COOPERATIVE MULTIPLE ACCESS TRANSMISSION USING PRECODING VECTORS

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ABSTRACT

We address in this paper the problem of transmitting data to multiple mobile stations (MS) using a decode-and-forward strategy. The links between the base station and relays are considered ideal and we study the downlink between relays and mobile stations. We propose a method to optimize the precoding vector in relays to cancel out multiple access interference and to maximize the signal to noise ratio at the mobile stations. Simulation results show that the maximum diversity advantage can be obtained, which is the product of the number of antennas at each relay by the number of relays minus the total number of system constraints. Furthermore, using Lagrange multipliers to optimize precoding vectors, we obtain a flexibility that enables us to consider different cases for signal to noise ratio at mobile stations.

1. INTRODUCTION

Today it becomes inevitable to establish high speed and reliable communications to mobile stations in dense urban environments. Recent relaying techniques show to be very promising in order to extend the wireless network coverage and has become a very hot research area. In general, the relaying protocols are divided into two main categories: Amplify-and-forward (AF) and decode-and-forward (DF) [1]. In the DF case, the relays decode the received signal then retransmit the information via a suitable processing to MS. In the case of successful decoding in a relay, the noise effect from the first link (base station to relay stations) is removed. To improve the performance, maximum ratio combining [2] together with distributed beamforming [3] are used to eliminate multiple access interference (MAI) and at the same time to maximize the signal to noise ratio (SNR) at receivers.

This paper addresses the downlink cellular network link to the mobile stations. It can also be applied to selective broadcast of multimedia data to network subscribers. In an urban environment, because of shadowing effect, a good link between base station (BS) and mobile stations is not guaranteed, even using multiple antennas in both transmitter and receiver. One attractive configuration is to use a base station communicating with some relays using high speed and reliable channels. This configuration is easily achievable because BS and relays are supposed not to move. They can be related by wireless link, satellite communications, cable, optical fiber or whatever else. Therefore a noise-free ideal link is assumed between BS and relays. In this paper we concentrate only on relay-mobile link.

In the configuration proposed in this paper, the relays, using multiple antennas, send the information corresponding to all the mobiles at the same time and at the same carrier frequency to all mobile stations using optimized precoding vectors. The objective is to maximize the signal to noise ratio at each mobile station and at the same time to minimize the multiple access interference. The relay-mobile channel is supposed to be frequency non selective Rayleigh fading channel. However, orthogonal frequency division multiplexing (OFDM) modulation can be used in the case of multipath channel and the principles will remain the same. Furthermore, we suppose that we are in the context of low mobility where the channel state information (CSI) can be obtained and fed back to relay station(RS).

The main contribution of this work lies in applying the Lagrange multipliers method to multiple-input multiple-output (MIMO) systems, which is a flexible approach to calculate precoding vectors and may be adopted for different schemes and strategies. This method allows us to obtain different and therefore adjustable SNR at receivers. The proposed method consists of linearizing a set of optimizing equations and to use matrix derivations to calculate the optimum weighting vectors. This paper also generalizes the results of [4], in which only two relay stations and two mobile stations were used, to the case of L relays and N mobile stations. Simulation results show that the proposed precoding vectors achieve the maximum diversity advantage that can be obtained, which is the product of the number of antennas at each relay by the number of relays minus the number of constraints. In this paper we study two different cases, i) each relay knows only its own channel to all MS, ii) each relay knows completely the CSI. We obtain for each case the optimum precoding vectors.

The rest of this paper is organized as follows: Section 2 gives a brief description of system model and introduces the equations that define the system. Section 3 uses the vector-based Lagrange multipliers method introduced in Appendix A to optimize the precoding vectors. Then in section 4, a special case where the same SNR is imposed at each mobile station is studied. Finally, section 5 presents the simulation results confirming the system performance. Some mathematical aspects are discussed in the appendices.

