PRACTICAL COMPRESS-AND-FORWARD COOPERATION FOR THE CLASSICAL RELAY NETWORK

Jing Jiang, John S. Thompson, Peter M. Grant,

Institute for Digital Communication,
Joint Reasearch Institute for Signal & Image Processing,
School of Engineering, University of Edinburgh,
Edinburgh, UK
(J.Jiang, John.Thompson, Peter.Grant)@ed.ac.uk

and Norbert Goertz

Institute of Communications and Radio-Frequency Engineering,
Vienna University of Technology
Vienna, Austria
norbert.goertz@nt.tuwien.ac.at

ABSTRACT

This paper proposes a practical compress-and-forward cooperation scheme with vector coding at the relay node for a three-terminal classical relay network. We discuss the framework of the relay receiver and analyse two practical vector coding algorithms for the cooperation, nearest neighbour quantization and lattice vector quantization. The error rate performance of the compress-and-forward cooperation and some other protocols under different SNRs is investigated. The impact of the quantization rate at the relay node is also characterised. It is shown that for a quantization rate larger than 2 bits/sample, the vector coding whether employing nearest neighbour quantization or lattice vector quantization, outerforms both the decode-and-forward protocol and scalar coding.

1. INTRODUCTION

It is well known that cooperative communication is a new and good way of improving the performance of wireless networks [1]. Multiple nodes in a network can cooperate by jointly encoding or decoding the transmission signals, to realise spatial diversity and increase channel rates [2, 3]. Cooperation protocols for wireless relay networks is currently a hot research topic [4]. These protocols determine what the individual relay should do, decode-and-forward (DF), amplify-and-forward (AF), or compress-and-forward (CF), after receiving the signal [5]. It was shown that CF protocol can be applied to a variety of wireless channels and always gives a rate gain over direct transmission [6]. In [7], the authors also showed that CF outperforms DF when the link between the relay and destination is better than that between the source and relay. In this paper, we consider a scenario where there are one transmitter and two clustered receivers, i.e. the relay is close to the destination. When the clustered nodes do cooperate, the CF protocol is a better choice providing higher communication rates than DF protocol. Hence here the focus in this paper is on the CF protocol.

The compress-and-forward protocol has the relay forwarding a quantized and compressed version of the received signal. The relay node can employ standard source coding, or the Wyner-Ziv coding (WZC) technique, when compressing the signal. The CF protocol with WZC at the relay following the rate distortion theory with side information [8], could support a slightly higher achievable rate in theory, compared with standard source coding [7]. However, for the WZC technique in practice, how to efficiently take advantage of the statistical dependence between the relay and the destination, and how to realise the theoretical performance limit of the CF protocol, are still open problems [9, 10]. If there exist multiple independent transmitters, the performance of CF protocol with WZC will be impaired by a larger compression noise when employing side information [11]. Since standard source coding is much simpler for the CF protocol and also performs well in practical scenarios, we choose to implement it at the relay.

In this work, we examine the improvement in bit error rate (BER) from a practical CF cooperation scheme with standard source coding at the relay node. For standard source coding, there are a number of algorithms to perform quantization, which can be divided into two kinds, scalar coding and vector coding. The scalar coding technique for compressing the signal has been studied in [12]. A more sophisticated coding technique, vector coding, for the CF protocol is desirable and still an open area of research. Our work differs from previous research in this area in that: i) we propose vector coding at the relay node, which is tailored for multi-dimensional signals; ii) we recommend two practical vector coding algorithms for the cooperation and examine their BER improvements from cooperation; and iii) we characterise the impact of quantization rate at the relay node.

The remainder of the paper is organized as follows. Section 2 presents the channel model. In section 3, we analyse the framework of the relay receiver, and propose that vector coding for CF protocol at the relay is a better choice. Two practical design algorithms for vector coding, nearest neighbour coding and lattice vector coding, are recommended in section 4. Section 5 shows some simulation results about BER improvements from CF cooperation, and section 6 concludes the paper.

2. CHANNEL MODEL

Consider a classical relay network with one transmitter (source) and two clustered receivers (relay and destination), as shown in Figure 1. We assume the nodes within a cluster are close together, but the distance between the transmitter and receiver cluster is large.

