DESIGN OF PERFECT-RECONSTRUCTION NONUNIFORM FILTER BANKS WITH LINEAR-PHASE PROPERTY

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ABSTRACT

This paper proposes a new method for designing nonuniform filter banks (NUFBs) with perfect-reconstruction (PR) and linear-phase (LP) properties. It is based on the recombination structure, where the outputs of the analysis filters in a uniform original filter bank (FB) are combined by the synthesis bank of recombination FBs. New matching conditions on the original and recombination FBs for the PR NUFBs to possess good frequency characteristics are derived. This simplifies the design of the PR LP RNUFBs to those of a set of appropriate PR LP uniform FBs, which greatly reduce the design complexity. The effectiveness of the proposed method is illustrated by the design of several PR LP RNUFBs with sampling factors (2/3, 1/3) and (1/3, 2/3[1]).

1. INTRODUCTION

The theory and design of nonuniform filter banks (NUFBs) have received considerable attention due to their flexibility in subband partitioning and processing. In applications where phase distortion is an important concern, filter banks with linear-phase (LP) analysis and synthesis are highly desirable. For example, in image processing, LP filter banks (FBs) are frequently employed to avoid possible phase distortion.

Among the various methods for designing NUFBs, only a few are applicable to the design of NUFBs satisfying the LP property [1-3], [6]. The tree-structure FB is a simple method for realizing PR LP NUFBs, but the choice of the decimation factors is usually limited and the system delay is rather long due to the cascade nature of the tree structure. Another class of design method is based on the direct structure in [1-3], where the analysis filters are designed directly [1] or by mean of iterative optimization algorithm. In [3], Wada suggested to employ FIR filters with complex coefficients to reduce the aliasing distortion in LP NUFBs. The disadvantage of these direct methods is that the FBs so obtained are only nearly PR. Another class of method for designing NUFBs is based on the recombination structure [4-6, 17]. In the recombination NUFBs (RNUFBs), certain channels of an *M*-channel uniform FB are combined by the synthesis filters of set of recombination FBs with smaller channel number in order to generate subbands with different decimation factors. The advantage of the RNUFB is that the PR property is structurally imposed as long as the original uniform FB and the recombination FBs are PR. In [5,17] the theory and design of PR cosine-modulated RNUFBs are studied in detail. Due to large number of design variables and hence complexity associated with a general NUFB, even LP FBs are difficult to design in practice. The use of cosine modulatd FBs (CMFBs) greatly reduces the number of design variables and complexity. Consequently, high quality PR RNUFBs can be obtained. In [6], the design of linear phase RNUFBs is considered where a matching condition for orthogonal PR FBs was derived. However, due to the difficulties in designing LP original and recombination FBs with large number of channels, the RNUFBs so obtained are nearly PR.

We note that the key difficulty in obtaining LP RNUFBs hinges on a design of the original and recombination PR LP FBs in the recombination structure. Among the existing methods for designing *M*-channel PR LP uniform FBs [7-15], the modulation-based approach is probably the simplest to apply [7-9]. However, the frequency support of these FBs is considerably different from that of uniform FBs. Therefore, it may not be suitable in RNUFBs. The lattice structure is another popular method for designing PR LP FBs, since the PR and linear phase property can be structural imposed [10-14]. Unfortunately, the frequency response becomes a very nonlinear function of a large number of lattice parameter, making the design very complicated and somewhat difficult to obtain good results as the filter length increases. In [15], a direct method for designing *M*-channel PR LP uniform FBs was proposed. By choosing properly the synthesis and analysis filters, the aliasing is first cancelled and the number of design variables can be reduced.