2. SYSTEM MODEL

The system is composed of one transmitter node with M antennas, L relay stations each with R antennas, and N single-antenna mobile stations. For simplicity we will focus our study for the case $L = 2$, then we will generalize the results for arbitrary L under section 4. The system model where $L = 2$ is given in Figure 1. Let us consider that a signal $\mathbf{s} = [s_1, s_2, \cdots s_N]^T$ is to be transmitted from the base station (BS in Figure 1) to mobile stations (MS_1 to MS_N in Figure 1). We desire that each mobile station receives only its intended data, i.e. MS_1 receives only s_1, MS_2 receives only s_2, and so on. The bottle neck of the system is the second hop, where data is transmitted from relays to each mobile station and we suppose that the link between the base station and the relays is ideal. Thus we assume that relays will perfectly receive and decode the signal $\mathbf{s}$. Then the signal is multiplied by some precoding vectors before being transmitted to mobile stations. Here we have used the same

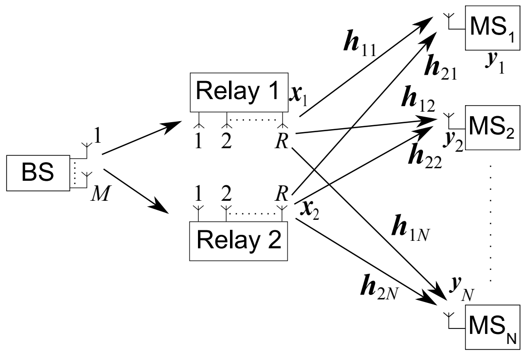

Figure 1: System model.

notations as in [4]:

$$\mathbf{x}_1 = \sum_{j=1}^{N} s_j \mathbf{w}_j^1 \quad , \qquad \mathbf{x}_2 = \sum_{j=1}^{N} s_j \mathbf{w}_j^2 \qquad (1)$$

where $\mathbf{w}_j^i$ of size $R \times 1$ represents the precoding vectors of i^{th} relay. $\mathbf{x}_i$ are then transmitted to mobile stations via the Rayleigh channels $\mathbf{h}_{ij}$, $i = 1, 2$, $j = 1 \cdots N$. The second hop being a Rayleigh flat fading channel of size $1 \times R$, the signal at the j^{th} mobile station can be expressed as:

$$\begin{aligned} y_j &= \mathbf{h}_{1j} \cdot \mathbf{x}_1 + \mathbf{h}_{2j} \cdot \mathbf{x}_2 + n_j \qquad (2) \\ &= \mathbf{h}_{1j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^1 + \mathbf{h}_{2j} \cdot \sum_{k=1}^{N} s_k \mathbf{w}_k^2 + n_j \quad , j = 1 \cdots N \end{aligned}$$

where $\mathbf{h}_{ij} \sim \mathcal{CN}(0, \mathbf{I}_R)$ and n_j denotes the Gaussian noise. As stated above, we want y_j to depend only on s_j, that is to say:

$$\sum_{k \neq j} s_k \mathbf{h}_{1j} \cdot \mathbf{w}_k^1 + \sum_{k \neq j} s_k \mathbf{h}_{2j} \cdot \mathbf{w}_k^2 = 0 \qquad (3)$$

The problem with (3) is that each relay needs the channel state information of the other relay in order to calculate its own precoding vectors. This implies that there should be an inter-connection link between the two relays. To overcome this limitation we can impose each of the terms at the left hand side of (3) to equal zero:

$$\sum_{k \neq j} s_k \mathbf{h}_{1j} \cdot \mathbf{w}_k^1 = 0 \quad , \qquad \sum_{k \neq j} s_k \mathbf{h}_{2j} \cdot \mathbf{w}_k^2 = 0 \qquad (4)$$

Of course the optimization constraints in (4) are more strict than those given by (3), but using (4) the precoding vectors can be calculated independently in each relay.

If either (3) or (4) is satisfied the resulting signal at each mobile station is given by:

$$y_j = s_j \left(\mathbf{h}_{1j} \cdot \mathbf{w}_j^1 + \mathbf{h}_{2j} \cdot \mathbf{w}_j^2 \right) + n_j \qquad (5)$$

with n_j denoting the additive white Gaussian noise at the j^{th} station. The precoding vectors must also guarantee that the term in parentheses at the right hand side of (5) is a real positive number to guarantee a coherent addition.