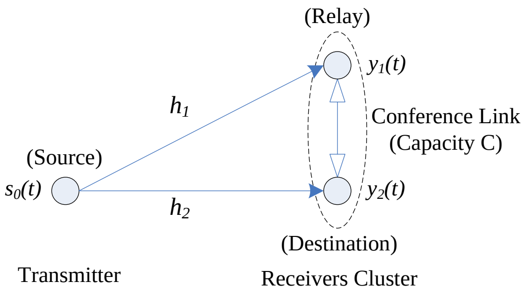

Figure 1: *System model of a classical relay network with one transmitter (source) and two clustered receivers (relay and destination).*

To focus on the performance of source coding techniques at the relay, we consider a simplified channel environment. We assume the channels from the transmitter to the two clustered receivers are quasi-static phase fading [13]: the channels have unit magnitude with independent and identically distributed (i.i.d.) random phase. Thus the channel gains are denoted by $h_i = e^{j\theta_i}, i = 1,2$, where $\theta_i \sim U[0,2\pi]$. The channel side information (CSI) is known to the receivers.

Let $s_0(t)$ denotes the source signal. We assume it is encoded and QPSK modulated before transmission. The signal energy per bit equals to $A^2/2r_b$, where A denotes the amplitude of the source signal, r_b denotes the bit rate which is twice the symbol rate r_s for QPSK signal. Let $\boldsymbol{y(t)} \triangleq [y_1(t), y_2(t)]^T$ denotes the corresponding received signals. In vector form, the data channel can be written as

$$\boldsymbol{y(t)} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} s_0(t) + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix}, \qquad (1)$$

where $n_1(t)$ and $n_2(t)$ are i.i.d zero-mean circularly symmetric complex Gaussian (ZMCSCG) white noise samples, with one-sided power spectral density (PSD) N_0.

As the clustered receivers are close together, it is reasonable to expect that the communication between the two receivers is much better and more stable than that between the transmitter and receivers. It is highly possible that we could achieve the channel capacity with high reliability on this short range link. Hence we assume the two receivers cooperate by way of an error-free conference link, with capacity C, as shown in Figure 1. We consider one-shot conferencing cooperation [14], which requires the destination to decode the signal from the relay which is sent over the conference link. In the CF cooperation protocol, the relay sends a compressed version of its observed signal to the destination. The destination then performs maximal-ratio combining (MRC) of the compressed signal and its own observation. As the relay chooses standard source coding to perform the CF protocol, the quantization rate at the relay will be equal to the capacity C of the error-free conferencing link.

3. COMPRESS-AND-FORWARD COOPERATION AT THE RELAY

The CF protocol has the relay forwarding a quantized version of the received signal. The relay node can employ different source coding techniques for compressing the signal. Vector quantization (VQ) is desirable for 2D QPSK source signals.

3.1 Vector Quantisation at the Relay

When implementing the CF protocol, the relay and the destination receives the i.i.d. $y_1(t)$ and $y_2(t)$, and $y_1(t)$ is compressed with a quantization rate and forwarded to destination. Here the quantization rate is equal to the capacity C of the error-free conferencing link. Then this system is equivalent to a system where destination has two antennas that receive the signals

$$\begin{bmatrix} y_1(t) + n_c(t) \\ y_2(t) \end{bmatrix}, \qquad (2)$$

where $n_c(t)$ is compression noise [6], which is independent of $y_1(t)$ and $y_2(t)$.

If the relay node chooses vector quantization to compress the signal with a quantization rate C, we could compute the power of $n_c(t)$ according to Shannon rate-distortion theory

$$\sigma^2_{c,standard} = \frac{E[|y_1(t)|^2]}{2^{2C}} = \frac{N_0 r_s + \frac{A^2}{2}|h_1|^2}{2^{2C}}. \qquad (3)$$

If the relay node employs the Wyner-Ziv Coding technique, the compression noise n_c has variance [13]

$$\sigma^2_{c,WZC} = \frac{N_0 r_s (\frac{A^2}{2}|h_1|^2 + \frac{A^2}{2}|h_2|^2 + N_0 r_s)}{(2^{2C}-1)(\frac{A^2}{2}|h_2|^2 + N_0 r_s)}. \qquad (4)$$

Considering 2D QPSK source signals, the quantization rate C should be at least 2 bits/sample. A more detailed discussion of the impact of C at the relay node will be presented in Section 4. For C smaller than 2 bits, the CF protocol will not give us any benefits over other protocols. For C equal to or larger than 2 bits, a comparison of the compression noises is shown in Figure 2.

