DUAL H∞ VS DUAL KALMAN FILTERS FOR M-AR PARAMETER ESTIMATION FROM NOISY OBSERVATIONS

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ABSTRACT

When multiple correlated data channels are simultaneously processed, multichannel-autoregressive (M-AR) processes may be combined with optimal filtering such as Kalman or H_∞ for prediction or estimation from noisy observations. However, the estimation of the M-AR parameters from noisy observations is a key issue to be addressed. Off-line or iterative approaches have been proposed recently, but their computational costs are high or some of them may diverge. Using on-line approaches such as EKF and SPKF is of interest, but the size of the state vector to be estimated is quite high. To reduce this size and the resulting computational costs, we suggest using dual optimal filters. In this paper, we study the relevance of cross-coupled Kalman filters and cross-coupled H_∞ filters. The comparative simulation study we carry out shows that our approach corresponds to a compromise between computational cost and performances in terms of pole estimation accuracy.

1. INTRODUCTION

Scalar autoregressive (AR) process has played a key role in signal processing, more particularly in the field of speech enhancement and coding, in spectral analysis for biomedical applications, in mobile communication systems, in radar processing, etc. Nevertheless, when the observations are disturbed by an additive measurement noise, the AR parameter estimates may be biased. To overcome this problem, one can use the "noise-compensated" Yule-Walker (NCYW) equations, which however require the preliminary estimation of the additive-noise variance. To deal with the estimations of both the AR process and the noise variance, various off-line or iterative methods have been proposed by Zheng [1], Bobillet et al. [2], etc. On-line methods can be based on extended Kalman filter (EKF) and the sigma-point Kalman filters (SPKF) [3]. As an alternative, one of us suggests using mutually-interactive Kalman filters based solutions to avoid a non-linear approach [4]. Once a new observation is available, the first filter uses the latest estimated AR parameters to estimate the signal, whereas the second filter uses the estimated signal to update the AR parameters. This approach can be viewed as a recursive instrumental variable-based method and hence has the advantage of providing consistent estimates of the parameters from noisy observations. To relax Gaussian assumptions required by Kalman filtering, cross-coupled H_∞ filter based solution was then studied [5].

Even if scalar AR model is often used, a multichannel-autoregressive (*M*-AR) process is more suited when multiple correlated data channels are simultaneously processed. This is for instance the case in biomedical applications when dealing with cardiovascular systems and analyzing the interactions between blood pressure, heart rate and respirations [6]. In radar processing when multiple antennas are used, one aims at rejecting the sea clutter to detect the target. Variants of the space-time adaptive processing (STAP) algorithm such as the parametric adaptive matched filter (PAMF) and the space-time autoregressive filter (STAR) [7] consist in modelling the sea clutter by a *M*-AR process. In the field of mobile communication systems based on CDMA or OFDM, the fading-channel processes can be estimated or predicted by modelling the channels with a *M*-AR process [8] [9]. In all cases, the p^{th} order *M*-AR process $\underline{s}(n)$ satisfies:

$$\underline{s}(n) = \begin{bmatrix} s_1(n) \\ \vdots \\ s_M(n) \end{bmatrix} = -\sum_{l=1}^{p} A^{(l)} \underline{s}(n-l) + \underline{u}(n) \qquad (1)$$

where $\left\{A^{(l)}\right\}_{l=1,\ldots,p}$ are the $M \times M$ AR parameter matrices and $\underline{u}(n)$ is a $M \times 1$ zero-mean white noise vector whose autocorrelation matrix Σ_u satisfies:

$$\Sigma_u = diag\left(\begin{bmatrix} \sigma^2_{u,1} & \cdots & \sigma^2_{u,M} \end{bmatrix}\right) \qquad (2)$$

In addition, the *M*-AR parameter matrices $\left\{A^{(l)}\right\}_{l=1,\ldots,p}$ are constrained so that the roots $\{p_i\}_{i=1,\ldots,pM}$ of:

$$\det A_p(z) \qquad (3)$$

lie inside the unit circle in the *z*-plane, where:

$$A_p(z) = I_M + A^{(1)}z^{-1} + A^{(2)}z^{-2} + \ldots + A^{(p)}z^{-p} \qquad (4)$$

In (4), z^{-1} denotes the backward shift operator and I_M is the $M \times M$ identity matrix.