3. CALCULATION OF PRECODING VECTORS

In this section the Lagrange method is used to calculate the appropriate precoding vectors as explained in Appendix A. The following notations are used in the rest of this paper:

$$\mathbf{H}_i = \left[\begin{array}{c|c|c|c} \mathbf{h}_1^{i\;T} & \mathbf{h}_2^{i\;T} & \ldots & \mathbf{h}_N^{i\;T} \end{array} \right]^T_{N \times R} \qquad , i = 1, 2 \quad (6)$$

$$\mathbf{W}_i = \left[\begin{array}{c|c|c|c} \mathbf{w}_1^i & \mathbf{w}_2^i & \ldots & \mathbf{w}_N^i \end{array} \right]_{R \times N} \qquad , i = 1, 2 \quad (7)$$

$$\boldsymbol{\mathcal{W}}_i = \left[\begin{array}{cccc} \mathbf{w}_1^{i\;T} & \mathbf{w}_2^{i\;T} & \ldots & \mathbf{w}_N^{i\;T} \end{array} \right]^T_{NR \times 1} \qquad , i = 1, 2 \quad (8)$$

The aim of this section is to calculate $\boldsymbol{\mathcal{W}}_i$ or equivalently $\mathbf{w}_j^i$ subject to a number of constraints that optimize an objective function. In the following subsections we will discuss and determine the constraints and the objective function.

3.1 Interference cancellation

As stated in section 2 precoding vectors must be able to eliminate the MAI by respecting (3) or (4). Since it is more desirable to calculate the precoding vectors of each relay independently, we will use the criteria expressed in (4). Equation (4) can be written in a matrix form using (6) and (8):

$$\mathbf{A}_{i1} \boldsymbol{\mathcal{W}}_i = \mathbf{0} \quad \text{with } \mathbf{A}_{i1} = \left(\mathbf{s}^T \otimes \mathbf{1}_{N \times 1} - \text{diag}(\mathbf{s}) \right) * \mathbf{H}_i \quad (9)$$

where $\otimes$ and $*$ respectively denote Kronecker and row-wise Kronecker products[1]. Equation (9) is a set of N linear complex equations for each relay that guarantee the cancellation of intersymbol interferences. In order to obtain the equivalent real equations we use the method introduced under Appendix B. Using (29) we will obtain a set of $2N$ real equations for each relay station:

$$\Re \left\{ \left(\mathbf{A}_{i1} \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} \right) \right\} \Re \left\{ \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix} \right) \right\} = \mathbf{0}_{2N \times 1}$$

This quation has two rows per mobile station, the first row is the real part of (9) and the second row is the imaginary part of (9). We will rewrite this equation to simplify future developments.

$$\hat{\mathbf{A}}_{i1} \hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{0}_{2N \times 1} \qquad (10)$$

Note that $\hat{\mathbf{A}}_{i1}$ and $\hat{\boldsymbol{\mathcal{W}}}_i$ are only composed of real values.

3.2 Coherent addition

If the interference canceling constraint is respected, according to (5), each mobile station MS_j receives a signal containing s_j from each relay. Since the signal received at the mobile station is the sum of these two signals, it is obvious that they must arrive in phase at the mobile station. In order to maintain the original constellation, without loss of generality, we impose the coefficient of s_j in (5) to be a real number. This can be written as:

$$\Im \left\{ \mathbf{h}_{ij} \cdot \mathbf{w}_j^i \right\} = 0, \qquad i = 1, 2, \quad j = 1 \cdots N \qquad (11)$$

Equations in (11) for each relay (i.e. $i = 1$ or 2) can be grouped in a single matrix equation:

$$\Im \left\{ \mathbf{A}_{i2} \boldsymbol{\mathcal{W}}_i \right\} = \Im \left\{ (\mathbf{I}_N * \mathbf{H}_i) \boldsymbol{\mathcal{W}}_i \right\} = 0, \qquad i = 1, 2 \qquad (12)$$

with * denoting again the row-wise Kronecker product. Using (29), this equation can be written as:

$$\Re \left\{ \left(\mathbf{A}_{i2} \otimes \begin{bmatrix} -j & 1 \end{bmatrix} \right) \right\} \Re \left\{ \left(\boldsymbol{\mathcal{W}}_i \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix} \right) \right\} = \mathbf{0}_{N \times 1}$$

$$\Rightarrow \qquad \hat{\mathbf{A}}_{i2} \hat{\boldsymbol{\mathcal{W}}}_i = \mathbf{0}_{N \times 1} \qquad (13)$$

Since (13) is only about the imaginary part of $\mathbf{h}_{ij} \cdot \mathbf{w}_j^i$, the matrix $\hat{\mathbf{A}}_{i2}$ has a single row per mobile station.

[1] Row-wise Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$ is a matrix each line of which is the Kronecker product of corresponding lines in $\mathbf{A}$ and $\mathbf{B}$.

