SNR OPTIMIZED RESIDUAL FREQUENCY OFFSET COMPENSATION FOR WIMAX WITH THROUGHPUT EVALUATION

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ABSTRACT

WiMAX with an OFDM physical layer employed is sensitive to carrier frequency offset. Even though most of this offset can be compensated with the initial training sequence, there still remains a residual frequency offset due to estimation errors. In this paper, we investigate pilot-based and data-aided residual frequency offset estimators and apply them to WiMAX. To improve the pilot-based method, we propose two compensation schemes which exploit the a-priori knowledge from the previous frame. For the data-aided method with general QAM schemes, the optimal weighting factors that maximize the SNR after combining are analytically derived. Throughput results show that most of the degradation due to the residual frequency offset can be compensated by our proposed low-complexity suboptimal methods.

1. INTRODUCTION

Since Orthogonal Frequency Division Multiplexing (OFDM) is well suited for bandwidth efficient data transmissions, it has been included in the physical layer of the WiMAX standard IEEE 802.16-2004 [1]. A potential drawback of OFDM, however, is its sensitivity to Carrier Frequency Offset (CFO).

Numerous papers dealing with carrier frequency synchronization in OFDM can be found (e.g. [2–5]). Some of the techniques have been applied to wireless LAN with multiple antennas [6, 7]. The basic idea is to split the CFO into the Fractional Frequency Offset (FFO), the Integer Frequency Offset (IFO) and the Residual Frequency Offset (RFO). The FFO and the IFO are estimated using a training sequence which has a specific structure [2–4]. To estimate the RFO, pilot-based and decision directed methods have been developed [4, 5]. To combine the estimated RFOs on different subcarriers and receive antennas, equal weighting factors were applied [6, 7] for PSK modulated signals. To the authors' knowledge, it has not been proven that an equal weights combiner gives optimal SNR, especially for modulation schemes other than PSK. Also, performance is usually expressed in terms of mean square error and bit-error-ratio comparisons but not in terms of coded physical layer throughput.

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In this work, we investigate pilot-based and decision directed RFO estimation schemes and apply them to WiMAX. In Section 3.1.3, two novel pilot based estimator structures that take the estimation results of the previous frame into account are introduced. In Section 3.2, we propose the combining factors that maximize SNR for the decision directed method. In Section 4, performance is expressed in terms of physical layer throughput due to synchronization errors. Unlike usual comparisons in terms of BER and MSE, such an evaluation is of more significance for a frame-based transmission system.

2. SYSTEM MODEL

In this section, we define the system model that is used in Section 3 to derive estimators for the RFO.

In an OFDM system, the CFO Δf_{CFO} is normalized to the subcarrier spacing f_{s} and denoted by $\varepsilon_{\text{CFO}} = \frac{\Delta f_{\text{CFO}}}{f_{\text{s}}}$. We denote the OFDM symbol index within one frame by l, the receive antenna index by m and the time index within one OFDM symbol by $n \in [(l-1)N + 1, lN + N_{\text{g}}]$, where N is the FFT size and N_{g} is the length of the Cyclic Prefix (CP). The received signal is referred to as $r_{l,n}^{(m)}$, the transmitted signal as $x_{l,n}^{(m)}$, the channel impulse response as $h_{l,n}^{(m)}$ and the additive Gaussian noise as $v_{l,n}^{(m)}$. We assume that the receiver as well as the transmitter are run by central oscillators, leading to an identical CFO at each antenna. Thus, the transmission with CFO can be described as convolution in the time domain as

$$r_{l,n}^{(m)} = \left\{ x_{l,n}^{(m)} * h_{l,n}^{(m)} + v_{l,n}^{(m)} \right\} \cdot e^{\frac{j2\pi\varepsilon_{\text{CFO}} n}{N}}. \quad (1)$$

In WiMAX, the Channel Impulse Response (CIR) is assumed to be quasi-static within one frame. Nevertheless, when the CFO is considered, the CIR becomes time variant. For the OFDM symbol l and $l+1$,

$$h_{l+1,n}^{(m)} = h_{l,n+N+N_{\text{g}}}^{(m)} = h_{l,n}^{(m)} e^{j2\pi\varepsilon_{\text{CFO}} \frac{N+N_{\text{g}}}{N}} \quad (2)$$

holds true. When only RFO (typically in the order of 10^{-3}) is considered and the CP is removed correctly, Eq.(2) can be expressed in the frequency domain as[1]

$$H_{l+1,k}^{(m)} = H_{l,k}^{(m)} \cdot e^{j2\pi\varepsilon_{\text{RFO}} \frac{N+N_{\text{g}}}{N}} \quad (3)$$