According to the comparative study carried out by Schlögl [10], the Nuttall-Strand method is the most relevant approach to estimate the *M*-AR parameters among the standard approaches (Yule-Walker equations, Levinson algorithm, etc.), when noise-free observations are available. However, when the *M*-AR process is disturbed by an additive white noise, the standard estimation methods, mentioned above, lead to biased estimates of the *M*-AR parameter matrices.

To avoid this drawback, the approach proposed in [11] is based on a set of two equations that the noise variances and the coefficients of the AR matrices should satisfy. The first one corresponds to the NCYW equations and the second makes it possible to express the noise variances from the coefficients of the AR matrices and the autocorrelation of the observations filtered by the inverse filter $A_p(z)$. Therefore, a Newton-Raphson algorithm is used to estimate the noise variances and the *M*-AR parameters are deduced by means of the NCYW equations. In [12], the extension of Zheng's method [1] to the multichannel case has recently been proposed. Nevertheless, this method may lead to a set of AR parameter matrix estimates corresponding to an unstable system when the SNR is low. In [13], Petitjean et al. have proposed an extension of [2]; although the approach provides significant results even for low SNR, the computational cost could be reduced. Concerning on-line methods, SPKF methods or EKF approach could be considered [13]. However, the size of the state vector to be estimated is $\left(M^2 p + Mp\right)\times 1$. Indeed, it stores both the coefficients of the AR parameter matrices and the *p* last values of the process $\underline{s}(n)$. The computational cost hence becomes high. To avoid using high dimension matrices, we propose to extend to the multichannel case the approach proposed in [5] and [4]. The proposed methods are then compared with existing methods such as [11], [12], [13] and SPKF methods.

In section 2, the dual optimal structure is presented. The results of the comparative study between dual Kalman filtering, dual H_∞ filtering and other approaches are given in section 3.

2. PROBLEM STATEMENT

2.1 Dual optimal filter structure

Let the *M*-AR process $\underline{s}(n)$ be disturbed by an additive zero-mean white noise vector $\underline{b}(n)$ uncorrelated with $\underline{u}(n)$, and with correlation matrix $\Sigma_b = diag\left(\left[\sigma_{b,1}^2 \quad \cdots \quad \sigma_{b,M}^2\right]\right)$:

$$\underline{y}(n) = \underline{s}(n) + \underline{b}(n) \tag{5}$$

The purpose of our method is to estimate the *M*-AR parameter matrices from the noisy observation vector $\underline{y}(n)$. To this end, our approach is based on dual optimal filters as shown in Fig.1. Thus, given the noisy observation vector $\underline{y}(n)$, the first optimal filter uses the latest estimated AR parameter matrices to estimate the *M*-AR process, while the second filter estimates the parameter matrices from the estimated process vector $\underline{\hat{s}}(n)$.

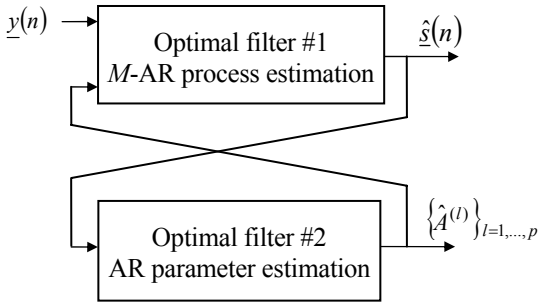

Figure 1 : Dual optimal filters for the estimation of the *M*-AR process and its parameter matrices.

Let us first define the following state vector, whose dimension is $Mp \times 1$:

$$\underline{x}(n) = \left[\underline{s}(n)^T \quad \cdots \quad \underline{s}(n-p+1)^T\right]^T \tag{6}$$

Hence, the state-space representation of the system (1) and (5) is given by:

$$\begin{cases} \underline{x}(n) = \Phi \underline{x}(n-1) + \Gamma \underline{u}(n) \\ \underline{y}(n) = H \underline{x}(n) + \underline{b}(n) \end{cases} \tag{7}$$

where the transition matrix Φ, Γ and H are respectively defined as follows:

$$\Phi = \begin{bmatrix} -A^{(1)} & -A^{(2)} & \cdots & -A^{(p)} \\ I_M & 0_M & \cdots & 0_M \\ 0_M & \ddots & 0_M & \vdots \\ 0_M & 0_M & I_M & 0_M \end{bmatrix} \tag{8}$$

and $$H = \Gamma^T = \left[I_M \quad 0_M \quad \cdots \quad 0_M\right] \tag{9}$$

When using H_∞ filters, one focuses on the estimation of a specific linear combination of the state vector components, as follows:

$$\underline{z}(n) = L\underline{x}(n) \tag{10}$$

where L is a $M \times Mp$ linear transformation operator. Here, as we aim at estimating the process $\underline{s}(n)$, this operator is selected to be $L = H = \Gamma^T = \left[I_M \quad 0_M \quad \cdots \quad 0_M\right]$.