In this paper, the LP RNUFBs proposed in [6] further extended. By analyzing the frequency characteristics of the RNUFBs, new matching conditions for biorthogonal and orthogonal FBs, which are necessary for constructing the proposed LP RNUFB with good frequency responses, are derived. With these conditions being satisfied, the design of LP PR NUFBs can be decomposed into the individual design of a set of PR LP uniform FBs, which greatly reduce the design complexity. The original and recombination FBs are then designed by the method in [15]. We also present several

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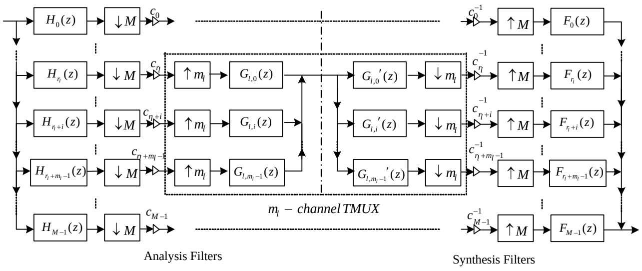
PR linear-phase recombination nonuniform filter banks so obtained, which to our best knowledge are reported at the first time in the literature.

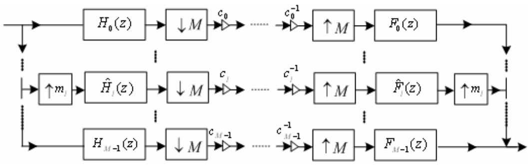
2. THE PROPOSED PR LP RNUFBS

2.1 Principle of Recombination NUFBs

Figure-1(a) shows the general structure of an L-channel RNUFB, where certain subchannels of a M channel uniform FB are recombined by the synthesis filters of transmultipliers with small channel number. For simplicity, only the *l-th* channel, l=0,…,L-1, is shown where an m_l channel transmultiplexer is employed. For simplicity, we only consider the case where m_l and M are coprime in the paper. The constants $c_0 \sim c_{M-1}$ and $c_0^{-1} \sim c_{M-1}^{-1}$ are used to control the magnitude response of the equivalent filter.



(a)



(b)

Figure-1. Structure of (a) recombination NUFB, (b) the equivalent of the recombination NUFB.

According to the noble identity, we know that the downsampler and upsampler can be interchanged if m_l and M are coprime. Hence, $H_{r_l+i}(z)$ and $G_{l,i}(z)$ can be moved across the upsampler and downsampler respectively, where $i = 0, \cdots, m_l - 1$ and r_l is the starting channel number of the M-channel original FB to be merged. Thus, the merged channels has an equivalent LTI filter representation as shown in Figure-1(b) [5]. Its z-transform can be written as

$$\hat{H}_l(z) = \sum_{i=0}^{m_l-1} c_{r_l+i} H_{r_l+i}(z^{m_l}) G_{l,i}(z^M)\,. \tag{1}$$

For notational convenience, we use $H(\omega)$ to denote its frequency response $H(e^{j\omega})$. Hence

$$\hat{H}_l(\omega) = \sum_{i=0}^{m_l-1} c_{r_l+i} H_{r_l+i}(m_l\omega) G_{l,i}(M\omega)\,. \tag{2}$$

A detailed theory and analysis of the PR RNUFBs can be found in [5]. An important advantage of RNUFBs is that if the original and recombination FBs are PR, then the entire system will be PR, after compensating for the different delays introduced by the transmultiplexers in each branch.

2.2 Design of Uniform PR LP Uniform Filter Banks

As the uniform PR LP original and recombination FBs will be designed using the method in [15], we now shall briefly review this method below.

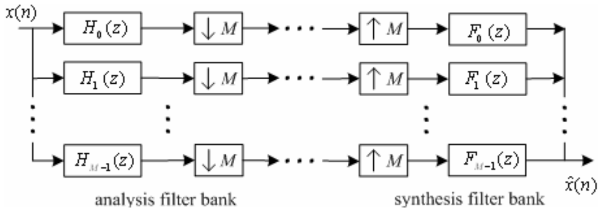


Figure-2. Structure of M-channel uniform filter banks.

