STOCHASTIC TRANSFER FUNCTION IN BAYESIAN INFERENCE FOR COMBUSTION INDICATOR ESTIMATION

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ABSTRACT

This paper deals with the use of Bayesian inference to regularize an inverse problem with a non linear transfer function. Bayesian inference is an useful tool to combine a statistical approach coming from a set of sensors and prior physical models of the system. Bayesian inference is applied to the estimation of combustion indicators. This is an inverse problem where only an indirect measurement from engine block vibrations is available and combustion models are necessary to extract relevant combustion parameters. Moreover the engine block is not a linear system and Bayesian inference can take into account this non linearity.

1. BAYES' THEOREM

Bayesian inference is used in different signal processing domains such as audio analysis [7], image analysis [10] or inverse problems [8]. The Bayesian inference approach is to check if an hypothesis $\underline{\Theta}$ is valid knowing evidence $\underline{Y}$ coming from a set of sensors. This can be written with probabilities

$$\mathbf{P}(\underline{\Theta}|\underline{Y}) = \frac{\mathbf{P}(\underline{Y}|\underline{\Theta})\mathbf{P}(\underline{\Theta})}{\mathbf{P}(\underline{Y})} \tag{1}$$

In the following notations, all vectors will be underlined and here $\underline{Y} = [Y_1 \dots Y_i \dots Y_{Nc}]^t$ where i is a sensor from a set of sensors $I = [1 : N_c]$. $\mathbf{P}(\underline{\Theta}|\underline{Y})$ is the posterior probability, $\mathbf{P}(\underline{\Theta})$ is called the *prior* probability, $\mathbf{P}(\underline{Y})$ is the evidence probability and $\mathbf{P}(\underline{Y}|\underline{\Theta})$ is the likelihood probability.

Equation (1) is maximized to obtain the hypothesis the most relevant and it reads

$$\arg\max_{\underline{\Theta}} \mathbf{P}(\underline{\Theta}|\underline{Y}) = \arg\max_{\underline{\Theta}} \left\{ \frac{\mathbf{P}(\underline{Y}|\underline{\Theta})\mathbf{P}(\underline{\Theta})}{\mathbf{P}(\underline{Y})} \right\} \tag{2}$$

The evidence probability $\mathbf{P}(\underline{Y})$ does not depend on combustion parameters, and from equation (2) we deduce a cost function C according to the relation

$$C(\underline{\theta}) = \arg\min_{\underline{\Theta}} \left\{ \frac{1}{\mathbf{P}(\underline{Y}|\underline{\Theta})\mathbf{P}(\underline{\Theta})} \right\} \tag{3}$$

The likelihood probability $\mathbf{P}(\underline{Y}|\underline{\Theta})$ is deduced from data coming from knock sensors, it traduces the appropriateness to the data. The second probability $\mathbf{P}(\underline{\Theta})$ traduces the prior knowledge about the problem. The determination of these probabilities depends on the application, thus it is necessary to explain the combustion indicator problematic in depth. Next section defines the combustion indicators and the convolutive model which links combustion parameters to sensor data. Section 3 and 4 are devoted to the estimation of $\mathbf{P}(\underline{Y}|\underline{\Theta})$ and $\mathbf{P}(\underline{\Theta})$ of equation (3). Section 5 deals with the experimental results and section 6 concludes this paper.

2. THE COMBUSTION INDICATORS

2.1 Introduction

The new Diesel engines using cleaner combustion modes must be controlled to get the maximum benefit of these systems. The standard variables for combustion control are the combustion timing and the combustion energy. These variables can be computed from the cylinder pressure measure but this method is too expensive and the sensor may drift. An innovative approach is to use low cost knock sensor which records vibrations circulating on the engine block. However mechanical vibrations may overlap vibrations coming from the combustion in time and in frequency domains. Moreover, it is difficult to deduce combustion variables from combustion signature on vibrations because the engine block transfer function depends on engine speed and load [14]. In The following all data are sampled according to the crank angle degree θ (CA). This base is independent of the engine speed and is measured by an optic encoder which measures the rotation speed of the crankshaft.