2.2 Purpose of Kalman or H_∞ filtering

Optimal recursive filters make it possible to recursively estimate the state vector $\underline{x}(n)$. In the following, let $\underline{\hat{x}}(n/l)$ denote the estimation of $\underline{x}(n)$ given $\left\{\underline{y}(i)\right\}_{i=0,\ldots,l}$.

Based on the state space model (7), two kinds of approaches can be used:

On the one hand, a Kalman filter provides an estimate $\underline{\hat{s}}(n/n) = \Gamma^T \underline{\hat{x}}(n/n)$ of the *M*-AR process $\underline{s}(n)$ by minimizing the trace of the following *a posteriori* error covariance matrix $P(n/n) = E[(\underline{x}(n) - \underline{\hat{x}}(n/n))(\underline{x}(n) - \underline{\hat{x}}(n/n))^T]$. Note that this covariance matrix satisfies a Ricatti equation.

On the other hand, the *a posteriori* H_∞ filter aims at estimating $\underline{\hat{s}}(n) = L\underline{\hat{x}}(n) = \Gamma^T \underline{\hat{x}}(n/n)$ by minimizing the H_∞ norm of the transfer operator that maps the noise vectors $\underline{u}(n)$, $\underline{b}(n)$ and the initial state error $\underline{e}_0 = \underline{x}(0) - \underline{\hat{x}}(0)$ to the estimation error $\underline{e}(n) = \underline{s}(n) - \underline{\hat{s}}(n)$, as follows:

$$J_\infty = \sup_{\underline{u}(n),\underline{b}(n),\underline{x}(0)} J \tag{11}$$

where:

$$J = \frac{\sum_{n=0}^{N-1} \underline{e}(n)^T \underline{e}(n)}{\underline{e}_0{}^T P_0^{-1} \underline{e}_0 + \sum_{n=0}^{N-1} \left(\underline{u}(n)^T Q_u^{-1} \underline{u}(n) + \underline{b}(n)^T R_b^{-1} \underline{b}(n) \right)} \tag{12}$$

with N the number of available data samples, $Q_u > 0$ and $R_b > 0$ are weighting matrices which are tuned by the designer to achieve performance requirements. In addition, $P_0 > 0$ denotes a positive matrix that reflects how small is the initial state error $\underline{e}_0$.

However, as a closed-form solution to the above optimal H_∞ estimation problem does not always exist, the following suboptimal design strategy is usually considered:

$$J_\infty < \gamma^2 \tag{13}$$

Where $\gamma > 0$ is a prescribed level of disturbance attenuation.

Following the method presented in [14], there exists an H_∞ estimator $\underline{\hat{s}}(n)$ for a given $\gamma > 0$ if there exits a stabilizing symmetric positive definite solution $P_\infty(n) > 0$ to the following Riccati-type equation:

$$P_\infty(n+1) = \Phi P_\infty(n) D(n)^{-1} \Phi^T + \Gamma Q_u \Gamma^T \tag{14}$$

where:

$$D(n) = I_{Mp} - \gamma^{-2} L^T L P_\infty(n) + H^T R_b^{-1} H P_\infty(n) \tag{15}$$

This leads to the following constraint:

$$P_\infty(n) D(n)^{-1} > 0 \tag{16}$$

If the condition (16) is fulfilled, the H_∞ filter exists.

It should be noted that the level attenuation factor γ should be carefully selected [14] to satisfy the condition in (16):

$$\gamma^2 > \max\left(\text{eig}\left[L^T L \left[P_\infty(n)^{-1} + H^T R_b^{-1} H \right]^{-1} \right] \right) \tag{17}$$

where $\max(\text{eig}[F])$ is the maximum eignvalue of the matrix F.

At that stage, one can either set γ^2 to a specific constant value that is high enough to satisfy (17) or adjust it according to (17) as follows:

$$\gamma^2(n) = \zeta \max\left(\text{eig}\left[L^T L \left[P_\infty(n)^{-1} + H^T R_b^{-1} H \right]^{-1} \right] \right) \tag{18}$$

with $\zeta > 2$.