[1] According to [2], the amplitude reduction and phase shift as well as the inter-carrier interference due to the frequency offset are small enough to be ignored.

due to the linearity of the Fourier transform. Here, $H_{l,k}^{(m)}$ is the channel frequency response at the l-th OFDM symbol, the k-th subcarrier and the m-th receive antenna. To simplify the notation in the following, we define

$$\tilde{\varepsilon}_{\mathrm{RFO}} = \frac{N + N_{\mathrm{g}}}{N} \varepsilon_{\mathrm{RFO}}. \tag{4}$$

3. RESIDUAL FREQUENCY OFFSET COMPENSATION

A conventional method of RFO estimation can be found in [4]. The idea is to derive the phase variation $\exp\{j2\pi\tilde{\varepsilon}_{\mathrm{RFO}}\}$ in two consecutive OFDM symbols by using

$$\begin{aligned} W_{l,k}^{(m)} &= R_{l-1,k}^{(m)} R_{l,k}^{(m)*} (X_{l-1,k}^{(m)} X_{l,k}^{(m)*})^* \\ &= (H_{l-1,k}^{(m)} X_{l-1,k}^{(m)} + V_{l-1,k}^{(m)}) \\ &\quad \cdot (H_{l-1,k}^{(m)} X_{l,k}^{(m)} e^{j2\pi\tilde{\varepsilon}_{\mathrm{RFO}}} + V_{l,k}^{(m)})^* (X_{l-1,k}^{(m)} X_{l,k}^{(m)*})^* \\ &= |H_{l-1,k}^{(m)}|^2 |X_{l-1,k}^{(m)}|^2 |X_{l,k}^{(m)}|^2 e^{-j2\pi\tilde{\varepsilon}_{\mathrm{RFO}}} + \tilde{V}_{l,k}^{(m)}, \end{aligned} \tag{5}$$

where $R_{l,k}^{(m)}$ denotes the received symbol, $X_{l,k}^{(m)}$ the transmitted symbol and $V_{l,k}^{(m)}$ the noise term in the k-th subcarrier of the l-th OFDM symbol and the m-th receive antenna. All additional noise terms (see Eq. (18)in Appendix. A) are contained in $\tilde{V}_{l,k}^{(m)}$.

For a WiMAX system with N_{R} receive antennas, N_{f} OFDM symbols per frame and N_{p} subcarriers used for estimation, each frame results in $N_{\mathrm{R}} \times (N_{\mathrm{f}} - 1) \times N_{\mathrm{p}}$ values of $W_{l,k}^{(m)}$ according to Eq. (5).

Considering that all subcarriers on all receive antennas experience the same CFO and that the output frequency of an oscillator does not change abruptly in time, the estimator can be improved by combining the results $W_{l,k}^{(m)}$ over l, k and m.

In the following, we first focus on pilot-based estimators and perform combining over all pilot symbols. Then, we will additionally make use of the data subcarriers in the estimation to further improve the results.

3.1 Pilot-based Approaches

For BPSK-modulated pilot symbols like in WiMAX [1], it is proved in Appendix A that equal weight combining yields a maximized SNR. Therefore, we apply weight one equally to all the pilot tones from all the receive antennas. The combined value W_l for the OFDM symbol l becomes

$$W_l = \sum_{m=1}^{N_{\mathrm{R}}} \sum_{k \in \mathcal{N}_{\mathrm{p}}} W_{l,k}^{(m)}, \quad l = 2, \cdots, N_{\mathrm{f}} \tag{6}$$

where $\mathcal{N}_{\mathrm{p}}$ denotes the subset of the pilot subcarrier indices. In the following, combining in time is carried out in four different approaches.