The structure of an M-channel maximally decimated uniform filter bank is shown in Figure-2, where $H_i(z)$ and $F_i(z)$, $i = 0,1,\cdots,M-1$, are the analysis and synthesis filters, respectively. The relationship between the z-transforms of the output signal $\hat{x}(n)$ and the input signal $x(n)$ is

$$\begin{aligned}\hat{X}(z) &= \frac{1}{M}\sum_{k=0}^{M-1}\sum_{l=0}^{M-1} H_k(zW^l)F_k(z)X(zW^l) \\ &= T(z)X(z) + \frac{1}{M}\sum_{k=0}^{M-1}\sum_{l=1}^{M-1} H_k(zW^l)F_k(z)X(zW^l),\end{aligned} \tag{3}$$

where $W = e^{-j2\pi/M}$ and

$$T(z) = \tfrac{1}{M}\sum_{k=0}^{M-1} H_k(z)F_k(z)\,,$$

is the distortion transfer function. To cancel the aliasing, all terms involving $X(zW^l)$, $l \neq 0$, on the right in equation (3) should be equal to zero. To this end, we can express the synthesis filters in terms of the analysis filers as follows:

$$F_k(z) = \frac{(-1)^k F_0(z)\det H_{-0k}}{\det H_{-00}}$$

for $k = 1, \cdots, M-1$, where

$$F_0 = \det H_{-00}\,,$$

and H_{-jl} is the Alias Component (AC) matrix with the $(j+1)^{\text{th}}$ row and the $(l+1)^{\text{th}}$ column deleted, and the AC matrix $\mathbf{H}$ is

$$\begin{pmatrix} H_0(z) & H_1(z) & \cdots & H_{M-1}(z) \\ H_0(zW) & H_1(zW) & \cdots & H_{M-1}(zW) \\ \vdots & \vdots & \ddots & \vdots \\ H_0(zW^{M-1}) & H_1(zW^{M-1}) & \cdots & H_{M-1}(zW^{M-1}) \end{pmatrix}.$$

Thus, $T(z)$ can be rewritten as

$$T(z) = \tfrac{1}{M}\sum_{k=0}^{M-1} (-1)^k H_k(z)\det H_{-0,k}\,. \tag{4}$$

Objective function: To get good filter quality, the square of the difference between the ideal and actual frequency responses over appropriate frequency bands for all analysis filters should be minimized.

Constraints: For PR system, the transfer function $T(z)$ should be constrained to a multiple of certain delays αz^{-n_0}. To simplify notation, the following auxiliary function is introduced

$$Q(z) = \sum_{n=0}^{N-1} q(n) z^{-n} = H_0(z) F_0(z), \quad (5)$$

where $N = \sum_{i=0}^{M-1} N_i - M - 1$, and N_i is the length of the filter $H_i(z)$. Substituting Eq. (5) into Eq. (4), $T(z)$ in time domain, i.e. $T(z) = \sum_{n=0}^{N-1} t(n) z^{-n}$ can be written as:

$$t(n) = \begin{cases} q(n), \ n = M(\frac{M-1}{2} + r), \text{ for some integer } r, \\ 0, \quad \text{otherwise.} \end{cases} \quad (6)$$

For a PR system, $T(z)$ should be symmetric and of odd length. This permits the optimization program to focus only on the first $(N+1)/2$ terms of $T(z)$, and only the center coefficient of $T(z)$ is nonzero. According to Eq. (6), the requirements would apply as well to $Q(z)$, which will further reduce the number of the variables constrained. More detailed can be found in [15].

2.3 The Matching conditions

We note that the original and recombination FBs are to be designed by the method described in Section 2.2. To ensure that the equivalent filters of the merged channels in the RNUFB have good frequency characteristics, we found that they have to satisfy certain matching conditions. These conditions are suitable for the case that the uniform FBs have analysis/synthesis filters with equal length. As we know [1], this kind of LP FBs with equal length have alternate symmetry property (i.e., successive analysis filters are symmetric and anti-symmetric). We first summary in this section these matching condition and the delay the analysis later to Section 3.

Matching conditions for the biorthogonal FBs:

If the original/recombination FBs are biorthogonal, and if the starting channel number r_l is even, then in order to get an equivalent filter with good filter quality, the biorthogonal original and recombination FBs should satisfy the following conditions:

Condition 1: $c_{r_l+i} = -c_{r_l+i+1}$,

Condition 2: The transition band of $G_{l,i}(z^M)$ should be similar to that of $F_{r_l+i}(z^{m_l})$.