Generally the convolutive model is applied to deduce combustion parameters

$$y(\theta) = (h_l * p(\underline{\Theta}))(\theta) + b(\theta) \tag{4}$$

with y the vibration recorded on knock sensor, p the in cylinder pressure which contains combustion parameters $\underline{\Theta}$ and an additive noise b depicting mechanical and measurement noises which can occur in the same frequency domain and in the same time domain than the combustion. h_l is a linear time invariant transfer function. Many signal processing algorithms have been proposed to extract Combustion indicators: from filtered vibration oscillations [16], from time-frequency analysis [4], from Cepstrum [15] but few papers have studied the variability of the transfer function of the engine block [3]. This variability is due to clearance between the different components of the engine and lubricant oil films [11]. In this paper, we propose to take into account engine block transfer function variability in the Bayesian inference. Moreover, the method obtains relevant combustion parameters with a frequency domain criterion and priors on combustion timing and combustion energy.

2.2 The Combustion indicators

We propose to describe the combustion with two indicators: the maximum pressure gradient which is an image of the combustion energy and the maximum pressure gradient crank angle to deduce the combustion timing (see Figure 1). Combustion parameters can be linked to the knock sensor data y with a convolutive model taking into account the transfer function variability [12]

$$y(\theta) = (h_l * p(\underline{\Theta}))(\theta) + b(\theta) + b_{nl}(\theta, p) \tag{5}$$

The difference between classical convolutive model of equation (4) and equation (5) is b_{nl} which countains all that the convolutive model cannot depict as the variability of the transfer function. However b_{nl} is correlated to the input of the system, p. h_l is the transfer function given by the best linear model at least squares sense(BLM). Cylinder pressure evolution depends mainly on three events: the

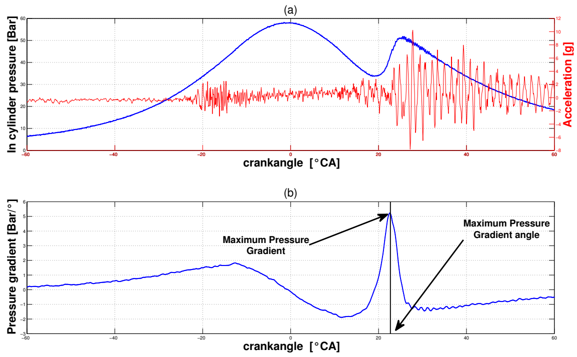

Figure 1: (a) In cylinder pressure and engine block acceleration from knock sensor (b) In cylinder pressure gradient and combustion indicators

motored phase, the combustion and the cavity resonances which depend on speed velocity in the combustion chamber and cylinder geometry. These events are separated in three different frequency bands. As it is depicted in Figure 2, the motored pressure is a low frequency based event ($< 1\ kHz$) , the combustion is a middle range frequency based event ($[1-3]\ kHz$) and cavity resonances are high frequency based event ($> 4\ kHz$). The study of the spectrum of the in cylinder pressure $P(\omega_k)$ in the combustion frequency band $[\omega_{f1} - \omega_{f2}]$ (see Figure 2) shows that pressure can be modeled at first order by

$$P(\omega_k) = K e^{-\frac{a\omega_k F_e}{N_{\text{FFT}}}} e^{-\frac{j2\pi\theta_0\omega_k F_e}{N_{\text{FFT}}}} \tag{6}$$

where ω_k is a frequency bin, N_{FFT} is the length of the discrete Fourier transform of p and F_e the sampling frequency. a is the duration of the combustion and is supposed invariant in the following. The combustion timing depends on θ_0 and the maximum pressure gradient depends on the parameter K. This cylinder pressure model describes only the rapid pressure rise due to the combustion in Figure 1. It is during this interval that the combustion parameters occur. Finally a relation between knock sensor data and combustion event in a domain of validity has been formulated. The goal is now to estimate $\underline{\Theta} = \{K, \theta_0\}$ with a set of knock sensors and prior models and to take into account the variability of the transfer function.

