THE PERCENTAGE OCCUPANCY HIT OR MISS TRANSFORM

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ABSTRACT

The Hit or Miss Transform (HMT) is a well known morphological technique which can be used for shape and object recognition in digital image processing. The standard HMT is a particularly powerful tool for locating objects which are noise free in both the background and foreground regions, do not exhibit internal texture and where objects have well defined edges. Often for various reasons, objects of interest do not exhibit such qualities and as a result may not be detected by the standard HMT. This paper proposes a percentage occupancy based Hit or Miss Transform for the detection of objects subject to noisy edges, internal texture, holes and non homogeneous intensity.

1. INTRODUCTION

Mathematical Morphology first introduced by Matheron [1] and Serra [2] provides an extremely powerful set of tools for various applications in image processing. Among these is the Hit or Miss Transform (HMT) [2] which is capable of identifying pixels which have certain geometric properties [3]. The HMT can be used to perform various operations including thinning and thickening, however, for this paper, only shape recognition is considered.

For the binary case, the HMT is capable of recognising shapes or objects in an image using a complementary pair of structuring elements (SEs) which search for the shape and its complement in an image and return a marker in the case of successful detection. The HMT has subsequently been generalised such that it can be applied to greyscale images [4].

The HMT is suitable for object recognition in both binary and greyscale images which are not corrupted by noise. The HMT fails in the presence of noise since signals which should be detected do not precisely match the geometry of the templates used to probe the image for those shapes.

Various attempts have been made to generalise the HMT such that it is capable of detecting objects even in the presence of noise. Raducanu and Graña [5] propose a technique based on decomposing a greyscale image into level sets which are considered as binary images. A HMT is then applied to each binary image and the reconstruction of the resultant images gives the greyscale HMT. Zhao and Daut [6] present a technique which uses upper and lower bounds to determine the SEs for use in the HMT using a priori knowledge of the shapes to be detected. This technique uses the skeletons of both the object to be recognised and its complement as SEs. While this technique allows variants of the shape to be detected in the presence of noise, it is prone to producing erroneous hits.

This paper presents a Percentage Occupancy Hit or Miss Transform (POHMT), which is a generalisation of the HMT such that it can detect desired shapes in the presence of noise. Unlike the HMT which requires that one template fits entirely inside the object to be detected and another entirely outside of it, the POHMT requires that only a predetermined area of both SEs need be occupied. This ensures that even in the presence of noise the desired features may still be extracted from the noisy image.

2. THE PERCENTAGE OCCUPANCY HIT OR MISS TRANSFORM

The HMT for shape recognition in binary images is defined as the intersection of two erosions; see [2], [3], [7] and [8]

$$A \otimes B = \left(A \Theta B_1\right) \cap \left(A^c \Theta B_2\right) \tag{1}$$

where A is a binary image and B denotes the structuring elements B_1 and B_2, which satisfy the condition