2.3 Recursive estimation of the state vector when using Kalman or H_∞ filtering

In both cases, the state vector and the *M*-AR process are estimated as follows:

$$\underline{\hat{x}}(n/n) = \Phi \underline{\hat{x}}(n-1/n-1) + K(n) \underline{\upsilon}(n) \tag{19}$$

where the so-called innovation process is given by:

$$\underline{\upsilon}(n) = \underline{y}(n) - H\Phi \underline{\hat{x}}(n-1/n-1) \tag{20}$$

However, the way the gain $K(n)$ in (19) is defined depends on the kind of filtering. Thus, when a Kalman filter is used, the gain, now noted $K_{Kal}(n)$, is given by:

$$K_{Kal}(n) = P(n/n-1) H^T [HP(n/n-1)H^T + \Sigma_b]^{-1} \tag{21}$$

where the covariance matrix is updated by using the following set of relations:

$$P(n/n-1) = \Phi P(n-1/n-1) \Phi^T + \Gamma \Sigma_u \Gamma^T \tag{22}$$

$$P(n/n) = P(n/n-1) - K_{Kal}(n) H P(n/n-1) \tag{23}$$

In addition, the covariance matrix of the innovation process $\underline{\upsilon}(n)$ satisfies:

$$C(n) = HP(n/n-1)H^T + \Sigma_b \tag{24}$$

When an H_∞ filter is used, the gain is denoted as $K_\infty(n)$ and is given by:

$$K_\infty(n) = P_\infty(n) D(n)^{-1} H^T R_b^{-1} \tag{25}$$

It should be noted that the matrix $P_\infty(n)$ can be seen as an upper bound of the error covariance matrix in the Kalman filter theory, i.e:

$$P_\infty(n) \geq P(n) = E[(\underline{x}(n) - \underline{\hat{x}}(n))(\underline{x}(n) - \underline{\hat{x}}(n))^T].$$

Remark 1:

Due to (15), the H_∞ estimator has a computational cost slightly higher than Kalman's one.

Remark 2:

If the weighting parameters Q_u, R_b and P_0 are respectively chosen to be Σ_u, Σ_b and the initial error covariance matrix of $\underline{x}(0)$ then the H_∞ filter reduces to the Kalman one when $\gamma \to \infty$.

However, $\left\{ A^{(l)} \right\}_{l=1,\ldots,p}$ must be available because they are used to define the transition matrix Φ. In addition, the covariance matrices of $\underline{u}(n)$ and $\underline{b}(n)$ or the weighting matrices Q_u, and R_b must be defined.

2.4 M-AR process estimation

To estimate the *M*-AR parameter matrices from the estimated process, equations (19) is used to express the estimated process as a function of the parameter matrices:

$$\begin{aligned} \underline{\hat{s}}(n/n) &= \Gamma^T \Phi \underline{\hat{x}}(n-1/n-1) + \Gamma^T K(n) \underline{\upsilon}(n) \\ &= -\Theta \underline{\hat{x}}(n-1/n-1) + \underline{\nu}(n) \end{aligned} \tag{26}$$

where:

$$\Theta = [A^{(1)} \quad A^{(2)} \quad \cdots \quad A^{(p)}] = \left[\begin{bmatrix} a_{11}^1 & \cdots & a_{1M}^1 \\ \vdots & \ddots & \vdots \\ a_{M1}^1 & \cdots & a_{MM}^1 \end{bmatrix} \cdots \begin{bmatrix} a_{11}^p & \cdots & a_{1M}^p \\ \vdots & \ddots & \vdots \\ a_{M1}^p & \cdots & a_{MM}^p \end{bmatrix} \right] \quad (27)$$

and $\underline{\nu}(n) = \Gamma^T K(n) \underline{\upsilon}(n)$. When a Kalman filter is used, its covariance matrix is equal to $\Sigma_\nu = \Gamma^T K_{Kal}(n) C(n) K_{Kal}(n)^T \Gamma$.