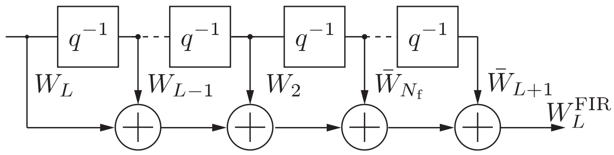

Figure 1: Sliding window averaging

3.1.1 Frame-wise Approach

In the frame-wise approach, estimation is carried out by averaging over all N_{f} OFDM symbols in the current frame. This yields the estimated RFO

$$\hat{\varepsilon}_{\mathrm{RFO,Frame}} = -\frac{1}{2\pi} \frac{N}{N + N_{\mathrm{g}}} \arg\left\{ \sum_{l=2}^{N_{\mathrm{f}}} W_l \right\}. \tag{7}$$

From the practical point or view, this approach has the drawback that the complete data frame has to be buffered until the first RFO estimate is obtained.

3.1.2 Symbol-wise Approach without Pre-knowledge

In order to produce an instantaneous estimate at each OFDM symbol, an alternative is to perform combining only over the first L received OFDM symbols in the current frame. In this way, the estimated RFO at the L-th OFDM symbol in the current frame is given by

$$\hat{\varepsilon}_{\mathrm{RFO},L} = -\frac{1}{2\pi} \frac{N}{N + N_{\mathrm{g}}} \arg\left\{ \sum_{l=2}^{L} W_l \right\}. \tag{8}$$

The estimation window is initialized at the beginning of each frame and then grows during the transmission. Starting from a "zero" phase at the beginning of each frame, every time a new OFDM symbol is received, the estimation result is updated and improved.

3.1.3 Symbol-wise Approaches with Pre-knowledge

In order to avoid the "zero start" phase, we use the estimation results from the previous frame as pre-knowledge for the current frame. Two methods of reasonable complexity and memory cost are proposed.

Method I: *Sliding Window Averaging*

To estimate the RFO at the L-th symbol, we utilize L symbols of the current frame and $N_{\mathrm{f}} - L$ symbols of the previous frame. Since the two frames have different FFO and IFO estimates, the a-priori RFO estimate of the previous frame has to be adjusted for the current frame, which is given by

$$\begin{aligned} \varepsilon_{\mathrm{RFO},l}^{\mathrm{adjust}} = \hat{\varepsilon}_{\mathrm{RFO},l}^{\mathrm{previous}} + \hat{\varepsilon}_{\mathrm{FFO}}^{\mathrm{previous}} + \hat{\varepsilon}_{\mathrm{IFO}}^{\mathrm{previous}} \\ - \hat{\varepsilon}_{\mathrm{FFO}}^{\mathrm{current}} - \hat{\varepsilon}_{\mathrm{IFO}}^{\mathrm{current}}. \end{aligned} \tag{9}$$

Correspondingly, the adjusted combined value $\bar{W}_l$ can be written as

$$\bar{W}_l = |W_l^{\mathrm{previous}}| \cdot \exp\left\{ -j2\pi\varepsilon_{\mathrm{RFO},l}^{\mathrm{adjust}} \cdot \frac{N + N_{\mathrm{g}}}{N} \right\}. \tag{10}$$

The FIR filter structure in Fig. 1 is designed for sliding window averaging, where $N_{\mathrm{f}} - 1$ values of W_l are taken

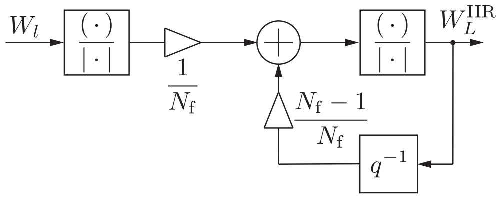

Figure 2: Forgetting factor averaging

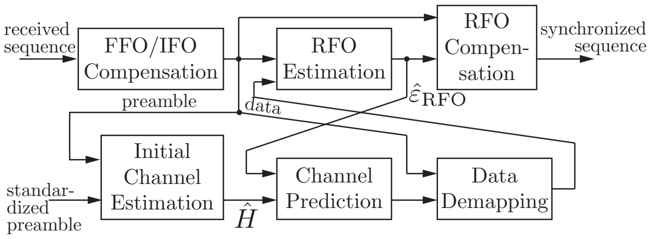

Figure 3: Data-aided residual frequency offset estimation

either from the previous frame or from the current. The RFO at the L-th OFDM symbol in the current frame is derived by

$$\begin{aligned}\hat{\varepsilon}_{\mathrm{FIR},L} &= -\frac{1}{2\pi}\frac{N}{N+N_\mathrm{g}}\arg\left\{W_L^{\mathrm{FIR}}\right\}\\ &= -\frac{1}{2\pi}\frac{N}{N+N_\mathrm{g}}\arg\left\{\sum_{l=2}^{L} W_l + \sum_{l=L+1}^{N_\mathrm{f}} \bar{W}_l\right\}. \quad (11)\end{aligned}$$