3.3 Power constraint and the objective function

When posed formally, the problem consists of finding $\mathcal{W}_i$ maximizing the SNR at mobile stations that satisfy a power constraint as well as (10) and (13). It means that we must define a transmission power limit P_i for each relay and choose the precoding vector $\mathcal{W}_i$ such that $\mathcal{W}_i^H \mathcal{W}_i \leq P_i$. This constraint is a quadratic function and as stated in Appendix A results in nonlinear system equations. The Lagrange multipliers method described in Appendix A applies only to problems with linear constraints and quadratic objective function. Note that when all other constraints are verified, maximizing SNR with respect to a fixed transmission power reduces to minimizing the transmission power while maintaining a desired signal to noise ratio at the receiver. The only difference is a scaling factor that will amplify (or attenuate) the $\mathcal{W}_i$ to the desired power level. Since all other constraints are linear, this scaling will not cause the precoding vectors fail to satisfy a constraint that they had satisfied before being scaled. If $\mathcal{W}'_i$ is a precoding vector which minimizes the transmission power subject to a desired SNR value, the scaled precoding vector can be easily calculated by $\mathcal{W}_i = \mathcal{W}'_i \sqrt{\frac{P_i}{|\mathcal{W}'^H_i \mathcal{W}'_i|}}$.

As stated above, we will fix a signal to noise ratio at reception and we will minimize the transmission power while maintaining the SNR at the desired level. SNR at a given mobile station is simply derived from (5) as:

$$\text{SNR}_{ij} = \frac{|\mathbf{h}_{ij}\mathbf{w}_j^i|^2}{E[n_j^2]} = \frac{|\mathbf{h}_{ij}\mathbf{w}_j^i|^2}{N_0} \tag{14}$$

where SNR_{ij} is the contribution of i^{th} relay in the signal to noise ratio at j^{th} mobile station. The advantage of this method is the flexibility of the choice of constraints. Several strategies and possibilities may be considered. One possible approach is maximizing $\sum \text{SNR}_{ij}$ while maintaining the transmission power below a predefined threshold. This can be achieved by imposing $\sum \text{SNR}_{ij} = \text{SNR}_{desired}$ and minimizing the required transmission power. Other possibility may be to impose the same SNR at all mobile stations and to maximize the signal to noise ratio at one mobile station. Other possible strategy is to impose a different SNR at each mobile station. For example we may use water-filling to assign a SNR proportional to equivalent channel at each mobile station. In fact, as long as the constraints are expressed as linear combinations of $\mathbf{w}_j^i$, this method may be useful. It is the designer's task to choose the set of constraints the best fits the situation.

Each constraint must be expressed as $\hat{\mathbf{A}}_{in}\hat{\mathcal{W}}_i = \mathbf{c}$ where $\hat{\mathcal{W}}_i$ is the precoding vector $\mathcal{W}_i$ with real and imaginary parts separated.

3.4 Solving the system

In this section we are going to use the method introduced under Appendix A to solve the above equations. The problem consists of minimizing the transmission power $\hat{\mathcal{W}}_i^T \hat{\mathcal{W}}_i$ while satisfying a set of linear constraints $\hat{\mathbf{A}}_{in}\hat{\mathcal{W}}_i = \mathbf{c}_n$. The algorithm is as follows:

- Form the system constraints equation $\hat{\mathbf{A}}_i^T \hat{\mathcal{W}}_i = \mathbf{c}$ with $\hat{\mathbf{A}}_i = \left[\ \hat{\mathbf{A}}_{i1}^T \mid \hat{\mathbf{A}}_{i2}^T \mid \ldots \mid \hat{\mathbf{A}}_{in}^T\ \right]$ and $\mathbf{c} = \left[\ \mathbf{c}_1^T \mid \mathbf{c}_2^T \mid \ldots \mid \mathbf{c}_n^T\ \right]^T$
- Write the equation to be solved, $\mathbf{A}_i\mathbf{u}_i = \mathbf{b}$, for $i = 1, 2$ with

$$\mathbf{A}_i = \begin{bmatrix} 2\mathbf{I}_{2N} & \hat{\mathbf{A}}_i \\ \hat{\mathbf{A}}_i^T & \mathbf{0} \end{bmatrix},\ \mathbf{u}_i = \begin{bmatrix} \hat{\mathcal{W}}_i \\ \boldsymbol{\lambda} \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} \mathbf{0}_{2N\times 1} \\ \mathbf{c} \end{bmatrix}$$

- Find the solution as $\mathbf{u}_i = \mathbf{A}_i^{-1}\mathbf{b}$ and take the first $2N$ elements of $\mathbf{u}_i$ for $\hat{\mathcal{W}}_i$.
- Since $\hat{\mathcal{W}}_i$ is the $\mathcal{W}_i$ with the real and imaginary parts separated, combine every two consecutive elements of $\hat{\mathcal{W}}_i$ into a complex number and form $\mathcal{W}_i$.
- From $\mathcal{W}_i$, find precoding vectors $\mathbf{w}_j^i$ with respect to (8).