L and L' are respectively the filter lengths of the original and recombination FBs. M and m_l are the channel number of the original and recombination FBs, respectively.

If the starting channel number r_l is odd, then the matching conditions read:

Condition 1: $c_{r_l+i} = c_{r_l+i+1}$.

Condition 2: The transition band of $G_{l,i}(z^M)$ should be similar to that of $F_{r_l+i}(z^{m_l})$.

Matching conditions for the orthogonal FBs:

In [6], the matching conditions when the starting channel number r_l is even were derived as follows:

Condition 1: $c_{r_l+i} = -c_{r_l+i+1}$.

Condition 2: The transition band of $G_{l,i}(z^M)$ should be similar to that of $H_{r_l+i}(z^{m_l})$.

For odd r_l, similar matching conditions can be deduced as follows:

Condition 1: $c_{r_l+i} = c_{r_l+i+1}$.

Condition 2: The transition band of $G_{l,i}(z^M)$ should be similar to that of $H_{r_l+i}(z^{m_l})$.

For simplicity, we shall choose c_i either as 1 or -1 in the sequel.

3. ANALYSIS OF THE MATCHING CONDITIONS

We now elaborate further on the matching conditions introduced in Section 2.3. It is well known that LP filters with symmetric and antisymmetic impulse responses can be written respectively as:

$$H(\omega) = e^{-j\omega(L-1)/2} H_R(\omega), \quad (7)$$

and

$$H(\omega) = je^{-j\omega(L-1)/2} H_R(\omega), \quad (8)$$

where L denotes the length of the filter $H(\omega)$ and $H_R(\omega)$ is its amplitude response.

It is known from the property of LP FIR filters that the impulse response of lowpass LP filters must be symmetric. Moreover, for uniform LP FBs having analysis/synthesis filters with equal length, the FBs should have alternate symmetry property. Therefore, if the starting channel number r_l is even, the filter $H_{r_l}(\omega)$ should be symmetric. That is, the equivalent filter expressed in Eq. (2) can be rewritten as

$$\begin{aligned} \hat{H}_l(\omega) = e^{-j\omega\frac{m_l(Lo-1)+M(Lr-1)}{2}} [& c_{r_l} H_{Rr_l}(m_l\omega) G_{R0}(M\omega) \\ & - c_{r_l+1} H_{R(r_l+1)}(m_l\omega) G_{R1}(M\omega) \\ & + c_{r_l+2} H_{R(r_l+2)}(m_l\omega) G_{R2}(M\omega) \\ & - \cdots c_{r_l+m_l-1} H_{R(r_l+m_l-1)}(m_l\omega) G_{R(m_l-1)}(M\omega)], \end{aligned} \quad (9)$$

where Lo and Lr denote the length of the analysis filters in the original FBs and the length of the synthesis filters in the recombination FB respectively. The term $e^{-j\omega\frac{m_l(Lo-1)+M(Lr-1)}{2}}$ is the phase response of the filter $H_l(\omega)$, and the remainder in the right side of Eq. (9) is the amplitude response. From Eq. (9), we know, $c_{r_l+i} = -c_{r_l+i+1}$ is necessary for the equivalent filter to possess good frequency characteristic.

Similarly, if the starting channel number r_l is odd, the filter $H_{r_l}(\omega)$ is antisymmetric. In this case, the equivalent filter expressed in Eq. (2) can be rewritten as

$$H_l(\omega) = jc_{r_l} e^{-j\omega\frac{m_l(Lo-1)+M(Lr-1)}{2}} [H_{Rr_l}(m_l\omega)G_{R0}(M\omega) + c_{r_l+1}H_{R(r_l+1)}(m_l\omega)G_{R1}(M\omega) + c_{r_l+2}H_{R(r_l+2)}(m_l\omega)G_{R2}(M\omega) + \cdots c_{r_l+m_l-1}H_{R(r_l+m_l-1)}(m_l\omega)G_{R(m_l-1)}(M\omega)]. \quad (10)$$