Method II: *Forgetting Factor Averaging*

An averaging with a forgetting factor can be implemented using an IIR filter. The following initial RFO is assumed

$$\begin{aligned}\hat{\varepsilon}_{\mathrm{RFO},1}^{\mathrm{initial}} = \hat{\varepsilon}_{\mathrm{RFO},N_\mathrm{f}}^{\mathrm{previous}} &+ \hat{\varepsilon}_{\mathrm{FFO}}^{\mathrm{previous}} + \hat{\varepsilon}_{\mathrm{IFO}}^{\mathrm{previous}}\\ &- \hat{\varepsilon}_{\mathrm{FFO}}^{\mathrm{current}} - \hat{\varepsilon}_{\mathrm{IFO}}^{\mathrm{current}}. \quad (12)\end{aligned}$$

The corresponding initial W_1 at the first OFDM symbol in the current frame is expressed as

$$W_1 = \exp\left\{-j2\pi\varepsilon_{\mathrm{RFO},1}^{\mathrm{initial}} \cdot \frac{N+N_\mathrm{g}}{N}\right\}. \quad (13)$$

At each OFDM symbol, a newly generated value of W_l goes into an IIR filter as shown in Fig. 2. The new value is weighted by $\frac{1}{N_\mathrm{f}}$ and the stored one by $\frac{N_\mathrm{f}-1}{N_\mathrm{f}}$. The RFO for the L-th OFDM symbol in the current frame is derived as

$$\hat{\varepsilon}_{\mathrm{IIR},L} = -\frac{1}{2\pi}\frac{N}{N+N_\mathrm{g}}\arg\left\{W_L^{\mathrm{IIR}}\right\}. \quad (14)$$

3.2 Data-aided Approach

In this section, estimation using pilot and data subcarriers is performed. Combining factors that result in the optimum SNR for Quadrature Amplitude Modulation (QAM) are proposed.

A data-aided scheme is shown in Fig. 3. In the upper branch, the *RFO Estimation* block evaluates Eq. (5) on non-zero subcarriers. The RFO at the OFDM symbol

Parameter	Value
Number of RX antennas	1,2,4,8
Number of TX antennas	1
Channel model	ITU Pedestrian B [8]
Number of channel realizations	500
Channel coding	RS-CC
Channel estimation	Least squares
Demapper	max-log-MAP

Table 1: Simulation Parameters

L in the current frame is derived by

$$\hat{\varepsilon}_{\mathrm{DA},L} = -\frac{1}{2\pi}\frac{N}{N+N_\mathrm{g}}\arg\left\{\sum_{l=2}^{L}\sum_{m=1}^{N_\mathrm{R}}\sum_{k\in\mathcal{N}} g_{l,k}\cdot W_{l,k}^{(m)}\right\}. \quad (15)$$

The pilot and data subcarrier indices required for the combining are contained in $\mathcal{N}$ and the optimal weighting factors $g_{l,k}$ are given by

$$g_{l,k} = \frac{1}{|\hat{X}_{l-1,k}|^2 + |\hat{X}_{l,k}|^2}, \quad (16)$$

where $\hat{X}_{l,k}$ is the demapped data symbol. In Appendix A, it is proved that these factors maximize the SNR in general, regardless of the symbol alphabet employed.

In order to demap the data symbols $\hat{X}_{l-1,k}^{(m)}$, an initial channel estimation is required at the beginning of each frame. According to Eq. (3), the channel frequency response in the L-th OFDM symbol in one frame can be predicted by

$$\hat{H}_{L,k}^{(m)} = \hat{H}_{L-1,k}^{(m)}\exp\left\{j2\pi\hat{\varepsilon}_{\mathrm{DA},L-1}\cdot\frac{N+N_\mathrm{g}}{N}\right\}. \quad (17)$$

Using this predicted channel frequency response, the data symbols are hard demapped and fed into the *RFO Estimation* block.

4. SIMULATION RESULTS

The simulation is carried out in a Matlab implementation [9][2] of the IEEE 802.16-2004 WiMAX standard [1].