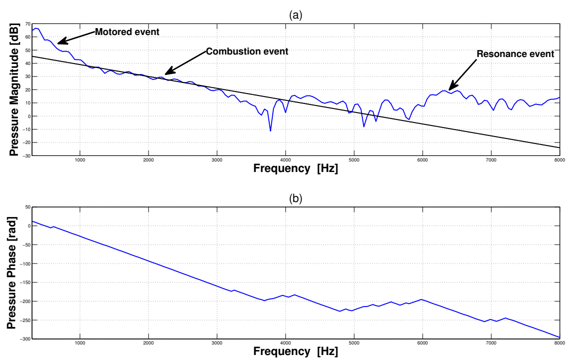

Figure 2: Pressure spectrum and model by straight line between $[1000\ 3000]\ Hz$ for : (a) magnitude and (b) phase

3. THE LIKELIHOOD PROBABILITY P($\underline{Y}|\underline{\Theta}$)

3.1 The convolutive model

This section is devoted to the estimation of the likelihood probability. This one can be deduced from the convolutive model which induces a relation between combustion parameters and sensor data. We use this relation in the frequency domain for several reasons

- The convolution product become a simple multiplication.
- The domain of validity of knock sensors is defined in the frequency domain.
- The central limit theorem simplify the probability laws of different variables. Indeed Fourier transforms of noise $\underline{B}_i$ and transfer function $\underline{H}_i$ tend to have Gaussian distributions.

For each sensor i,the convolutive model is expressed by

$$\underline{Y}_i = \underline{P}.\underline{H}_i + \underline{B}_i \tag{7}$$

Here $\underline{H}_i = \underline{H}_0 + \underline{H}_{nl}$ is the transfer function and the sum of two terms :

- $\underline{H}_0$ given by the BLM and it is the deterministic part of the transfer function.
- $\underline{H}_{nl}$ is added to $\underline{H}_0$, it is a random variable which takes into account all that the linear transfer function cannot describe.

$\underline{Y}_i = [Y_i(\omega_{f1}) \cdots Y_i(\omega_{f2})]^t$ is the vector of sensor data, $\underline{P}$ the diagonal matrix of cylinder pressure,$\underline{H}_i$ the vector of transfer function random values and $\underline{B}_i$ the vector of noise random values.

3.2 The transfer function $\underline{H}_i$

In this study, it is necessary to estimate the transfer function from the combustion chamber to the sensor location. According to the combustion domain of validity (see section 2.2), transfer function have to be determined only in a little frequency band. One of the main difficulty of the estimation of combustion parameters from accelerometer is the fact that an engine block transfer function is not a linear and time invariant system [11]. Its parameters depend on engine speed and load. In the literature there is three main approaches

- A time approach which consists in the estimation of different coefficients of ARMA filter. Poulimenos [13] takes into account the signal non stationarity and introduces TARMA filters.
- A frequency approach. Bendat [2] proposes to study autospectrum and cross-spectrum to determine the transfer function and Antoni *et al.* [1] apply it to internal combustion engine. Although the transfer function variability is not taken into account.
- A Ceptrum approach. Gao *et al.* [6] use Cepstrum to estimate the transfer function.

Equations coming from the Bayesian inference can take into account the transfer function variability. Only a transfer function template H_0 is necessary processed with an off line analysis of in cylinder pressure p and knock sensor y by estimating their centered autospectrum and cross-spectrum on N_p engine cycles

$$H_0(\omega_k) = \frac{\sum_{n=1}^{N_p} P_n(\omega_k) Y_n^*(\omega_k)}{\sum_{n=1}^{N_p} |P_n(\omega_k)|^2} \tag{8}$$

Transfer function variability is taken into account by the standard deviation σ_H which is experimentally estimated over 30 engine set points at different engine speed and load.