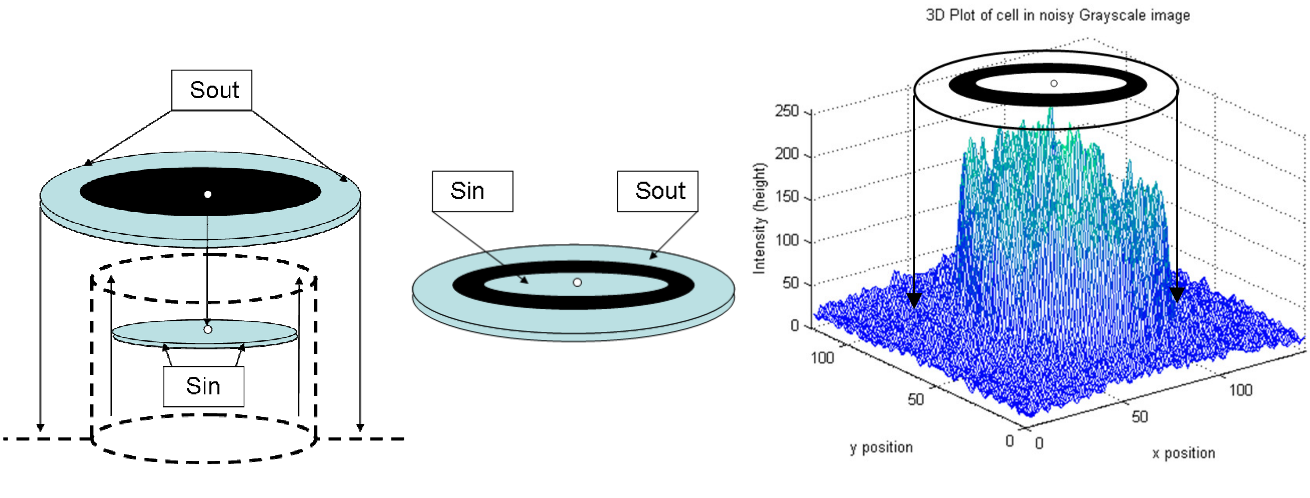
$$B_1 \cap B_2 = \phi .$$

The HMT can be extended and applied to greyscale images by substituting the binary sets in (1) for functions and the two binary erosions for two greyscale erosions of image *A* using flat structuring elements b_1 and b_2. The greyscale erosion of image *f* using a flat structuring element *b* becomes

$$\left[f \;\Theta\; b\right](x, y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}. \tag{2}$$

The erosion of *f* by *b* at any location (x,y) in the image is defined as the minimum value of the image in the region coincident with *b* when the origin of *b* is at position (x,y) [8]. From (1) and (2) the greyscale HMT of image *f* by flat structuring elements b_1 and b_2 can be defined as

$$A \otimes B = \min_{(s_1,t_1)\in b_1}\{f(x+s_1, y+t_1)\} \;+ \min_{(s_2,t_2)\in b_2}\{-f(x+s_2, y+t_2)\}.$$



(a) (b) (c)

Figure 1 - Illustration of the operation of the HMT and the POHMT (a) Inner and outer SEs being used to detect a desired shape (b) The required intersection of both SEs needed to detect an object of interest (c) Profile of a noisy image containing an object of interest and the SEs probing this image.

Objects in f which match the geometry of both structuring elements can be marked at locations where $A \otimes B > 2^n - 1$ for an n bit per pixel greyscale image. This ensures that only locations in the image where both the object and its complement are found will be considered a hit.

An alternative description of the HMT is that both structuring elements probe the image for places which exactly match their geometry. The structuring element used for the first erosion returns a hit when it fits entirely inside an object in the image from underneath. The second erosion returns a hit when it fits entirely around an object in the image from above. Adding the two images resulting from both erosions results in "hits" at places in the image having an intensity value greater than 255 (using 8 bits per pixel, greyscale images).

Fundamentally, a "hit" i.e. an object which is detected and marked by the HMT, is one which satisfies the conditions stated above and hence the reference points of the inner and outer SEs must intersect each other at some point within the image. This description is illustrated in Figure 1 where the precise meaning of the term intersect is described.

Figure 1a shows a pair of SEs which could be used to detect a circular object using the standard HMT. S_{in} is a solid disk and S_{out} is a solid ring. The dotted line in the Figure represents a noise free shape which is to be detected. Clearly, in this case, the inner SE; S_{in} fits entirely inside the shape and the outer SE; S_{out} fits entirely around the shape. Ultimately, for the HMT to detect this shape, the reference points in the SEs (marked by white dots in the figure) must intersect. That is, the reference points of both S_{in} and S_{out} must reach at least one common point in the height of the object to be detected. For the case shown in Figure 1a, it is possible for the reference points of both SEs to intersect at all points in the height of the shape since neither SE is restricted by noise. This shape and any shape which is larger than S_{in} and smaller than S_{out} will be detected by the HMT when using these SEs.

In the case that the shape, its edges, or both are corrupted by noise, the inner SE may be prevented from fitting entirely inside a shape by the noise, or the outer SE prevented from fitting entirely outside. These undesired effects of the noise prevent the reference points of both SEs intersecting and as a result the HMT cannot be used to detect shapes which are corrupted by noise. For the same reasons objects which have internal texture similar to noise cannot be detected by the standard HMT.