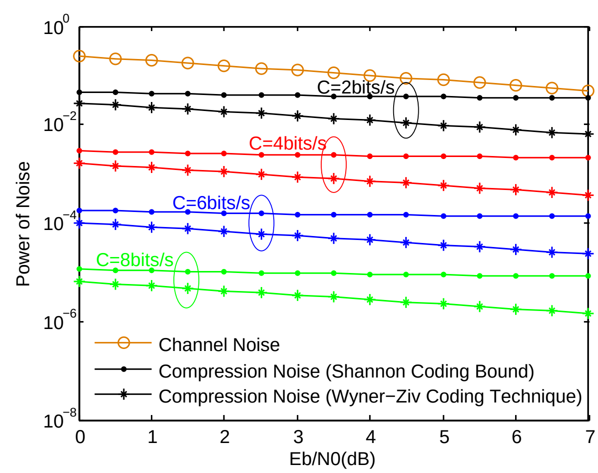

Figure 2: *Power comparison between channel noise and compression noise at the relay node.*

It can be seen that compared with the power of channel noise, the powers of the two compression noises are too small to impair significantly the achievable channel rates or BER performance, especially for higher quantization rates C. Since vector coding is much simpler than WZC technique in practice, we choose it for compressing the signals at the relay.

3.2 Relay Framework

The relay receiver is shown in Figure 3. For purpose of analysis, we consider the operation of the receiver during the signal interval (0, T_s), with $T_s = 1/r_s$. In QPSK, one of four possible waveforms are transmitted during each signalling interval. These waveforms are:

$$s_0(t) = A\cos(\omega_c t + \varphi), \text{ with } \varphi = [\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}]. \quad (5)$$

We denote the signal components at the output of the correlators by $s_{01}(T_s)$ for I-channel and $s_{02}(T_s)$ for Q-channel respectively, with the values calculated as

$$s_{01}(T_s) = \frac{1}{T_s}\int_0^{T_s} h_1 s_0(t)\cos\omega_c t dt = \pm\frac{A}{2}e^{j\theta_1}\cos\frac{\pi}{4}; \quad (6)$$

$$s_{02}(T_s) = \frac{1}{T_s}\int_0^{T_s} h_1 s_0(t)\sin\omega_c t dt = \pm\frac{A}{2}e^{j\theta_1}\sin\frac{\pi}{4}. \quad (7)$$

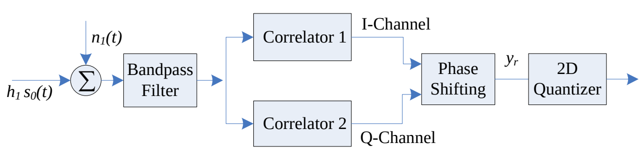

Figure 3: *Relay Receiver Block Diagram.*

We denote the noise components at the output of the correlators by $n_{11}(T_s)$ for I-channel and $n_{12}(T_s)$ for Q-channel respectively. We could calculate their values similarly with equation (6) and (7), just replacing $h_1 s_0(t)$ with $n_1(t)$. Since $n_1(t)$ is i.i.d ZMCSCG white noise with one-sided PSD N_0, we can show that $n_{11}(T_s)$ and $n_{12}(T_s)$ are also independent Gaussian random variables, with zero mean and equal variance given by

$$E\{[n_{11}(T_s)]^2\} = E\{[n_{12}(T_s)]^2\} = \frac{N_0}{4T_s}A^2 \quad (8)$$

As the channel side information is known to the receivers, a phase shifting device at the relay could be used to eliminate the effect of the channel phase. The signal after the phase shifting is 2-dimensional(2D), denoted by y_r. For each dimension, its probability distribution is a Gaussian distribution with mean $\pm\frac{\sqrt{2}A}{4}$, and variance $\frac{N_0}{4T_s}A^2$. The probability density function (PDF) of y_r could be shown as

$$p(y_r) = \frac{1}{\sqrt{\pi A^2 \frac{N_0}{2T_s}}}\exp\{\frac{-(y_r - (\pm\frac{\sqrt{2}A}{4} \pm \frac{\sqrt{2}A}{4}i))^2}{\frac{N_0}{2T_s}A^2}\} \quad (9)$$

With knowledge of the PDF of y_r, we could design a desired codebook for the quantizer at the relay. Then the relay will compress the signals y_r through this quantizer, and send the compressed signals to the destination.