3.3 The Likelihood probability

It is possible to deduce from convolutive model (equation (7)) a relation between probability density function (pdf) of different random variables.

$$f_{\underline{Y}}(\underline{Y}|\underline{\Theta}) = f_{\underline{Y}_i}(\underline{P}.\underline{H}_i + \underline{B}_i|\underline{\Theta}, I) \tag{9}$$

with $f_{\underline{Y}}$ the probability density function connected with $\underline{Y}$. Let the noise dependency appear in equation (9)

$$f_{\underline{Y}}(\underline{Y}|\underline{\Theta}) = \int_{-\infty}^{\infty} f_{\underline{Y}_i|\underline{B}_i}(\underline{\underline{P}}.\underline{H}_i + \underline{B}_i|\{\underline{B}_i, \underline{\Theta}\}, I) f_{\underline{B}_i}(\underline{B}_i|\underline{\Theta}, I) d\underline{B}_i \quad (10)$$

Noise B_i is independent of combustion parameters, thus equation (10) can be simplified

$$f_{\underline{Y}}(\underline{Y}|\underline{\Theta}) = \int_{-\infty}^{\infty} f_{\underline{PH}_i}(\underline{\underline{P}}.\underline{H}_i|\{\underline{\Theta}\}, I) f_{\underline{B}_i}(\underline{B}_i, I) d\underline{B}_i \quad (11)$$

with the relation $f_{\underline{PH}_i}(\underline{\underline{P}}\underline{H}_i(\underline{\Theta})|\{\underline{\Theta}\}) = f_{\underline{PH}_i}(\underline{Y}_i - \underline{B}_i|\{\underline{\Theta}\})$, a convolution product appears between the two pdf in equation (11)

$$f_{\underline{Y}}(\underline{Y}|\underline{\Theta}) = f_{\underline{PH}_i} * f_{\underline{B}_i}(\underline{Y}_i, I) \quad (12)$$

Now we can express $f_{\underline{H_i P}}$ with the pdf related with $\underline{H}_i$ called $f_{\underline{H}_i}$

$$f_{\underline{PH}_i}(\underline{X}) = f_{\underline{H}_i}(\underline{\underline{P}}^{-1}\underline{X}) \det(\underline{\underline{P}}) \quad (13)$$

$f_{\underline{H_i P}}$ can be replaced by $f_{\underline{H}_i}$ in equation (12)

$$f_{\underline{Y}}(\underline{Y}|\underline{\Theta}) = \int_{-\infty}^{\infty} f_{\underline{H}_i}(\underline{\underline{P}}^{-1}(\underline{Y}_i - \underline{B}_i), I) f_{\underline{B}_i}(\underline{B}_i, I) \det(\underline{\underline{P}}) d\underline{B}_i \quad (14)$$

Now we use the commutativity property of the convolution product, set $\underline{X}_i = \underline{\underline{P}}^{-1}\underline{B}_i$ and assume noises and transfer functions between each sensors are independents. The independence between transfer function of each sensors has no physical meaning although this assumption is retained because it carries the less information according to the maximum entropy principle

$$\mathbf{P}(\underline{Y}|\underline{\Theta}) = \prod_{i=1}^{N_c} \int_{-\infty}^{\infty} \mathbf{P}_{\underline{H}_i}(\underline{X}_i) \mathbf{P}_{\underline{B}_i}(\underline{Y}_i - \underline{\underline{P}}(\underline{\Theta})\underline{X}_i) d\underline{X}_i \quad (15)$$

Equation (15) provides a relation between the likelihood probability and the noise and random transfer function probabilities. Supposing these two probabilities follow circular complex multidimensional Gaussian laws according to the central limit theorem, it is possible to deduce the likelihood probability law expression. Moreover, we assume that the variance of complex Gaussian laws of transfer function and noise are the same for the real and the imaginary parts for each frequency For a given frequency bin ω_k

$$\mathbf{P}_H(H(\omega_k)) = \frac{e^{-\frac{|H(\omega_k) - H_0(\omega_k)|^2}{\sigma_H^2}}}{\pi \sigma_H}, \quad \mathbf{P}_B(B(\omega_k)) = \frac{e^{-\frac{|B(\omega_k)|^2}{\sigma_B^2}}}{\pi \sigma_B} \quad (16)$$

