written as

$$S_i(x_i) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3, \quad \text{(A-3)}$$

for $i = 1, ..., n-1$, where a_i, b_i, c_i and d_i represent 4n unknown coefficients. With the three conditions in the definition, $4n-2$ equations are known. To solve the polynomial group (A-3), two more conditions can be imposed upon the problem.

A.2 Natural Spline Interpolation for Calibration

Natural spline is used to calculate the $4n$ coefficients in (A-3). So the two conditions are

$$S''(x_0) = S''(x_n) = 0 \tag{A-4}$$

An example for the interpolation of calibration data is given here.

We suppose that the $(n+1) \times (n+1)$ measurements are taken for

$$\theta = \theta_{meas_1}, \theta_{meas_2}, \ldots, \theta_{meas_{n+1}}$$

and

$$r = r_{meas_1}, r_{meas_2}, \ldots, r_{meas_{n+1}}.$$

From (4), (5) and (6), we can calculate$(n+1) \times (n+1)$ estimates of $\mathbf{Q}(\theta, r)$. We note that $\mathbf{Q}(\theta, r)$ is supposed to be diagonal matrix under the hypothesis that the mutual coupling is neglected. Each diagonal element of $\mathbf{Q}(\theta, r)$

$$q_m(\theta, r) \text{ for } m = -M, \ldots, 0, 1, \ldots, M$$

is a function of (θ, r). The interpolation is then performed to each $q_m(\theta, r)$ separately via two steps.

1. Interpolate $q_m(\theta, r)$ with respect to θ for $r = r_{meas_1}, r_{meas_2}, \ldots, r_{meas_{n+1}}$. We obtain then $(Kn+1) \times (n+1)$ estimates of $q_m(\theta, r)$, where $K > 1$ indicates the density of interpolation.
2. Interpolate $q_m(\theta, r)$ with respect to r for $\theta = \theta_{new_1}, \theta_{new_2}, \ldots, \theta_{new_{Kn+1}}$, where $q_m\left(\theta_{new_i}, r_{meas_j}\right)$ for $i = 1, 2, \ldots Kn+1$ and $j = 1, 2, \ldots, n+1$ is obtained from the previous step.

Finally, $(Kn+1) \times (Kn+1)$ estimates of $q_m(\theta, r)$ for $\theta = \theta_{new_1}, \theta_{new_2}, \ldots, \theta_{new_{Kn+1}}$ and $r = r_{new_1}, r_{new_2}, \ldots, r_{new_{Kn+1}}$ are obtained via $Kn+n+2$ times spline interpolation.