4. VECTOR CODING DESIGN FOR THE CF PROTOCOL

When implementing the CF protocol, the relay chooses to employ vector quantization (VQ). VQ which is based on the principle of block coding, is desirable for 2D QPSK source signals. Here we recommend two algorithms to perform the vector coding.

4.1 Nearest Neighbour Quantization

An important special class of VQ, called Voronoi or nearest neighbour VQ, has the feature that the codebook is optimal in the sense of minimising average distortion [15]. Its advantage is that the encoding process does not require any explicit storage of the geometrical description of the cells. Assuming a mean squared error (MSE) distortion measure, the condition to identify the nearest neighbour VQ codebook entry could be described as:

$$S_m = \{y_r : \|y_r - c_m\|^2 \le \|y_r - c_{m'}\|^2, \forall m' = 1, 2, ..., M\}, \quad (10)$$

where S_m denotes the encoding region associated with codevector c_m, and M denotes the desired number of codevectors in the codebook which equals to 2^C. This condition says that the encoding region S_m should consists of all vectors that are closer to c_m than any other codevector. Furthermore, for the MSE criterion, the codevector c_m should be average of all those signal vectors that are in the encoding region:

$$c_m = \frac{\sum_{y_r \in S_m} y_r}{\sum_{y_r \in S_m} 1}, m = 1, 2, ..., M. \quad (11)$$

The objective of the relay is to design this kind of codebook, with the knowledge of signal vectors y_r and the desired number of codevectors. Here we propose the LBG algorithm [15] which is based on the iterative use of codebook modification, to design the nearest neighbour VQ. The equations (10) and (11) are the two key steps of the LBG algorithm. In this paper we use the splitting technique where an initial codevector is set as the average of the received signal vectors. This codevector is then split into two. The iterative algorithm is then run with the two codevectors as the initial codebook. We could use the equation (10) to design the partition, and then use equation (11) to update the codebook. The final two codevectors are split into four and the process is repeated until the desired number of codevectors is obtained. Finally the relay node obtains the complete codebook for quantization.

4.2 Hexagonal Lattice Quantization

In contrast to nearest neighbour VQ, which requires exhaustive search algorithm and implies a high computational complexity, lattice VQ has been developed to reduce the complexity of codebook design [16]. The codebook for lattice VQ has a special structure that allows faster encoding, while paying the price that the quantizer is suboptimal for a given set of signal vectors. For lattice VQ, the encoding regions S_m are regular lattices, either rectangular or hexagon. In fact the rectangular lattice VQ has the same performance as employing optimal scalar quantization on each dimension. So considering that a hexagonal covering of the 2D space is more efficient than a rectangular partitioning, the hexagonal lattice quantizer could be an alternative to the vector quantizer.

With knowledge of signal vectors y_r and the number of codevectors, designing a hexagonal lattice VQ is much simpler. We just need to consider the entire covering region for the signal vectors. Design one hexagonal encoding region and then use it to fill the 2D space until the desired number is obtained. We should make sure the hexagonal lattices cover most of the expected signal vectors. The codevector c_m is also obtained according to (11), which is the average of all those signal vectors in the S_m. A more detailed comparison of the codevectors of nearest neighbour VQ and hexagonal lattice VQ can be seen in Figure 5 of Section 5.

The relay employs nearest neighbour VQ or hexagonal lattice VQ to design a codebook, and then forwards the encoded signals and the whole codebook to the destination. We assume the destination could decode the source coded signals correctly, and then it implements a maximum ratio combiner to combine the two received signals from source and relay, and finally makes a decision on the transmitted signal s_0.