H_0 is the mean value vector of the transfer function. For a given frequency bin, the likelihood probability becomes

$$\mathbf{P}(Y_i(\omega_k)|\underline{\Theta}) = \frac{1}{\pi^2 \sigma_B \sigma_H} \int_{-\infty}^{\infty} e^{-\frac{|X - H_0(\omega_k)|^2}{\sigma_H^2}} e^{-\frac{|Y(\omega_k) - P(\omega_k)X|^2}{\sigma_B^2}} dX \quad (17)$$

Equation (17) can be further simplified to

$$\mathbf{P}(Y_i(\omega_k)|\underline{\Theta}) = \frac{1}{\pi \sqrt{(P(\omega_k))^2 \sigma_H^2 + \sigma_B^2)}} e^{-\frac{|Y(\omega_k) - H_0(\omega_k)P(\omega_k)|^2}{\sigma_B^2 + P(\omega_k)^2 \sigma_H^2}} \quad (18)$$

The final expression of likelihood probability is summarized in equation (19). Random variables are supposed independent for different frequency bins. This Assumption is the case when the less information is conveyed by convolutive model.

$$\mathbf{P}(\underline{Y}|\underline{\Theta}) = \prod_{i=1}^{N_c} \prod_{\omega_k = \omega_{f1}}^{\omega_{f2}} \frac{e^{-\frac{|Y_i(\omega_k) - P(\omega_k)H_{0i}(\omega_k)|^2}{\sigma_{Bci}^2 + |P(\omega_k)|^2 \sigma_{Hci}^2}}}{|P(\omega_k)|^2 \sigma_{Hci}^2 + \sigma_{Bci}^2} \quad (19)$$

σ_{Bci} and σ_{Hci} are the standard deviations of the complex random variables B_i and H_i of the sensor i.

4. PRIOR PROBABILITY $\mathbf{P}(\underline{\Theta})$

For each combustion parameter we suggest a model taken from the literature. Then prior probability can be expanded

$$\mathbf{P}(\underline{\Theta}) = \mathbf{P}(\theta_0)\mathbf{P}(K|\theta_0) \quad (20)$$

Where $\mathbf{P}(\theta_0)$ and $\mathbf{P}(K|\theta_0)$ are respectively deduced from a combustion occurrence model and a combustion energy model. Prior will come from models already used in other applications in engine control. Thus the necessary extensive data set to tune parameter models is already available at the beginning of this study.

4.1 Combustion occurrence model $\mathbf{P}(\theta_0)$

The choice of the model depends on the engine application. For example, Swan has proposed in [17] a model to deduce the start of combustion angle θ_{soc}. This model has four inputs : the intake pressure p_{ivc} and temperature T_{ivc}, the start of injection angle θ_{inj} and the burnt gas ratio (BGR). There is also 5 parameters γ, T_a, $C1$, $C2$ and n. The start of combustion is estimated when the integral of equation (21) is equal to one (to be solved for θ_{soc})

$$\int_{\theta_{inj}}^{\theta_{soc}} \frac{C_1 (P_{ivc} v(\theta)^{\gamma})^n e^{-(\frac{Ta}{T_{ivc} v(\theta)^{\gamma-1}})}}{1 + C_2.BGR} d\theta = 1 \quad (21)$$

This model gives the start of combustion θ_{soc}, however the combustion parameter to estimate is the maximum pressure gradient angle θ_0. Indeed the difference $\theta_0 - \theta_{soc}$ is unknown. We consider it as a random variable. The variability of $\theta_0 - \theta_{soc}$ has been measured on 85 engine set points by measuring θ_{soc} given by the model and θ_0 given by the in-cylinder pressure. Figure 3 shows the difference. From these data, a log-normal law is fitted according to the Kolmogorov-Smirnov test (error of 0.06 for a log normal law instead of 0.12 for a normal law, 0.26 for an exponential law and 0.09 for a chi-square law).

