

# AN LMMSE-BASED MERGING APPROACH FOR SUBPIXEL-BASED DOWNSAMPLING

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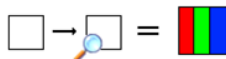
## ABSTRACT

Subpixel-based downsampling improves resolution at the cost of color fringing artifacts. In this paper, we propose a novel multiple descriptions approach for subpixel-based downsampling. Each description is treated as a noisy observation of the original image and an optimal linear estimator based on the LMMSE criteria is applied in the DCT domain to combine multiple descriptions. Simulation result shows that the proposed method gives sharper resultant images while being free of color fringing artifacts with compared to the resultant images of the conventional downsampling methods.

**Index Terms**— Subpixel-based Downsampling, Linear Minimum Mean Square Error(LMMSE)

## 1. BACKGROUND ON SUBPIXEL-BASED DOWNSAMPLING

Subpixel rendering takes advantage of the fact that each pixel on a color LCD is actually composed of individual red, green, and blue subpixel stripes to anti-alias text and gives greater details than simple pixel rendering. We typically think of a pixel as an inseparable region with a single color. However, the subpixels are visibly separated as shown in Fig.1 when viewed at short distance with the help of a magnifying glass.

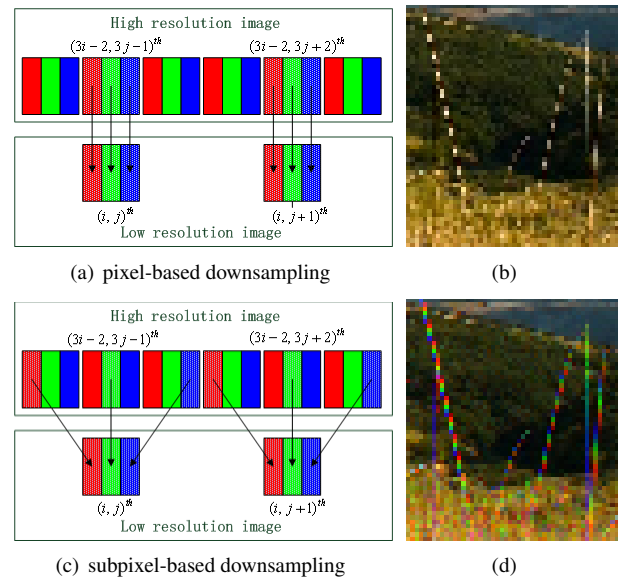


**Fig. 1.** A single white pixel appears as an inseparable region with solid color (left), and magnified showing separate R, G, B subpixels (right)

Nowadays, there is a tremendous need to display high resolution images/video on low resolution display terminals. A traditional way is simple downsampling by selecting one out of every N pixels which incurs severe aliasing artifacts in regions with high spatial frequency. So we usually apply an anti-aliasing filter to suppress aliasing artifacts. However, it smooths the result at the cost of unpleasant blurring artifacts, as only the low frequency information can be retained

in the process. Since the number of individual reconstruction points in a LCD can be increased by three times, application of subpixel rendering in downsampling scheme may lead to improvement in apparent resolution.

Let us assume an input  $3M \times 3N$  high resolution image is to be displayed on a  $M \times N$  low resolution device. Daly et al note a simple subpixel-based downsampling scheme which decimates red, green, and blue subpixels alternately in horizontal [3]. More specifically, the  $(i, j)$  pixel in downsampled image consists of subpixels  $(R_{3i-2,3j-2}, G_{3i-2,3j-1}, B_{3i-2,3j})$  as shown in Fig.2(c), where  $R_{3i-2,3j-2}$  is the red component of the  $(3i - 2, 3j - 2)^{th}$  pixel of original image and so on.



**Fig. 2.** (a) direct pixel-based downsampling (b) result of pixel-based downsampling where “grass” is broken (c) direct subpixel-based downsampling (d) result of subpixel-based downsampling where “grass” is continuous but with color fringing artifacts

Fig.2 shows the resultant images of two downsampling schemes. It is interesting to see that direct subpixel-based downsampling may potentially preserve more high frequency

details compared with the direct pixel-based downsampling counterpart thanks to the increase in the number of individual reconstruction points, leading to clearer and sharper down-sampled images. However, applying subpixel techniques to color images/video is non-trivial by reason that each subpixel can signal only red, green or blue information, instead of the original full color. As a result, direct application of subpixel approaches to images/video downsampling may cause color fringing problem as illustrated in Fig. 2(d) with very annoying perceptual artifacts.