In [6] a technique using the skeleton of the desired object as the inner SE and the skeleton of the complement of the object as the outer SE is proposed. Although this technique is capable of detecting desired objects in the presence of noise, the geometric difference between the two SEs can lead to many erroneous hits. These occur in images which contain artefacts the dimensions of which lie within the lower and upper bounds of the SEs. If the skeletons of the object and its complement are used as SEs, and the skeleton of the inner SE is a single pixel or a small group of pixels, then even noise in the image could be mistaken for an object. This technique is also difficult to automate since a priori knowledge of the desired shape is always required. The same can be said for simply reducing the size of the inner SE and enlarging the area of the outer SE. If these parameters are set such that the outer SE is much larger than the inner, this would lead to undesired objects or noise being detected by the HMT.

To this point, both the inner and outer structuring elements have been considered to be solid objects as shown in Figure 1a. For the desired object to be detected, the inner SE, S_{in} must remain entirely beneath the desired signal and the outer SE, S_{out} completely outside it. If therefore there is one noisy pixel or more in the shape to be recognised, the standard HMT, by this strict definition will fail. Similarly, if there is at least one noise spike at any point around the edge of the desired object, where the outer SE erodes the pixels,

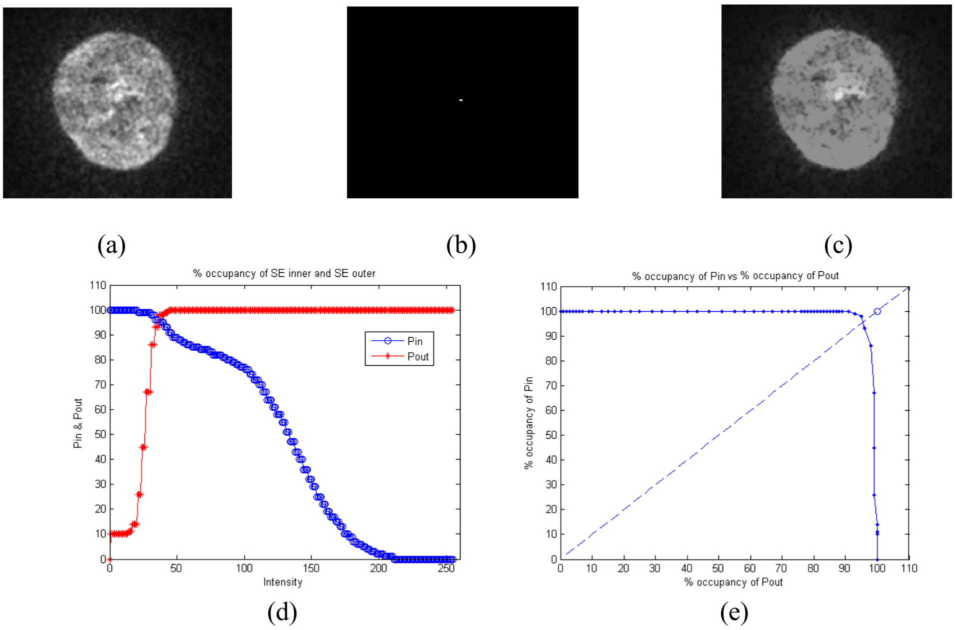


Figure 2 - Noisy object being detected by the POHMT and plots of the data used to set P_{in} and P_{out}. (a) Noisy image of cancer cell. (b) Marker for the cancer cell produced by the POHMT (c) Reconstructed cancer cell using marker (d) PO of SEs vs intensity for noisy image. (e) P_{in} vs P_{out} for noisy image and the point showing noise free case.

the standard HMT will fail.

This paper proposes a percentage occupancy based Hit or Miss Transform (POHMT) which allows a predefined portion of the inner SE, the outer SE, or both to be "punctured" by noise or texture in the signal. Figure 1b illustrates the intersection of the reference points of the SEs S_{in} and S_{out}. For the POHMT, S_{in} and S_{out} can be considered as an intersected pair of SEs which scans the image looking for places where both S_{in} and S_{out} are occupied by a predefined percentage which is set using a priori knowledge of the shape to be detected and an estimate of the noise or texture in the image. This technique is illustrated in Figure 1c which shows the profile of a noisy object and the intersected pair of SEs scanning the image for a height in the noisy object which occupies a particular percentage of both SEs simultaneously. If the percentage occupancy (PO) parameters are set correctly, the noisy object will be detected and marked by the POHMT.