The simulation result are given under section 5.

4. A SPECIAL CASE

In this section an alternative method is derived for a special case where we impose the same signal to noise ratio at all mobile stations. We will use the same notations as in section 3. Using (6) and (7), one can rewrite (2) as:

$$\begin{aligned} \mathbf{y} &= (\mathbf{H}_1\mathbf{W}_1 + \mathbf{H}_2\mathbf{W}_2)\,\mathbf{s} + \mathbf{n} \qquad (15) \\ &= \left[\ \mathbf{H}_1 \quad \mathbf{H}_2\ \right] \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix} \mathbf{s} + \mathbf{n} = \mathbf{H}_{N\times 2R}\mathbf{W}_{2R\times N}\mathbf{s} + \mathbf{n} \end{aligned}$$

where $\mathbf{y}$ represents the column vector formed of all received signals at the mobile stations and $\mathbf{n}$ denotes the vector of reception noises. In order to assure that (15) satisfies (5) and the signal to noise ratios at all mobile stations are equal, we may write:

$$\mathbf{HWs} = g\mathbf{s} \tag{16}$$

where g is a constant that indicates the system gain. Note that if a matrix $\mathbf{W}$ satisfies (16) then every matrix $\mathbf{W}' = a\mathbf{W}$ will also satisfy (16) with $g' = ag$. For simplicity we will choose a $\mathbf{W}'$ for which $g' = 1$. Thus we must solve $\mathbf{HW}'\mathbf{s} = \mathbf{s}$ for $\mathbf{W}'$. One possible solution[2] is:

$$\mathbf{H}_{N\times 2R}\mathbf{W}'_{2R\times N} = \mathbf{I}_N \tag{17}$$

Equation (17) has an answer if and only if $\mathbf{H}$ has full row rank (i.e. $R \geq N/2$). In this case $\mathbf{W}'$ is the Moore-Penrose pseudo inverse of $\mathbf{H}$. Thus $\mathbf{W}'$ can be calculated as a function of $\mathbf{H}$ using

$$\mathbf{W}' = \mathbf{H}^H \left(\mathbf{HH}^H\right)^{-1} \tag{18}$$

In order to estimate the diversity gain of the system using (18), we note that each mobile station (e.g. MS_1) receives $2R$ replicas of the message (e.g. s_1). However, the system must cancel the interferences of $N-1$ undesired messages (e.g s_2 to s_N). Thus, the diversity order would be $2R-N+1$. As stated under section 2, the problem with (17) is that it requires each relay to have the channel information of other relay. Thus once again we may require that each of the relays satisfy

$$\mathbf{H}_{i(N\times R)}\mathbf{W}'_{i(R\times N)} = \mathbf{I}_N \qquad ,i = 1, 2 \tag{19}$$

In this case since both $\mathbf{H}_1$ and $\mathbf{H}_2$ must have full row rank, the number of relay antennas must be equal or grater than the number of mobile stations ($R \geq N$). Then $\mathbf{W}'_1$ and $\mathbf{W}'_2$ can be calculated independently in each relay:

$$\mathbf{W}'_i = \mathbf{H}_i^H \left(\mathbf{H}_i\mathbf{H}_i^H\right)^{-1} \qquad ,i = 1, 2 \tag{20}$$

If the number of antennas in each relay is equal to the number of mobile stations, then pseudo inverse reduces to normal inverse and we can use $\mathbf{W}' = \mathbf{H}^{-1}$.