In summary, exploiting subpixel in downsampling brings both opportunity as well as problem. The opportunity is that we can potentially increase the apparent resolution of a patterned display by treating the subpixels separately. The potential luminance resolution is the subpixel resolution. The problem is that treating each subpixel purely as a luminance source, while ignoring color, can cause a large amount of color distortion.

In [1], a five-tap filter is proposed to smooth the result of the subpixel-based downsampling, however, the low pass filter relieves color at the cost of image blurring, and can be only adopted as an enhancement technique for achromatic image. Based on psychophysical experiments, [2] defines an error metric in frequency domain, and derive the filter coefficients by minimizing this metric. [3] proposes an algorithm based on HVS to remove visible chrominance aliasing.

In this paper, we present a novel method for subpixel-based downsampling. At first, we try to use multiple descriptions to represent the original image. Each description is treated as a noisy observation of the original image. Then, an LMMSE based merging algorithm is developed which gives improved image quality after reconstruction of descriptions.

This paper is organized as follows: in section 2, we derive our proposed LMMSE based merging algorithm for subpixel-based downsampling; simulation result is shown in section 3; and finally, section 4 concludes this work.

## 2. PROPOSED MULTIPLE DESCRIPTIONS APPROACH

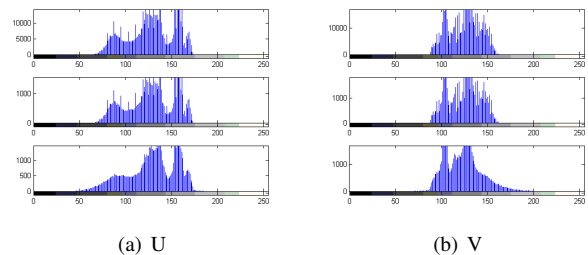
For the simplicity of presentation, let us define the method of direct pixel-based downsampling as PIXEL, pixel-based downsampling with an anti-aliasing filter as SMOOTH, and direct subpixel-based downsampling as SHARP.

### 2.1. Multiple Descriptions

For subpixel-based downsampling, we need to achieve these goals: preserving luminance resolution (**sharpness**) and suppressing chrominance distortion (**color fringing artifacts**). Therefore, we consider luminance and chrominance separately. Here we choose YUV color space, where luminance and chrominance could be represent by Y and U,V respectively.

It is interesting that none of the methods PIXEL, SMOOTH, or SHARP is perfect. For example, the resultant images of PIXEL suffer luminance aliasing artifacts; SMOOTH removes aliasing artifacts but causing unpleasant blurring; SHARP alleviates blurring at the cost of annoying color fringing artifacts.

Although the luminance component of PIXEL suffers aliasing artifacts, the chrominance components of PIXEL almost have the same distribution as the original image which is shown in Fig.3(a) and Fig.3(b). In the resultant image of SHARP, the distributions of UV components are expanded to a much wider range, causing severe chrominance distortion. Therefore, in our simulation, we will take the U, V components of PIXEL as those of output images.



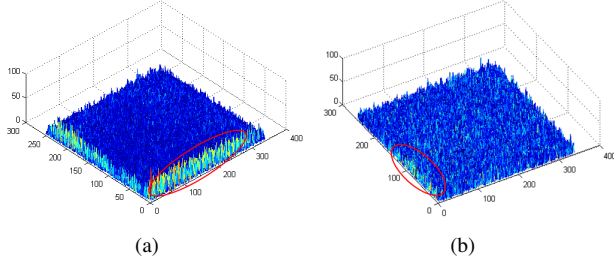
**Fig. 3.** histogram distributions of UV components: (upper) original image; (middle) PIXEL; (down) SHARP

Direct subpixel-based downsampling (SHARP) preserves much more luminance details than pixel-based downsampling (PIXEL). However, the luminance component of SHARP is still not perfect. Fig.4 is the resultant image of combining the Y component of SHARP with the U, V components of PIXEL. We observe that the resultant image is free of chrominance distortion, but the “grass” and “branch” are still broken, details are not well preserved. The Y component of SHARP could only partly represent original luminance information.