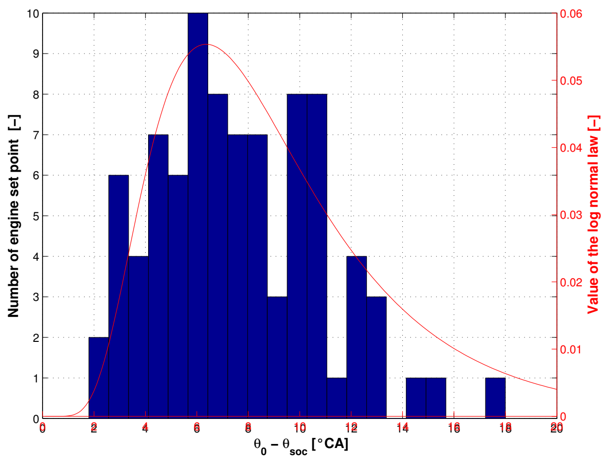

Figure 3: Histogram of the difference $\theta_0 - \theta_{soc}$ and log normal probability law proposed to fit data.

Finally

$$\mathbf{P}(\theta_0) = \frac{e^{-\frac{(\ln(\theta_0 - \theta_{soc}) - \mu_0)^2}{2\sigma_0^2}}}{\sqrt{2\pi}\sigma_0(\theta_0 - \theta_{soc})}, \text{ for } \theta_0 - \theta_{soc} > 0 \quad (22)$$

with $\sigma_0 = 0.55\ ^{\circ}CA$ and $\mu_0 = 2.1\ ^{\circ}CA$.

4.2 Energy of combustion model $\mathbf{P}(K|\theta_0)$

The maximum entropy principle proposed by E.T. Jaynes in [9] establishes the probability $\mathbf{P}(b|\theta_0)$ the less compromising in sense that we will use only information given by the constraints. it consists in maximizing entropy $H(\mathbf{P}) = \int \mathbf{P}(x) \ln \mathbf{P}(x) dx$ with regards to the constraints on $\mathbf{P}$. Indeed some knowledge about the energy of the combustion is available: engine thermodynamic laws give K

$$K = \frac{(\gamma - 1)\frac{\partial Q(\theta_0)}{\partial \theta}}{\frac{3\sqrt{3\pi}\gamma}{a^2} V(\theta_0) + \frac{2}{a}\frac{\partial V(\theta_0)}{\partial \theta}} \tag{23}$$

V is the volume of the combustion chamber and $\frac{\partial Q(\theta_0)}{\partial \theta}$ is the rate of heat release (ROHR) which is unknown. A model proposed by Chmela [5] predicts the value of the ROHR

$$\frac{dQ}{C_{Lhv} M_{inj} - Q} = C(1 - BGR)^{\beta} e^{\frac{\phi}{\sqrt[3]{V}}} \tag{24}$$

Where C_{Lhv} is the lower heating value depending on the fuel, M_{inj} is the mass of fuel, β and C are two constants depending on the engine. Although, ϕ, the turbulence in the cylinder is considered as a random variable because it can take many different values for the same engine and bring the predominant uncertainty in the Chmela model. Finally, constraints on $\mathbf{P}(K|\theta_0)$ are enumerated here :

1. The maximum of pressure gradient K is a non negative random variable.
2. The mean value of random variable K is estimated with 85 engine set points by comparing value of maximum pressure gradient coming from the in cylinder pressure and the value given by the model. $E(\ln K) = \phi_0$
3. The variance of K can be estimated following the same procedure than in point 2. $E(\ln K)^2 = \sigma_\phi^2$

According to the maximum entropy principle, the probability function satisfying (1),(2) and(3) is a log normal law.

$$\mathbf{P}(K|\theta_0) = \frac{e^{-\frac{(\ln(K) - \phi_0)^2}{2\sigma_\phi^2}}}{\sqrt{2\pi}\sigma_\phi K}, \text{ for } K > 0 \tag{25}$$