When $\mathbf{W}'_i$ is found, the relays will scale the precoding vectors to available transmission power. Thus $\mathbf{W}_i$ is calculated such that the power of transmitted signal of each relay equals P_i, that is to say:

$$\mathbf{W}_i = \frac{\sqrt{P_i}\,\mathbf{W}'_i}{\|\mathbf{W}'_i\|} \tag{21}$$

[2]The only solution which holds for all $\mathbf{s}$

If $\mathbf{W}_i$ is calculated according to (21), the received signal at each mobile station will depend only on the data symbol intended to that specific mobile station, thus the system can be seen as N separate channels. By substituting (20) and (21) in (15) we will obtain $\mathbf{y} = \mathbf{y}_1 + \mathbf{y}_2 + \mathbf{n}$, where $\mathbf{y}_1$ and $\mathbf{y}_2$ denote the contribution of the first and the second relays in the signal transmission and are given by

$$\begin{aligned} \mathbf{y}_i &= \frac{\sqrt{P}\mathbf{H}_i\mathbf{H}_i^H\left(\mathbf{H}_i\mathbf{H}_i^H\right)^{-1}}{\|\mathbf{H}_i^H\left(\mathbf{H}_i\mathbf{H}_i^H\right)^{-1}\|}\mathbf{s} \\ &= \frac{\sqrt{P}}{\|\mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\|}\mathbf{s} \end{aligned} \quad (22)$$

As we can see in (22), the system may be considered as N independent channels each of which transmitting one signal s_j. In this case, like the case with complete CSI at both relays, each mobile station receives $2R$ replicas of the message, but here each relay must independently cancel $N-1$ undesired messages. Therefore the total number of system constraints is $2 \times (N-1)$, and diversity gain would be $2R - 2(N-1)$.

The results can be simply generalized to the case of L relays. System equations remain unchanged with the only difference that we will have the channel vectors $\mathbf{H}_1$ to $\mathbf{H}_L$ and the precoding vectors $\mathbf{W}_1$ to $\mathbf{W}_L$. For the case that each relay knows only its own CSI, we can calculate the precoding vector $\mathbf{W}'_i$ of the i^{th} relay as follows:

$$\mathbf{W}'_i = \mathbf{H}_i^H\left(\mathbf{H}_i\mathbf{H}_i^H\right)^{-1} \qquad , i = 1, \cdots, L \quad (23)$$

Then the signal at the destination is given by (24).

$$\mathbf{y} = \sum_{i=1}^{L}\mathbf{y}_i + \mathbf{n} \quad (24)$$

Again, the system may be considered as N parallel channels each of which transmitting the signal s_j to the jth mobile station MS_j, with a diversity gain of $LR - L(N-1)$.

For the case that all relays have the complete CSI, the precoding vectors are calculated from (18) and the diversity will be $LR - N + 1$.

5. SIMULATION RESULTS

This section introduces some simulation results that confirm the equations in the previous sections. All simulations are obtained for QPSK modulation using Monte Carlo method in MATLAB for the case where the same SNR is imposed at all mobile stations.

As stated under section 2, there are two possible scenarios depending on whether or not the relay stations are provided with the knowledge of channel state information of other relays. Figure 2 shows the system performance in both cases when 2 three-antenna relay stations cooperate in sending a message toward two mobile stations. It shows that if the channel information is available to both relays, lower BER and higher diversity is obtained ($2\times3-2+1 = 5$). This is at the cost of more complexity in the transmission protocol. On the other hand, if each of the relays knows only its respective channel information, the system is more practical at the cost of higher BER and lower diversity ($2\times(3-1) = 4$).

Figure 3 shows the BER as a function of E_b/N_0 for different number of mobile stations ($N = 2\cdots5$), all for a given number of relays ($L = 2$) and relay antennas ($R = 4$). Using more relay antennas compared to mobile stations results in lower bit error rate and higher diversity gain. We can see that when the mobile stations outnumber relay antennas, an error floor appears in the curves. Note that in this figure

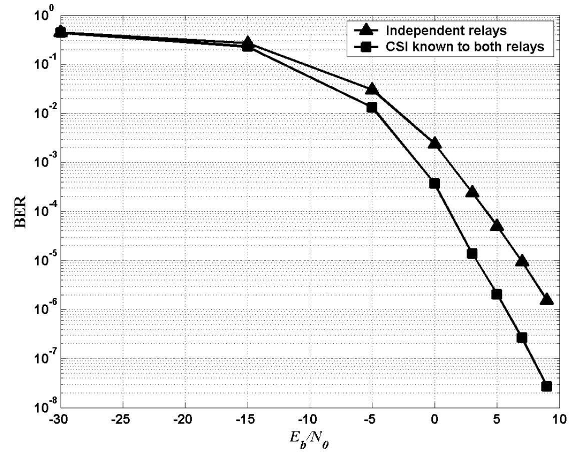

Figure 2: System performance ($L = 2$, $N = 2$, and $R = 3$)for i) when CSI is known to both relays and ii) when each relay only has its own relative CSI

relays are considered to have no knowledge of the link between other relay and mobile stations. The diversity is thus $2\times4-2(2-1) = 6$, 4, and 2 when number of mobile stations is respectively 2, 3, and 4.