**Fig. 4.** Resultant image of combining luminance of SHARP with chrominance of PIXEL

We calculate luminance residue between original image and SMOOTH/SHARP, then draw it in frequency domain as shown in fig.5. It is obvious that the residue distributions of SMOOTH and SHARP are located in different frequency area, which means that the error correlation coefficient between the two residues is small.



**Fig. 5.** Residue distributions in frequency domain: (a) residue for SMOOTH (b) residue for SHARP

From previous analysis, we conclude there are three characters for subpixel-based downsampling: each one of PIXEL, SMOOTH and SHARP alone provides low but acceptable quality; PIXEL, SMOOTH and SHARP have similar noise levels; The error correlation coefficient among residues are small. PIXEL, SMOOTH and SHARP may be treated as three descriptions of the original image [4].

## 2.2. LMMSE Based Merging

Suppose we have  $n$  different descriptions  $D = [d_1, d_2, \dots, d_n]^T$ . Each one is a kind of noisy observation of the same original image  $x$ . Then, the errors between descriptions and original image are given by

$$e_i = d_i - x, i = 1, 2, \dots, n \quad (1)$$

We also define  $W = [w_1, w_2, \dots, w_n]^T$  under the constraint of  $\sum_{i=1}^n w_i = 1$  as the optimal weights used to combine different descriptions. The distortion between original image and linear combination of descriptions can be written as:

$$\xi = |x - W^T D| \quad (2)$$

Substituting  $D$  in equation (2) with equation (1), the distortion  $\xi$  can be rewritten as

$$\begin{aligned} \xi &= |x - W^T D| \\ &= |x - w_1(x + e_1) - w_2(x + e_2) - \dots - w_n(x + e_n)| \\ &= w_1(e_1 - e_n) + w_2(e_2 - e_n) + \dots + w_{n-1}(e_{n-2} - e_n) + e_n \end{aligned} \quad (3)$$

For the weights in the sense of linear minimum mean square error (LMMSE), the expectation of square error  $E[\xi^2]$  should be minimized. The problem is formulated as below

$$\begin{aligned} \min_W \quad & E[\xi^2] \\ \text{s.t.} \quad & \xi = w_1(e_1 - e_n) + \dots + w_{n-1}(e_{n-2} - e_n) + e_n \\ & \sum_{i=1}^n w_i = 1 \end{aligned} \quad (4)$$

After differentiating  $E[\xi^2]$  with respect to each  $w_i, i = 1, 2, \dots, n-1$ , we get

$$\begin{cases} E[(e_1 - e_n)(w_1(e_1 - e_n) + \dots + w_{n-1}(e_{n-2} - e_n) + e_n)] = 0 \\ E[(e_2 - e_n)(w_1(e_1 - e_n) + \dots + w_{n-1}(e_{n-2} - e_n) + e_n)] = 0 \\ \vdots \\ E[(e_{n-1} - e_n)(w_1(e_1 - e_n) + \dots + w_{n-1}(e_{n-2} - e_n) + e_n)] = 0 \end{cases}$$

Solving above  $n-1$  equations together with the constraint of  $\sum_{i=1}^n w_i = 1$ , we obtain the optimal weights:

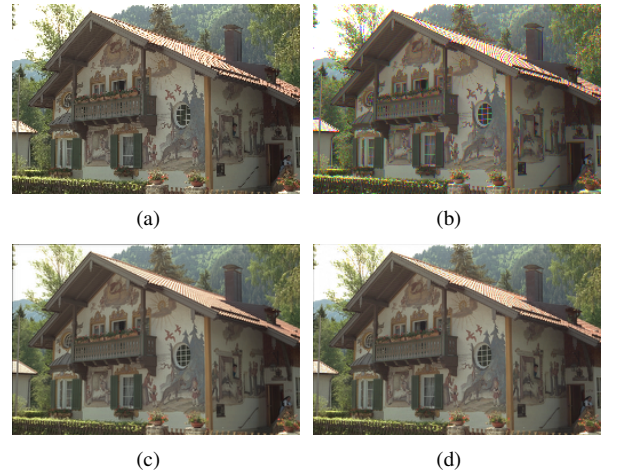
$$\begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_{n-1} \end{pmatrix} = C \begin{pmatrix} \sigma_n^2 - \sigma_1 \sigma_n \rho_{1n} \\ \sigma_n^2 - \sigma_2 \sigma_n \rho_{2n} \\ \vdots \\ \sigma_n^2 - \sigma_{n-1} \sigma_n \rho_{(n-1)n} \end{pmatrix} \quad (5)$$