The POHMT allows the first erosion to return a hit if P_{in}% of the inner SE remains beneath the signal while (100-P_{in})% is above the signal where $0 \leq P_{in} \leq 100$. Similarly, a hit from the second erosion is returned if P_{out}% of the outer SE is above the signal and (100- P_{out})% is below it where $0 \leq P_{out} \leq 100$.

For the HMT to detect an object, the reference points of both the inner and outer SEs must reach at least one common point within the height of that object. In noisy images the POHMT therefore allows S_{in} to continue to move up inside an object until it is less than P_{in}% occupied by the signal. S_{out} is permitted to move down through the object from above until it is less than P_{out}% occupied by the signal. This technique ensures that even in the presence of noise or internal texture within an object of interest, both the inner and outer SEs can reach at least one common point in the object in order to detect a hit.

Figure 2a shows a noisy image of a cancer cell in which the edges of the cell are blurred and the cell itself has internal texture and is immersed in noise. To detect this cell, the variables P_{in} and P_{out} are set a priori based on an estimate of the noise, texture or both present in the image. By interrogating the pixels within the shape to be recognised and the pixels around its edges, it is possible to estimate the noise in both the foreground and background regions of the image. The internal texture of objects of interest can be analysed and estimated by treating the texture as noise. If there is more noise on the foreground than the background, or if the object to be recognised has varying texture, P_{in} can be set lower than P_{out}. Conversely, if there is more noise or texture on the background than the foreground then P_{out} can be set lower than P_{in}.

To correctly set P_{in} and P_{out}, two SEs, one a solid disk, smaller than the diameter of the cell, and the other a ring larger in diameter than the cell were used to determine the PO of both SEs when eroding this object. The PO of S_{in} and S_{out}, at all intensity levels ℓ $(0 \leq \ell \leq 2^n\text{-}1)$ can be calculated by firstly using

$$O_{in\ \ell} \in_{(s_1,t_1)\in SE} \{ f(x+s_1, y+t_1) \geq \ell \} \tag{3}$$

$$O_{out\ \ell} \in_{(s_2,t_2)\in SE} \{ f(x+s_2, y+t_2) < \ell \} \tag{4}$$

where $O_{in\ \ell}$ and $O_{out\ \ell}$ are the sets of occupied points in each SE at each level ℓ. The PO of S_{in} and S_{out} can then be calculated by substituting $O_{in\ \ell}$ and $O_{out\ \ell}$ into