By stacking the columns of the matrix Θ^T on top of each others, the resulting $M^2 p \times 1$ state vector can be expressed as:

$$\underline{\theta}(n) = [[a_{11}^1 \quad \cdots \quad a_{1M}^1] \quad \cdots \quad [a_{11}^p \quad \cdots \quad a_{1M}^p] \quad \cdots \quad [a_{M1}^1 \quad \cdots \quad a_{MM}^1] \quad \cdots \quad [a_{M1}^p \quad \cdots \quad a_{MM}^p]]^T \quad (28)$$

Hence, equation (26) can be rewritten as follows:

$$\underline{\hat{s}}(n/n) = -\hat{X}(n-1/n-1)\underline{\theta}(n) + \underline{\nu}(n) \quad (29)$$

where $$\hat{X}(n-1/n-1) = I_M \otimes \underline{\hat{x}}(n-1/n-1)^T \quad (30)$$

with $\otimes$ denotes the matrix Kronecker product.

When the *M*-AR process is assumed stationary, the AR parameters are time-invariant and, hence, satisfy the following relationship:

$$\underline{\theta}(n) = \underline{\theta}(n-1) \quad (31)$$

Thus, equations (29) and (31) define a state space representation for the estimation of the AR parameters. A second optimal filter is then used to recursively estimate $\underline{\theta}(n)$. If a second H_∞ filter is chosen to recursively estimate $\underline{\theta}(n)$, the AR parameter estimation error is defined as $\underline{e}_\theta = -\hat{X}(n-1)\left(\underline{\theta}(n) - \underline{\hat{\theta}}(n)\right)$.

2.5. Other parameters to be estimated

An iterative estimation of the covariance matrix Σ_u can be derived from the Kalman filtering equations (22)-(23) as follows [4]:

$$\Sigma_u(n) = \lambda \Sigma_u(n-1) + (1-\lambda) F[P(n/n) - \Phi P(n-1/n-1)\Phi^T + K(n)\underline{\upsilon}(n)\underline{\upsilon}(n)^T K(n)^T] F^T \quad (32)$$

where $F = [\Gamma^T \Gamma]^{-1} \Gamma^T = [I_M \quad 0_M \quad \cdots \quad 0_M]$ and λ is the forgetting factor.

When tuning the weighting parameters Q_u, R_b and P_0 for the H_∞ filter, we suggest to recursively tune Q_u as follows:

$$Q_u(n) = \lambda Q_u(n-1) + (1-\lambda) L[P_\infty(n) - \Phi P_\infty(n-1)\Phi^T + K_\infty(n)\underline{\upsilon}(n)\underline{\upsilon}(n)^T K_\infty(n)^T] L^T \quad (33)$$

Here, the weighting matrix R_b is assigned to Σ_b. Moreover, as there is no a priori knowledge about the initial state error, the weighting matrix P_0 is assigned to the identity matrix I_{Mp}.

3. SIMULATION RESULTS

We have run several simulation tests based on various *M*-AR processes. Here, let us consider a 2nd order ($p = 2$) two-channel ($M = 2$) AR process:

$$\underline{s}(n) = -A^{(1)}\underline{s}(n-1) - A^{(2)}\underline{s}(n-2) + \underline{u}(n) \quad (34)$$

where the AR coefficient matrices are those defined by Hasan in [11]:

$$A^{(1)} = \begin{pmatrix} -0.71 & 0.32 \\ -0.88 & -0.24 \end{pmatrix} \text{ and } A^{(2)} = \begin{pmatrix} 0.57 & -0.15 \\ -0.49 & -0.30 \end{pmatrix}$$

In that case, the *M*-AR parameter matrices lead to four roots of $\det A_p(z)$, namely $p_1 = 0.941 \times e^{j1.125}$, $p_2 = 0.941 \times e^{-j1.125}$, $p_3 = 0.599$ and $p_4 = -0.461$.

In addition, $\underline{u}(n)$ is the two-channel stationary Gaussian white noise, uncorrelated between channels and with unit variance on each channel. The additive noise $\underline{b}(n)$ is also a two-channel stationary Gaussian white noise, uncorrelated with $\underline{u}(n)$.

A comparative study is carried out between [11], [12], [13], and SPKF methods (i.e. UKF and CDKF). Since UKF and CDKF provide the same results, we only present those obtained with UKF. Two simulation protocols are considered:

1. 1024 samples are available and the signal-to-noise ratio (SNR) varies from 0 to 20 dB.
2. the SNR is set to 10dB and the number of samples is equal to 64 or 128 or 256 or 512 or 1024. In every case, the results are averaged over 1000 realizations.

The criterion we considered is the mean square error (MSE) on the modulus and the argument of the roots of $\det A_p(z)$. For the sake of clarity, we only provide simulation results dedicated to p_1 and p_3. Note that the same comments can be done for p_2 and p_4, respectively.