5. EXPERIMENTAL RESULTS

5.1 Cost function coming from the Bayesian inference

The cost function C to minimize can be deduced by gathering the likelihood probability and prior probabilities described in the two last sections. To simplify the cost function we have chosen to minimize the logarithm of the probabilities

$$\begin{aligned} C(\underline{\Theta}) &= \sum_{i=1}^{N_c} \sum_{\omega_k=\omega_{f1}}^{\omega_{f2}} \frac{|Y_i(\omega_k) - H_0(\omega_k)P(\omega_k, \underline{\Theta})|^2}{|P(\omega_k, \underline{\Theta})|^2 \sigma_{Hci}^2 + \sigma_{Bci}^2} \\ &+ \sum_{i=1}^{N_c} \sum_{\omega_k=\omega_{f1}}^{\omega_{f2}} \ln(|P(\omega_k, \underline{\Theta})|^2 \sigma_{Hci}^2 + \sigma_{Bci}^2) \\ &+ \frac{1}{2\sigma_0^2}(\ln(\theta_0 - \theta_{soc}) - \mu_0)^2 + \ln(\theta_0 - \theta_{soc}) \\ &+ \frac{1}{2\sigma_\phi^2}(\ln(K) - \phi_0)^2 + \ln(K) + C_0 \end{aligned} \tag{26}$$

C is undefined if $\theta_0 \leq \theta_{soc}$ and $K \leq 0$.

5.2 Experimental setup and results

Bayesian algorithm has been tested on a four cylinder Diesel-HCCI engine for speeds from 1000 rpm to 2500 rpm. They are two knock sensors available located on the two sides of the engine. The in cylinder pressure is also available to validate the algorithm.

Three cases appear depending on signal to noise ratio(SNR) on knock sensors.

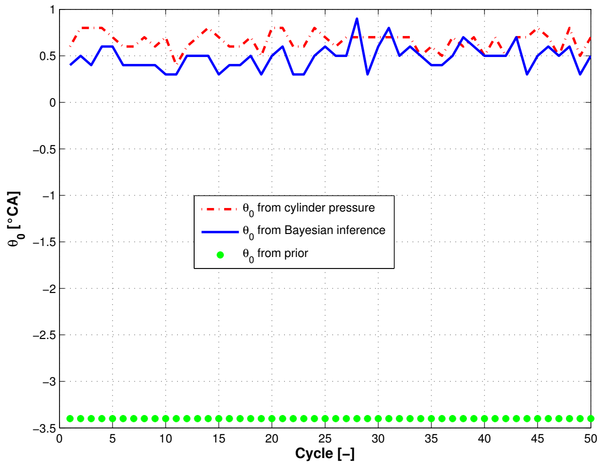

Figure 4: Cycle to cycle maximum pressure gradient angle θ_0 calculated with in cylinder pressure (blue line), estimated with Bayesian inference (red dotted line) and given by prior model (green circled line) on a 2000 RPM engine set point

- **case 1** The SNR is high and the likelihood probability is predominant in the cost function. It induces a cycle to cycle relevant extraction of combustion parameters (Figure 4) .This case happens generally at low speed (1000 and 1500 rpm) where mechanical noise has a little influence on global vibrations circulating on the engine block.
- **case 2** The SNR is low then the prior probability is predominant in the cost function. It is possible to extract combustion parameters however the little cycle to cycle variations are not detectable (Figure 5). This case happens generally at high speed (2000 and 2500 rpm) where mechanical noise has a large influence on global vibrations circulating on the engine block.
- **case 3** The SNR is low and the combustion parameters given by the models are wrong. In this case, the extraction of combustion parameters is not relevant.

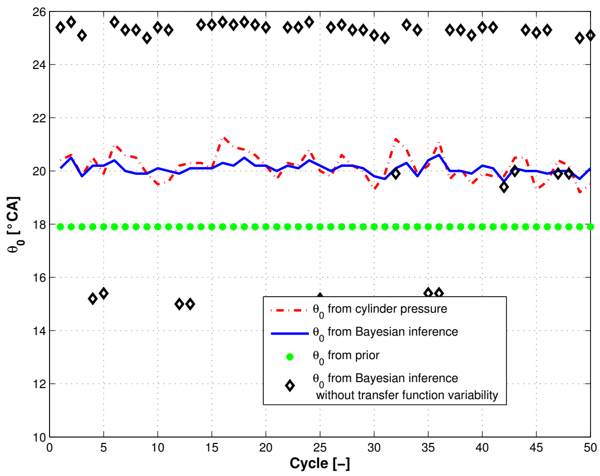

Figure 5: Influence of transfer function variability on a 2000 RPM engine set point. In some engine set point supposing the transfer function invariant causes the detection of non combustion phenomena.