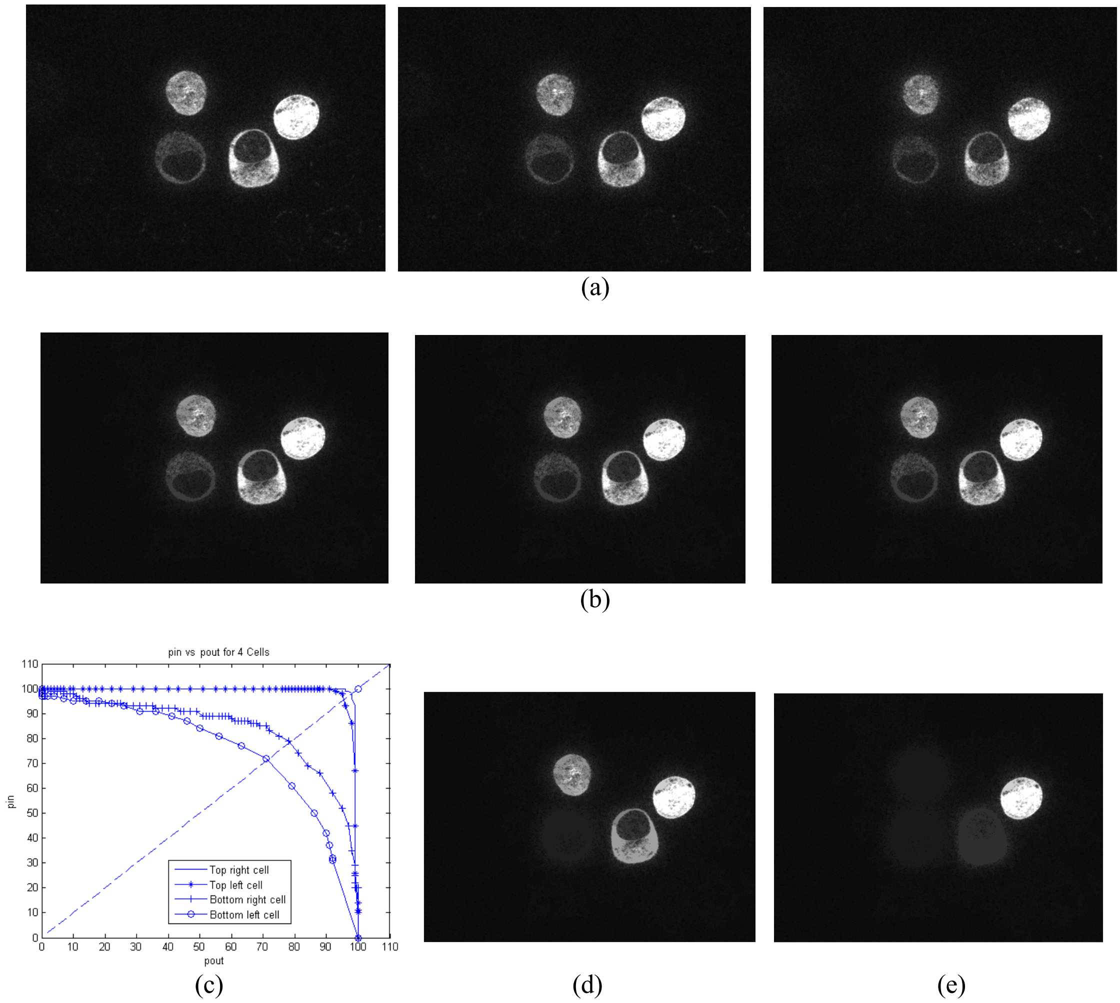


Figure 3 – Experimental Results (a) Noisy images of cancer cells (b) the results of applying the POHMT and reconstructing the cells (c) Plot of P_{in} and P_{out} for all four objects of interest in all three images (d) Reconstruction of three detected cells (e) Reconstruction of a single detected cell

$$PO_{\ell} = \frac{card(O_{\ell})}{card(SE)} \times 100 \qquad (5)$$

where *card* denotes the cardinality of a set.

By plotting the calculated POs at each intensity level ℓ it is possible to set a threshold (T) for the POHMT. From Figure 2d it is evident that the inner SE is initially fully occupied however the PO decreases rapidly as it moves higher inside the cell. The PO falls to 0 when the entire inner SE is above the highest point in the cell. At first, the outer SE is only partially occupied as it is blocked by the noise around the edges of the cell. As the SE is moved down, the PO of the outer SE increases until it is fully occupied. From Figure 2d it is clear that the reference points of the SEs intersect when they are both 96% occupied. Setting T=96 the POHMT can be calculated using

$$I_{(x,y)}^{\ out} \in \left\{ \left(card(PO_{FG}) \cap card(PO_{BG})\right) \geq T \right\} \qquad (6)$$

where I is a binary image containing markers in all places detected as a hit by the POHMT.

Figure 2e shows an alternative plot of the PO of both SEs where P_{in} is plotted against P_{out}. The advantage of plotting P_{in} against P_{out} is that it is possible to approximate the level of noise in the image by a measure of the extent to which this curve deviates from the noise free case shown in Figure 2e. That is the point marked in Figure 2e by a circle, this represents the case where both SEs would be 100 percent occupied at the same point in the image and hence the reference points of both SEs intersect. In this case the standard HMT would detect the desired object. For this case, the standard HMT can be implemented as a special case of the POHMT by setting P_{in} and P_{out} to be 100%. Setting P_{in} and P_{out} using the plot shown in Figure 2e has an additional advantage in that it allows these parameters to take on different values from each other based on the level of noise in the background when compared with that of the foreground.