To evaluate the performance of RFO compensation schemes, we introduce a constant normalized CFO of $\pi \approx 3.1416$. All RFO compensation schemes described in Section 3 are implemented. The fractional part is corrected using the method described in [4]. The integer part is corrected perfectly. Therefore, the remaining RFO only depends on the estimation error of the fractional part. The symbol timing is perfectly aligned. More simulation parameters are listed in Table 1. For each channel realization, at each SNR, seven Adaptive Modulation and Coding (AMC) schemes are transmitted. When calculating the throughput, only the number of bits in correctly received frames is counted. The AMC feedback is assumed to be optimal, that is, the AMC scheme that achieves the largest throughput at a

[2] freely available at `http://www.nt.tuwien.ac.at/wimaxsimulator`. In the downloadable version, the carrier frequency is perfectly synchronized.

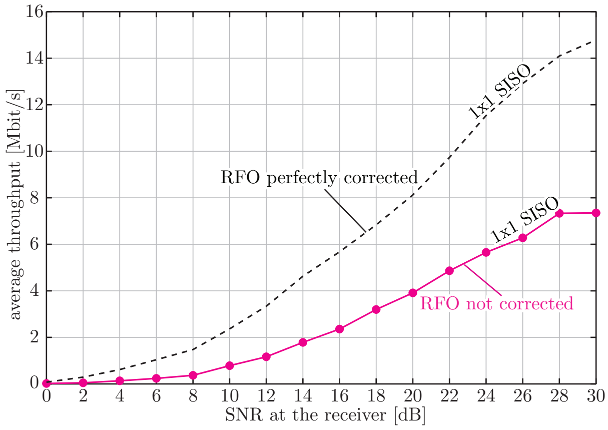

Figure 4: Influence of the RFO

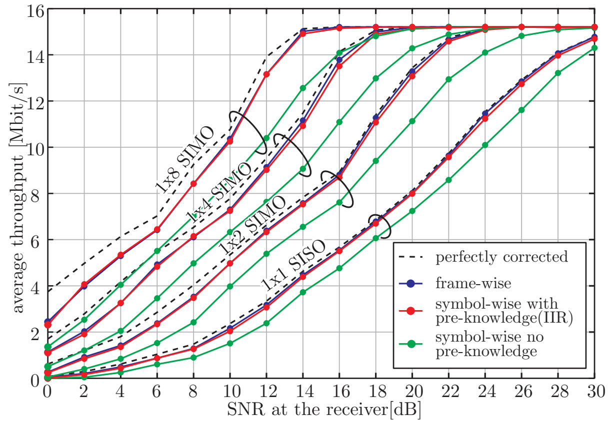

Figure 6: Pilot-based approaches

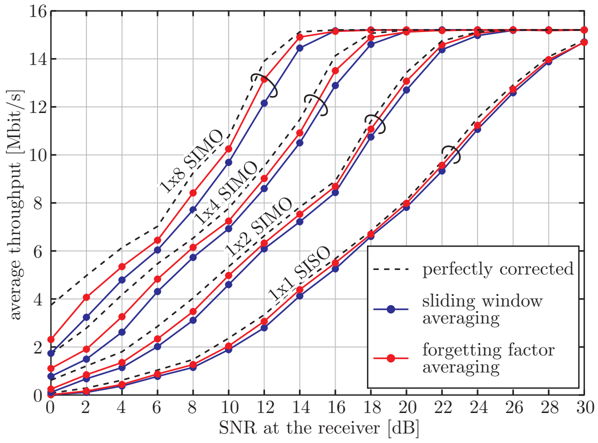

Figure 5: Symbol-wise approaches with pre-knowledge

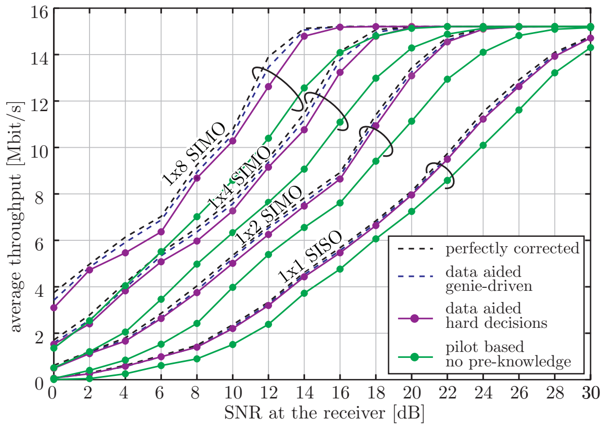

Figure 7: Data-aided approach

specific channel realization at a specific SNR is selected.