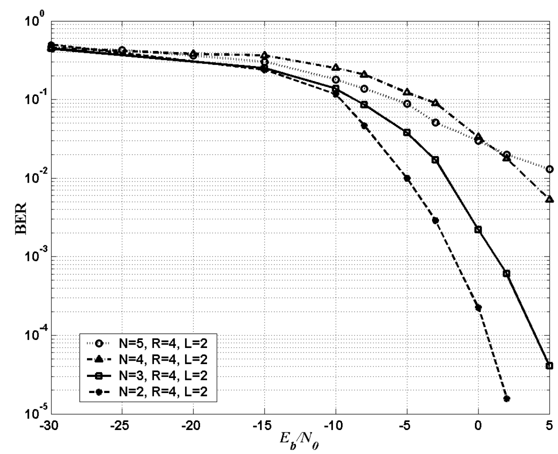

Figure 3: System performance for two independent 4-antenna relay stations with different mobile station numbers

Figure 4 depicts the system performance for different number of relay stations. All curves are obtained for 3-antenna relay stations and two mobile stations. The only difference is the number of relay stations contributing in signal transmission. As we can see higher relay numbers results in better system performance. the relays are considered to be independent, thus the diversity is of order $LR-L(N-1)$.

6. CONCLUSION

We have applied the Lagrange multipliers method to calculate the optimum precoding vectors (i.e the set of precoding vectors eliminating the multiuser interference and maximizing the signal to noise ratio at the mobile stations). Lagrange multipliers method featured a high flexibility to introduce the desired constraints and optimization equations.

We have demonstrated that if we want the SNR at all mobile stations to be the same, the precoding vectors may

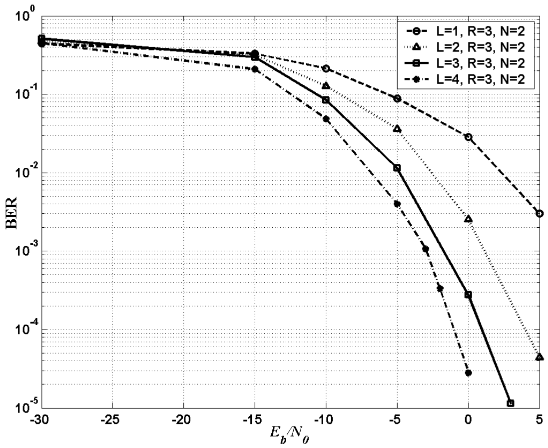

Figure 4: System performance for different number of relay stations (independent relays)

be calculated using the Moore-Penrose pseudo inverse. In this case we have considered two different schemes. In the first scheme all relays know all channel states, thus the whole system is modelled by a single matrix equation. In the second scheme relays are not inter-connected and thus each relay only knows its own relative channel to mobile stations. The latter is more practical but it has less degree of freedom resulting in reduced performance. Diversity gain is evaluated for both cases. Simulation results confirmed the accuracy of given models and equations.

A. LAGRANGE MULTIPLIERS METHOD

The method of Lagrange multipliers is a method for finding the local extrema of a function subject to constraints. For example if we want to maximize $f(x, y)$ subject to $g(x, y) = c$, we introduce a variable λ and try to maximize $f(x, y) - \lambda\left(g\left(x, y\right) - c\right)$. It is easy to perceive that the final set of equations will be linear, if and only if g is a linear function of x and y while f is a quadratic function of these two variables. Let us consider a case with a large number of parameters and constraints, that satisfies the above description and thus yields linear equations. This section introduces an approach for application of the Lagrange multipliers method to such cases using vector and matrix calculus.