Clearly, $c_{r_l+i} = c_{r_l+i+1}$ is necessary for the equivalent filter to possess good frequency characteristics. On the hand, for PR uniform FBs, we should have

$$\sum_{i=0}^{M-1} H_i(z)F_i(z) = cz^{-n_0}.$$

By replacing z by z^{m_l}, we obtain

$$\sum_{i=0}^{M-1} H_i(z^{m_l})F_i(z^{m_l}) = cz^{-m_l n_0}. \quad (11)$$

The corresponding amplitude response is:

$$\left|\sum_{i=0}^{M-1} H_i(m_l\omega)F_i(m_l\omega)\right| = c, \quad (12)$$

or equivalently:

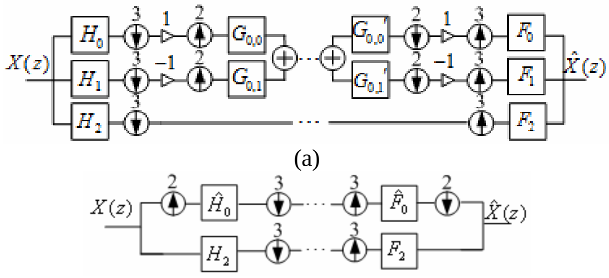
$$\left|\sum_{i=0}^{r_l-1} H_i(m_l\omega)F_i(m_l\omega) + \sum_{i=r_l+m_l}^{M-1} H_i(m_l\omega)F_i(m_l\omega) + \sum_{i=0}^{m_l-1} H_{r_l+i}(m_l\omega)F_{r_l+i}(m_l\omega)\right| = c. \quad (13)$$

As long as the stopband attenuation of the original FB is high, we can ensure that the passband of $\sum_{i=0}^{m_l-1} H_{r_l+i}(m_l\omega)F_{r_l+i}(m_l\omega)$ in the region of $(\frac{r_l}{Mm_l}\pi, \frac{r_l+m_l}{Mm_l}\pi)$ is flat. Thus, by adjusting the design parameters of $G_{l,i}^u(z)$ to make the shape of the filter $G_{l,i}(M\omega)$ similar to that of $F_{r_l+i}(m_l\omega)$, (in order to get $G_{l,i}(M\omega) = F_{r_l+i}(m_l\omega)$), the equivalent filter $\hat{H}_l(z) = \sum_{i=0}^{m_l-1} c_{r_l+i} H_{r_l+i}(z^{m_l})G_{l,i}(z^M)$ will possess good frequency characteristics.

Since the orthogonal FBs are special cases of biorthogonal FBs, in which the amplitude shape of the synthesis filters are the same as the corresponding analysis filters. Similar results can readily be obtained.

4. DESIGN EXAMPLES

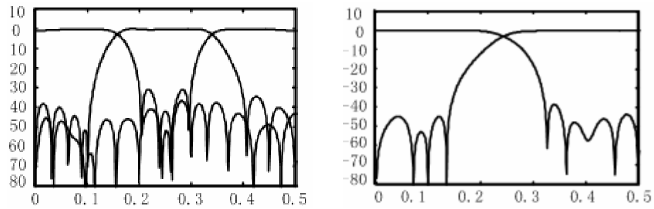
Example 1: LP RNUFBs with sampling factors (2/3, 1/3): The 3-channel original FB and 2-channel recombination FB, shown in Figure-3(a), are obtained by the method mentioned in Section 2.2. The lengths of the analysis filters $H_i(z)$ and $G_{0,i}(z)$ are 39 and 26, respectively. The passband and stopband cutoff frequencies of the resulting filters are as follows: for $H_0(z)$: $\omega_p = 0.240\pi$, $\omega_s = 0.476\pi$; for $G_{0,0}(z)$: $\omega_p = 0.400\pi$, $\omega_s = 0.600\pi$; for $H_1(z)$: ω_{s1}=0.233π , ω_{p1}=0.433π , ω_{p2}=0.567π , ω_{s2}=0.767π ; and for $G_{0,1}(z)$: $\omega_s = 0.255\pi$, $\omega_p = 0.639\pi$. Figure-3(b) shows the equivalent structure of this LP RNUFBS. Figures-4(a) and (b) depict the magnitude responses of the original and recombination FBs.