Using the same SEs as before and setting P_{in} and P_{out} to 96%, the noisy cancer cell is detected and a marker is placed in the resultant image (I) of the POHMT as shown in figure 2b. This marker can then be used to reconstruct the cell using a morphological opening by reconstruction [8]. The result of this reconstruction is shown in Figure 2c.

It should be noted that by simply reducing the dimensions of the inner SE or increasing the dimensions of the outer SE by 4% will not provide the same results as the POHMT.

3. EXPERIMENTAL RESULTS

To verify the operation of the POHMT three test images of cancer cells were used and the performance of the POHMT was compared against that of the standard HMT using the same images and structuring elements. The three images, as well as the results of applying the POHMT and reconstructing the cells are shown in Figure 3.

By observation it is clear that there are four cells in each image. All four cells have different characteristics within the image and characteristics of the cells vary slightly between images. These variations are in terms of shape, location and intensity. The amount of noise also varies between images as does the degree of blurring. All of these variations are visible in Figure 3a.

The geometry of the structuring elements was set using a priori knowledge of the shapes and sizes of the cells determined by observation. Both SEs were chosen such that all four cells could be detected in all three images without changing the geometry of the SEs or P_{in} and P_{out}. S_{in} was a flat disk measuring 90 pixels in diameter such that it could fit inside that smallest cell (bottom left). S_{out} was a ring with an inner diameter of 110 pixels set to encompass the largest object in the image (bottom right).

These SEs were used to interrogate the pixels in and around each cell in the image (as discussed in Section 2) in order to set the parameters P_{in} and P_{out}. For this experiment P_{in} and P_{out} were both set to a 70% occupancy requirement in accordance with Figure 3c such that all four cells should be detected. The plot shown in Figure 3c shows the data gathered from the images shown in Figure 3a (right). This plot is the only one shown since it is taken from the noisiest image and hence, to detect all four objects in all three images, P_{in} and P_{out} must be set using this information which represents the worst case in terms of noise and texture. Figure 3b illustrates that the POHMT is capable of detecting all four cells under extremely noisy conditions. For the same image set, and using the same SEs, the standard HMT did not detect any cells in any of the three images due to the noise corrupting both the foreground and background regions of the image.

Figure 3c shows a plot of the data collected when interrogating the pixels inside and around each of the four cells. Clearly, by changing the parameters P_{in} and P_{out} the POHMT can be made to operate as a selective/discriminatory filter. By setting P_{in} and P_{out} to 78% only the three reconstructed cells shown in Figure 3d are extracted where the noisiest cell with the largest hole (bottom left) has not been selected. By a similar technique the noisiest, most textured cells in the image can be discarded and only the cell in the top right of the image is extracted on its own. This technique can be used to isolate and extract individual cells or different groupings of these in the image by varying the required level of PO. The detection and reconstruction of the cell in the top right of the image is shown here in Figure 3e.

4. SUMMARY & CONCLUSIONS

This paper presents a generalisation of the HMT in the form of a POHMT. The POHMT is robust to noise in the foreground and background regions of an image and in such conditions it is capable of detecting objects which cannot be detected by the standard HMT. Additionally, the POHMT is suitable for use in noise free images to extract textured features or objects containing holes, where, in some cases the HMT would fail.

The POHMT is a generalisation of the HMT and hence the HMT can be implemented as a special case of the POHMT by setting parameters P_{in} and P_{out} to 100%. Further to this, it has been shown that the POHMT, using one pair of SEs can be set to operate in a selective/discriminatory fashion. This feature of the POHMT is made possible by the collection of data which represents the level of noise and texture in foreground objects and their edges and the flexibility of the algorithm when setting the percentage occupancy parameters.

Currently, the algorithm is computationally expensive however optimisation techniques and a comparison of efficiency with other techniques will be presented in a further paper.

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