5. NUMERICAL RESULTS

In this section, we present the the bit error rate performance of practical CF cooperation protocol for a three terminal classical relay network with QPSK source signals. The simulations are set up in accordance with the assumptions of the channel model in Section II and the analysis about the probability distribution of the relay received signals in Section III. The simulation results are computed via the Monte Carlo method. We assume $r_s = 1$ baud, and the signal-to-noise ratio (SNR) is defined here as E_b/N_0. We use 10^4 training vectors for both nearest neighbour VQ and hexagonal lattice VQ.

The BER performance of decode-forward, optimal scalar quantization (SQ), 2D hexagonal lattice VQ, and 2D nearest neighbour VQ, under different SNR assumptions, are shown in Figure 4. The bit error rates are compared against the lower bound of corresponding SIMO system as if the cooperating nodes were colocated and connected via a wire. With such colocated receivers, the channel becomes an ideal SIMO system with a two-antenna receiver. The bit error rates are also compared to the performance of the system with Shannon coding bound. According to equation (2), the compression noise $n_c(t)$ is considered for this kind of system, with variance calculated via equation (3). The destination then performs MRC of the two received signals as shown in equation (2), and finally makes a decision on the transmitted signal.

Figure 4 shows that, the CF protocol, whether using scalar or vector quantization, is expected to perform better than the DF protocol, because the relay and the destination are close together [7]. The 2D hexagonal lattice VQ, can achieve similar error rates comparable to the optimal one, nearest neighbour VQ. When the SNR is increasing, 2D nearest neighbour VQ offers much more improvement than the 2D optimal SQ, but is bounded by SIMO system.

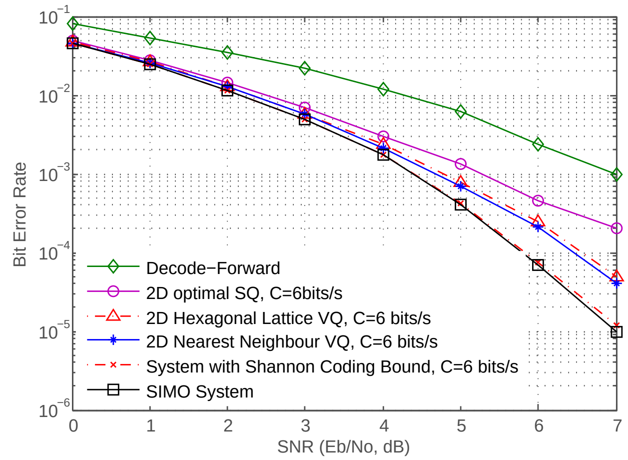

Figure 4: *Error performance of different protocols for the classical relay system with QPSK source signals ($r_s = 1$ baud).*

When the quantization rate C equals to 6 bits/s, we compare the distributions of the codevectors in the codebook for the hexagonal lattice VQ and the nearest neighbour VQ in Figure 5. The codebook for the nearest neighbour VQ is adapted to the received signal vectors, which is optimal in the sense of minimising average distortion. The codebook of hexagonal lattice VQ is designed when $SNR = 0$ dB, which is not changed for different SNRs. Even though the hexagonal lattice VQ is suboptimal for a given set of signal vectors, it is much simpler to design and can achieve similar error performance compared with the optimal VQ.

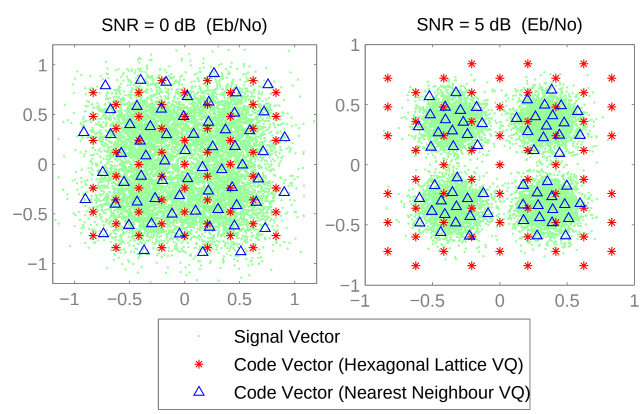

Figure 5: *The codevectors distributions for hexagonal lattice VQ and nearest neighbour VQ, under different SNRs. (QPSK source signals, $r_s = 1$ baud, $C = 6$ bits/s.)*

Considering the quantization rates 4 bits/s, 6 bits/s and 8 bits/s, we compare the performance of the 2D nearest neighbour VQ and 2D hexagonal lattice VQ in Figure 6. We also consider the performance of nearest neighbour VQ when the quantization rate $C = 2$ bits/s, and it is obvious that the CF protocol degrades to the DF case. The relay node then performs as a simple data demodulator. As C increases, both the nearest neighbour VQ and hexagonal lattice VQ perform better and come closer to the lower bound. The 2D hexagonal lattice VQ can achieve error rates comparable to the optimal but more complicated nearest neighbour VQ.