(a)

(b)

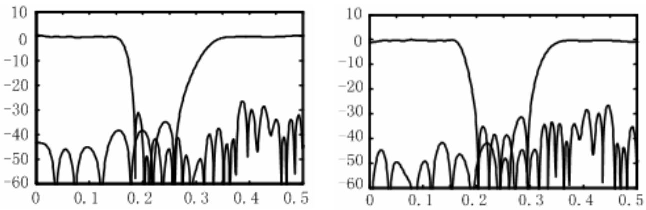
Figure-3. LP recombination NUFB with sampling factors (2/3, 1/3). (a) the indirect structure, and (b) the equivalent structure.



(a) (b)

Figure-4. Magnitude responses of the analysis filters of (a) the 3-channel original FB, and (b) the 2-channel recombination FB.

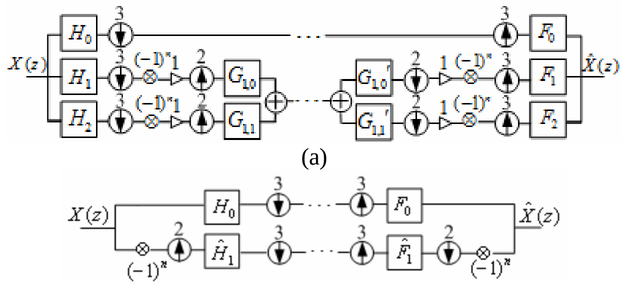
The analysis/synthesis filters in the equivalent structure are shown in Figure-5(a) and (b), respectively. For the equivalent FB, the amplitude distortion and the aliasing error are 2.554×10^{-15} and 7.729×10^{-16}, respectively.



(a) (b)

Figure-5. NUFB with samplers (2/3, 1/3). Magnitude responses of the equivalent (a) analysis filters, and (b) synthesis filters.

Example 2 LP RNUFBs with sampling factors (1/3, 2/3): It is known that this kind of recombination NUFBs with odd starting channel number r_l cannot be realized even for ideal filters due to the shuffling of frequencies [16]. ("Shuffling" denotes the process in which a part of the signal's spectrum has been translated to another part in the spectrum.) Fortunately, the shuffling of frequencies can be corrected by multiplying the sequence $\{(-1)^n\}$ to the input of the recombination FB. This is because multiplying with the sequence $\{(-1)^n\}$ is equivalent to shifting the signal by π in frequency domain [4, 5]. Here, we consider a NUFB with samplers (1/3, 2/3) as an example. Figure-6 shows the corresponding structure of this FB.

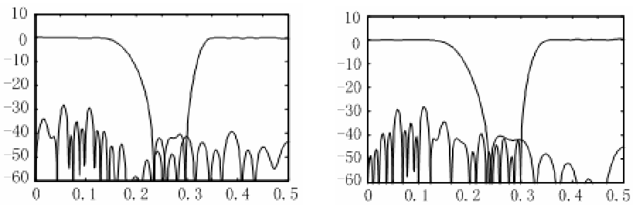


(a)

(b)

Figure-6 Recombination NUFB with sampling factors (1/3, 2/3). (a) the indirect structure, and (b) the equivalent structure.

Figure-7 shows the magnitude responses of the analysis/synthesis filters of the equivalent FB. The amplitude distortion and the aliasing error are 1.599×10^{-14} and 1.308×10^{-14}, respectively, which can be considered a PR system in practice.



(a) (b)

Figure-7 NUFB with samplers (1/3, 2/3). Magnitude responses of the equivalent (a) analysis filters, and (b) synthesis filters.

5. CONCLUSION

In this paper, we propose a method for designing PR LP RNUFBs with fractional samplers. It is based on the recombination structure and hence the design of the RNUFB is simplified to the design of appropriate uniform FBs. Using the proposed matching conditions and the design method in 17], it can be found that PR LP RNUFBs with good filter characteristics can be obtained by the proposed method.

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