where  $C$  is the covariance matrix of  $e_1, e_2, \dots, e_n$ .  $\sigma_i^2 = E[e_i^2], i = 1, 2, \dots, n$  are the error variances and  $\rho_{in} = E[e_i e_n] / \sigma_i \sigma_n, i = 1, 2, \dots, n-1$  are the error correlation coefficients.

For the sake of error distribution parameters, one way is to obtain the distortion between original image and descriptions. However, the size of the original large image is  $3M \times 3N$  while the sizes of descriptions are  $M \times N$ , we actually have no original small image  $x$ . In our experiments below, we use the resultant images of the method proposed in [5] as an estimation of original small image  $x$ .

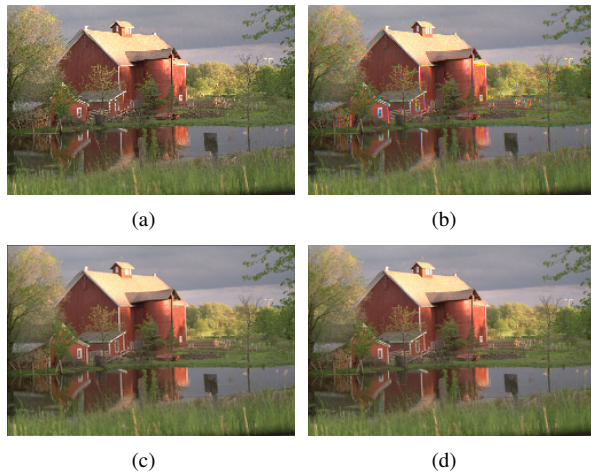
## 3. SIMULATION RESULTS

Simulation results below give a subjective measurement about visual quality. The measurement includes: luminance resolution (**sharpness**) and chrominance distortion (**color fringing artifacts**).



**Fig. 6.** (a): direct pixel-based downsampling (b): direct subpixel-based downsampling (c): pixel-based downsampling with antialiasing filter (d): proposed method

Fig.6(a) and Fig.6(b) show that pixel-based downsampling causes luminance aliasing artifacts while direct subpixel-based downsampling preserves more details at the expense of color fringing which are obvious at the broken lines as well as the continuous but colorful lines of the “window”, respectively; Applying traditional anti-aliasing filter causes blurring, as shown in Fig.6(c); From Fig.6(d), we can find that our proposed method not only suppresses color fringing artifacts but also gives a sharper result compared to the conventional downsampling methods, especially in regions with high details such as window, roof and trees. Similar conclusions could be obtained from Fig.7.



**Fig. 7.** (a): direct pixel-based downsampling (b): direct subpixel-based downsampling (c): pixel-based downsampling with antialiasing filter (d): proposed method

As mentioned in previous Section, we use the resultant images of [5] as an estimation of original small image  $x$ . The method in [5] preserves much more high frequency details but with some color fringing artifacts, while our proposed method is free of chrominance distortion (color fringing artifacts) at some expense of sharpness. It implies that the two goals of chrominance distortion reduction and apparent luminance resolution improvement may not be simultaneously achievable. Fig.8 depicts that our proposed method provides a good trade off between the opportunity and the problem of subpixel-based downsampling.

Unfortunately, showing the results in printed form may have problem, since the result depends on the display itself as well as the distance between the observer and LCD screen. Since the procedure is downsampling and we have no reference small images which implies that the traditional SNR measure is not appropriate. To find an effective objective measure is an urgent problem to be solved.



(a)



(b)

**Fig. 8.** (a): image used as estimation (b): proposed method

#### 4. CONCLUSION

In this paper, we propose a novel multiple descriptions approach for subpixel-based downsampling. Each description is treated as a noisy observation of the original image and an optimal linear estimator based on the LMMSE criteria is applied in the DCT domain to combine multiple descriptions. Simulation results demonstrate that our proposed method provides a good trade off between the apparent luminance resolution improvement and color fringing artifacts reduction.

#### 5. REFERENCES

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