As a reference, we plot the throughput curve when the RFO is not corrected. Fig. 4 shows for the Single-Input-Single-Output (SISO) case that the throughput loss is around 50%.

For the two symbol-wise approaches with pre-knowledge described in Section 3.1.3, a comparison is shown in Fig. 5. Compared to those of the sliding window averaging scheme, the curves of the forgetting factor averaging scheme are closer to the perfect case . Therefore, in the later evaluation, only the forgetting factor averaging scheme is considered.

The throughputs of the pilot-based schemes are displayed in Fig. 6. The frame-wise approach always shows the best performance, especially in the high SNR region. Compared to the symbol-wise approach without pre-knowledge, the a-priori estimates provide considerable gain in overall throughput. Typically, at the 12 Mbit/s throughput level, there is approximately 2 dB gain for the Single-Input-Multiple-Output (SIMO) cases. However, in the low SNR region, compared to the ideal case, the loss of all three methods becomes larger with increasing number of receive antennas .

The throughput curve for the data-aided compensation scheme described in Section 3.2 is shown in Fig. 7. As a reference, the throughput curves of a genie-driven estimator, which assumes correct demapping of all data symbols, are provided. The additional data subcarriers give approximately 2 dB gain compared to the pilot-based method for all SNR levels. Although the throughput also degrades with increasing number of receive antennas, the loss compared to the perfect correction is smaller than for the pilot-based methods.

5. CONCLUSION

In this work, we investigate pilot-based and data-aided RFO estimation techniques for WiMAX. Combining factors that maximize SNR for QAM constellations are derived. Simulation results show that by either taking pre-knowledge or involving demapped data into the RFO estimation, the throughput loss due to the RFO can be almost fully compensated. However, as the number of receive antennas increases, all schemes show degradation in performance.

A. PROOF OF COMBINING WITH MAXIMIZED SIGNAL-TO-NOISE RATIO

In this section, the optimum combining factors $g_{l,k}$ that maximize the SNR are derived. The proof is given with respect to the frequency dimension, but can be extended straightforwardly to the time and the space dimension.

We rewrite Eq. (5) and extend the noise terms:

$$\begin{aligned}
W_{l,k}^{(m)} &= R_{l-1,k}^{(m)} R_{l,k}^{(m)*} X_{l-1,k}^{(m)*} X_{l,k}^{(m)} \\
&= \left(X_{l-1,k}^{(m)} H_{l-1,k}^{(m)} + V_{l-1,k}^{(m)}\right) \\
&\qquad \cdot \left(X_{l,k}^{(m)} H_{l,k}^{(m)} + V_{l,k}^{(m)}\right)^{*} X_{l-1,k}^{(m)*} X_{l,k}^{(m)} \\
&= \left(X_{l-1,k}^{(m)} H_{l-1,k}^{(m)} + V_{l-1,k}^{(m)}\right) \\
&\qquad \cdot \left(X_{l,k}^{(m)} H_{l-1,k}^{(m)} e^{j2\pi\tilde{\varepsilon}} + V_{l,k}^{(m)}\right)^{*} X_{l-1,k}^{(m)*} X_{l,k}^{(m)} \\
&= |X_{l-1,k}^{(m)}|^2 \cdot |X_{l,k}^{(m)}|^2 \cdot |H_{l-1,k}^{(m)}|^2 \cdot e^{-j2\pi\tilde{\varepsilon}} \\
&\quad + |X_{l-1,k}^{(m)}|^2 \cdot X_{l,k}^{(m)} H_{l-1,k}^{(m)} V_{l,k}^{(m)*} \\
&\quad + |X_{l,k}^{(m)}|^2 \cdot X_{l-1,k}^{(m)*} H_{l-1,k}^{(m)*} V_{l-1,k}^{(m)} \cdot e^{-j2\pi\tilde{\varepsilon}} \\
&\quad + V_{l-1,k}^{(m)} V_{l,k}^{(m)*} X_{l-1,k}^{(m)*} X_{l,k}^{(m)}. \qquad (18)
\end{aligned}$$