In table 1, every method can be considered as reliable. EIV based method are of interest as they do not require the values of Σ_u and Σ_b. EKF and UKF lead to the smallest MSE. Our approaches provide quite similar results, but they have the advantage of having a smaller computational cost. Indeed, the computational cost of the EKF and SPKF methods is of the order of $O\left(\left(M^2 p + Mp\right)^3\right)$ whereas the dual filters have complexity of the order of $O\left(\left(M^2 p\right)^2\right) + O\left((Mp)^3\right)$.

When γ is fixed and high enough to always satisfy (17), cross-coupled Kalman filter and cross-coupled H_∞ filters provide similar results. When selecting the noise attenuation level γ automatically according to equ. (18), the resulting MSE becomes higher. Therefore, the main difficulty when using H_∞ filters is the selection of the attenuation level γ.

When the SNR becomes lower, Hasan's method [11] and ILSV [12] are not necessary reliable and may provide parameter estimates that lead to unstable system.

Methods	MSE on the estimated modulus of p_1	MSE of the estimated modulus of p_3
Hasan [11]	$3{,}45 \times 10^{-3}$	$16{,}4 \times 10^{-3}$
ILSV [12]	$20{,}0 \times 10^{-5}$	$24{,}2 \times 10^{-3}$
EKF with true values of Σ_u and Σ_b [3], [13]	$7{,}0 \times 10^{-5}$	4×10^{-3}
UKF with true values of Σ_u and Σ_b [3], [13]	$7{,}0 \times 10^{-5}$	$4{,}6 \times 10^{-3}$
EIV (method 1 labelled SR [13])	$11{,}7 \times 10^{-5}$	$9{,}6 \times 10^{-3}$
EIV (method 2 labelled HOYW [13])	$10{,}5 \times 10^{-5}$	$9{,}6 \times 10^{-3}$
Cross-coupled Kalman filtering (see section 2)	$8{,}0 \times 10^{-5}$	$11{,}8 \times 10^{-3}$
Cross-coupled H_∞ filtering (fixed value for γ)	$8{,}0 \times 10^{-5}$	$11{,}8 \times 10^{-3}$
Cross-coupled H_∞ filtering (varying value for γ)	$2{,}5 \times 10^{-3}$	$34{,}4 \times 10^{-3}$

Table 1: MSE of the modulus of some roots of $\det \hat{A}_p(z)$. SNR=10 dB, 1024 samples, and 1000 realizations.

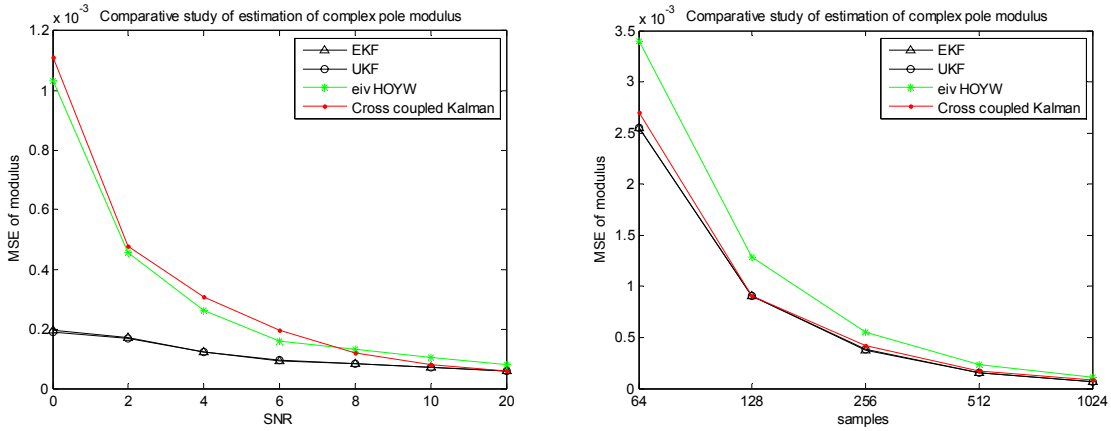

Figure 2: MSE of the modulus of p_1. Left: protocol #1 ; right: protocol #2.

According to figure 2, EKF and UKF provide similar results. Our cross-coupled structure based on Kalman filtering still provides good performances in terms of MSE. Although the Gaussian assumptions are required unlike H_∞ filters, this method can be very useful. Therefore, we would rather suggest using the cross-coupled Kalman filtering for the on-line estimation of the *M*-AR parameters.

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