Let $\mathbf{u} = [u_1 \cdots u_N]^T$ be a set of N independent variables. We want to minimize the quadratic function $f(\mathbf{u}) = \mathbf{u}^T\mathbf{u}$ subject to P linear constraints expressed as $\mathbf{a}^T\mathbf{u} = \mathbf{c}$, where $\mathbf{a}$ is an $N \times P$ matrix and $\mathbf{c}$ is a column vector of length P. We introduce a vector variable $\boldsymbol{\lambda} = [\lambda_1 \cdots \lambda_P]^T$ and an objective function $\Lambda(\mathbf{u}, \boldsymbol{\lambda}) = \mathbf{u}^T\mathbf{u} - \boldsymbol{\lambda}^T \cdot \left(\mathbf{a}^T\mathbf{u} - \mathbf{c}\right)$. In order to find the extrema of $\mathbf{u}$ respecting the constraints, we will derivate $\Lambda(\mathbf{u}, \boldsymbol{\lambda})$ with respect to $\mathbf{u}$ and then $\boldsymbol{\lambda}$:

$$\frac{\partial \Lambda}{\partial \mathbf{u}} = 2\mathbf{I}_N\mathbf{u} + \mathbf{a}\boldsymbol{\lambda} = 0, \quad \frac{\partial \Lambda}{\partial \boldsymbol{\lambda}} = \mathbf{a}^T\mathbf{u} - \mathbf{c} = 0$$

$$\nabla\Lambda = \begin{bmatrix} \frac{\partial \Lambda}{\partial \mathbf{u}} \\ \hdashline \frac{\partial \Lambda}{\partial \boldsymbol{\lambda}} \end{bmatrix} = \begin{bmatrix} 2\mathbf{I}_N\mathbf{u} + \mathbf{a}\boldsymbol{\lambda} \\ \hdashline \mathbf{a}^T\mathbf{u} - \mathbf{c} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hdashline \mathbf{0} \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{c:c} 2\mathbf{I}_N & \mathbf{a} \\ \hdashline \mathbf{a}^T & \mathbf{0} \end{array}\right] \begin{bmatrix} \mathbf{u} \\ \hdashline \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hdashline \mathbf{c} \end{bmatrix} \tag{25}$$

Thus the optimizing set of variables may be calculated by solving a set of linear equations. This method is used in section 3 to find the optimum precoding vectors minimizing the required power of relays.

B. COMPLEX TO REAL MATRIX CONVERSION

The transmitted data $\mathbf{s}$, channel coefficients $\mathbf{h}_{ij}$, and precoding vectors $\mathbf{w}_j^i$ are complex vectors. These values lead in complex equations that are not easy to handle. It is thus preferred to break each complex variable into two real variables and deal with real variables.

Let us write a complex number c as $c_R + jc_I$. In this case the equation $a \cdot b = c$, with a, b, and c being complex scalars will be written as:

$$\begin{bmatrix} c_R \\ c_I \end{bmatrix} = \begin{bmatrix} a_R & -a_I \\ a_I & a_R \end{bmatrix} \begin{bmatrix} b_R \\ b_I \end{bmatrix} \tag{26}$$

Using the same principle, a complex matrix equation such as $\mathbf{AB} = \mathbf{C}$ where $\mathbf{A}$ is a complex matrix and $\mathbf{B}$ and $\mathbf{C}$ are complex column vectors can be written as follows:

$$\begin{bmatrix} \mathbf{A}_R & -\mathbf{A}_I \\ \mathbf{A}_I & \mathbf{A}_R \end{bmatrix} \begin{bmatrix} \mathbf{B}_R \\ \mathbf{B}_I \end{bmatrix} = \begin{bmatrix} \mathbf{C}_R \\ \mathbf{C}_I \end{bmatrix} \tag{27}$$

Or:

$$\Re\left\{\left(\begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix} \otimes \mathbf{A}\right)\right\} \Re\left\{\left(\begin{bmatrix} 1 \\ -j \end{bmatrix} \otimes \mathbf{B}\right)\right\} = \Re\left\{\left(\begin{bmatrix} 1 \\ -j \end{bmatrix} \otimes \mathbf{C}\right)\right\} \tag{28}$$

Where $\otimes$ denotes the Kronecker product. For the further simplicity of the application intended in this paper, we prefer to use (29) instead of (28) which leads to the same equations:

$$\Re\left\{\left(\mathbf{A} \otimes \begin{bmatrix} 1 & j \\ -j & 1 \end{bmatrix}\right)\right\} \Re\left\{\left(\mathbf{B} \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)\right\} = \Re\left\{\left(\mathbf{C} \otimes \begin{bmatrix} 1 \\ -j \end{bmatrix}\right)\right\} \tag{29}$$

Equation(29) has the advantage of having the real and imaginary parts of an element next together.

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