Again, the received symbol is refereed to as $R_{l,k}^{(m)}$, the transmitted symbol as $X_{l,k}^{(m)}$ and the noise term as $V_{l,k}^{(m)}$. In the following proof, we assume an SNR that is large enough to allow for neglecting the quadratic noise term in Eq. (18). Also, the antenna index (m) is left out for simplicity. We denote the signal term by $W_{l,k}^{\mathrm{S}}$ and the noise terms by $W_{l,k}^{\mathrm{N}}$ which can be identified from Eq. (18) as

$$\begin{aligned}
W_{l,k}^{\mathrm{S}} &= |X_{l-1,k}|^2 \cdot |X_{l,k}|^2 \cdot |H_{l-1,k}|^2 \cdot e^{-j2\pi\tilde{\varepsilon}}, \qquad (19) \\
W_{l,k}^{\mathrm{N}} &= |X_{l-1,k}|^2 \cdot X_{l,k} H_{l-1,k} V_{l,k}^{*} \qquad (20) \\
&\quad + |X_{l,k}|^2 \cdot X_{l-1,k}^{*} H_{l-1,k}^{*} V_{l-1,k} \cdot e^{-j2\pi\tilde{\varepsilon}}.
\end{aligned}$$

Thus, the combining process can be expressed as

$$U_l = \sum_k g_{l,k} W_{l,k} = \sum_k g_{l,k} W_{l,k}^{\mathrm{S}} + \sum_k g_{l,k} W_{l,k}^{\mathrm{N}}. \qquad (21)$$

We assume additive Gaussian noise $\mathcal{CN} \sim (0, \sigma_v^2)$, $\sigma_v^2 = E\{|V_{l,k}|^2\}$. The signal energy S and the noise energy N after the combiner can be written as

$$\begin{aligned}
S &= \left|\sum_k g_{l,k} W_{l,k}^{\mathrm{S}}\right|^2 \\
&= \left|\sum_k g_{l,k} \cdot |X_{l-1,k}|^2 |X_{l,k}|^2 |H_{l-1,k}|^2\right|^2, \qquad (22) \\
N &= E_v\left\{\left|\sum_k g_{l,k} W_{l,k}^{\mathrm{N}}\right|^2\right\} \\
&= \sigma_v^2 \sum_k g_{l,k}^2 \cdot |H_{l-1,k}|^2 \cdot |X_{l-1,k}|^2 \cdot |X_{l,k}|^2 \\
&\qquad \cdot (|X_{l-1,k}|^2 + |X_{l,k}|^2). \qquad (23)
\end{aligned}$$

Furthermore, by defining

$$\begin{aligned}
p_{l,k} &= g_{l,k} \cdot |H_{l-1,k}| \cdot |X_{l-1,k}| \cdot |X_{l,k}| \\
&\qquad \cdot \sqrt{|X_{l-1,k}|^2 + |X_{l,k}|^2}, \qquad (24) \\
q_{l,k} &= |H_{l-1,k}| \cdot |X_{l-1,k}| \cdot |X_{l,k}| \\
&\qquad \cdot \frac{1}{\sqrt{|X_{l-1,k}|^2 + |X_{l,k}|^2}}, \qquad (25)
\end{aligned}$$

the SNR can be maximized by applying the Cauchy-Schwarz inequality:

$$\begin{aligned}
\mathrm{SNR}_l^{(m)} &= \frac{S}{N} = \frac{\left(\sum_k p_{l,k} \cdot q_{l,k}\right)^2}{\sigma_v^2 \cdot \sum_k p_{l,k}^2} \le \frac{\sum_k p_{l,k}^2 \cdot \sum_k q_{l,k}^2}{\sigma_v^2 \sum_k p_{l,k}^2} \\
&= \frac{1}{\sigma_v^2} \sum_k q_{l,k}^2 \qquad (26)
\end{aligned}$$

The equality is fulfilled *iff*

$$p_{l,k} = \alpha q_{l,k}. \qquad (27)$$

This leads to the solution that

$$g_{\mathrm{opt}\ l,k} = \frac{\alpha}{|X_{l-1,k}|^2 + |X_{l,k}|^2}, \qquad (28)$$

where α is an arbitrary scalar.

Therefore, it is proved that by applying the combining factors $g_{l,k}$ to the k-th subcarrier, the maximum SNR after the combiner can be achieved in the l-th OFDM symbol for each receive antenna.

Specifically, when the transmit signal is Phase-Shift Keying (PSK) modulated, where

$$|X_{l,k}| = constant, \quad \text{for arbitrary}\, l, k \qquad (29)$$

holds, equal weights lead to the maximized SNR.

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