Considering the transfer function as a random variable presents two main advantages

- Algorithm can give relevant combustion parameters for engine

Engine set point	$\overline{CA}_{50}$	$\overline{t_0}$	σ_{ca50-t_0}
Unit	[deg]	[deg]	[deg]
1000rpm - 3 bar	6.9	6.9	0
1000rpm - 8 bar	10.3	10.1	0.1
1500rpm - 3 bar	9.2	8.9	0.2
1500rpm - 8 bar	12.1	12.2	0.2
2000rpm - 3 bar	12.3	12.4	0.3
2000rpm - 8 bar	12.6	12.2	0.5
2500rpm - 3 bar	11.8	11.6	0.6
2500rpm - 8 bar	13.6	13.2	0.5
Engine set point	$\overline{MPG}$	$\overline{Ek} = \frac{3\sqrt{3\pi K}}{a^2}$	σ_{MPG-Ek}
Unit	$[bar/\text{deg}]$	$[bar/\text{deg}]$	$[bar/\text{deg}]$
1000rpm - 3 bar	5.4	5.1	0.2
1000rpm - 8 bar	3.9	4.1	0.1
1500rpm - 3 bar	3.6	3.6	0.2
1500rpm - 8 bar	3.1	3.2	0.2
2000rpm - 3 bar	2.5	2.2	0.3
2000rpm - 8 bar	2.3	2.1	0.3
2500rpm - 3 bar	2.2	2.0	0.5
2500rpm - 8 bar	1.9	2.1	0.4

Table 1: Combustion parameter extraction for different engine set points. $\overline{CA}_{50}$ and $\overline{MPG}$ are the reference combustion timing and maximum pressure gradient extracted from in cylinder pressure averaged on 50 engine cycles, $\overline{t_0}$ and $\overline{Ek}$ are given by the Bayesian inference averaged on 50 engine cycle and σ_{ca50-t_0} and σ_{MPG-Ek} are the standard deviation of the difference between CA_{50} and t_0 and between MPG and Ek.

set point transfer function far from H_0 given by the best linear model (BLM).

- H_0 can be chosen with the less setting parameters as possible. Real time applications are then possible. Transfer function template can be taken equal to H_0 as it has been explained in section 3.2. However, when the cylinder pressure is unavailable only a fixed transfer function gain and a fixed delay can be defined for transfer function template and H_{nl} variability will be a setting parameter.

Extraction combustion variable results are summarized in Table (1) for different engine speed and load. The cycle to cycle extraction (case 1) is possible for the majority of engine set points. However for low combustion energy engine set point, only a mean extraction (case 2) is available and for very low combustion energy engine set point the combustion parameter extraction is not relevant (case 3). The extraction is harder for high speed because mechanical noises increase with the speed. The transfer function variability influence is shown in Figure 5. Knock sensors filtered between $[\omega_{f1} - \omega_{f2}]$ provide informations coming from three different sources : the piston slap (15 deg after the top dead center), the combustion (20 deg after the top dead center) and intake valve closure from another cylinder (26 deg after the top dead center). The three sources have the same energy level in this frequency band. Taking into account the transfer function variability induces that prior has more influence and then global cost function minimum switch on the true combustion indicator.

6. CONCLUSION

Bayesian inference provides a well suitable methodology for combustion indicators extraction in an internal combustion engine. Indeed, the extraction of combustion parameters from vibration block encounter several problems such as a variability of the engine block transfer function or a perturbation of data due to mechanical noise vibrations. Bayesian inference can take into account these difficulties and combines the data information with prior combustion models already used for other engine applications. The combination induces a relevant extraction of combustion parameters on a large engine set point range and for engine transient. This study gives opportunity to have a cycle to cycle information about combustion timing and energy for Diesel engine control purpose. Detection of cylinder balancing, of misfire, diagnostic of different devices such as injection system are thereby possible.

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