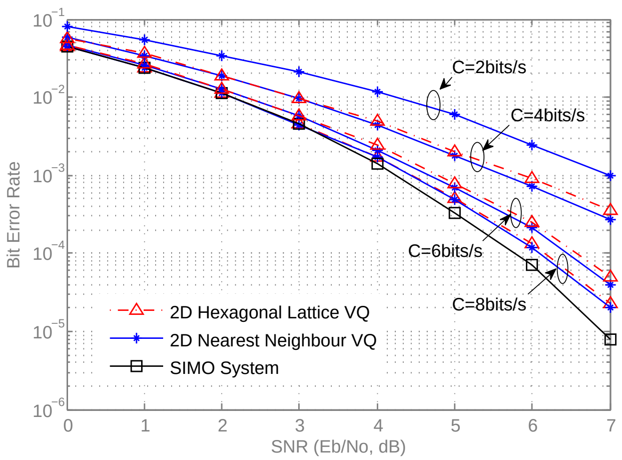

Figure 6: *Error performance of the CF protocol with vector coding at the relay, under different quantization rates. (Classical relay system with QPSK source signals, $r_s = 1$ baud.)*

6. CONCLUSIONS

Considering the classical relay network with one transmitter and two clustered receivers with phase fading channels, we proposed the vector coding technique for compress-and-forward cooperation at the relay node. We presented a framework for the relay receiver, and analysed the probability density function of the received signals. With this knowledge, two codebook design algorithms are recommended for VQ, nearest neighbour VQ and hexagonal lattice VQ.

Furthermore, we investigated the error rate performance of the CF cooperation under different SNRs. The impact of quantization rate at the relay node was also characterised. It was shown that for a quantization rate larger than 2 bits/sample, the vector coding outerformed the DF protocol and scalar coding in the sense of error performance. As for the two design algorithms, we found that the nearest neighbour VQ had the feature that the codebook is optimal, while the hexagonal lattice VQ was much simpler and could achieve similar error rates. Thus the two algorithms are both appropriate choices for designing the vector coding at the relay node in practice.

We note that our research work has focused on the case of a classical relay network; extension to more than three terminals will be included in our further work. Moreover, practical channel coding schemes will be implemented at the source node. A Wyner-Ziv coder, i.e. a source-channel coding scheme, will be considered as well for the relay node.

7. ACKNOWLEDGEMENT

Jing Jiang thanks the support from the UK/China Scholarships for Excellence programme. The authors acknowledge the support of the Scottish Funding Council for the Joint Research Institute with the Heriot-Watt University which is a part of the Edinburgh Research Partnership.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Info. Theory*, vol. 50, no. 10, pp. 3062–3080, Dec. 2004.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part I and Part II," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927-1948, Nov. 2003.

[3] Y. Fan, J.S. Thompson, "MIMO Configurations for Relay Channels: Theory and Practice," *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1774-1786, 2007.

[4] J. N. Laneman and G. W. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity In Wireless Networks," *IEEE Trans. Info. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[5] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.

[6] A. Host-Madsen, "Capacity bounds for cooperative diversity," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1522–1544, Apr. 2006.

[7] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.

[8] A. Wyner and J. Ziv, "The rate-distortion function for source coding with side information at the decoder," *IEEE Trans. Inf. Theory*, vol. IT-22, no. 1, pp. 1–10, Jan. 1976.

[9] A. Nosratinia, T.E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, Vol. 42, no. 10, pp. 74–80, Oct. 2004.

[10] Z. Liu, V. Stankovic, and Z. Xiong, "Wyner-Ziv coding for the half-duplex relay channel," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP '05)*, March 2005, Vol. 5, pp. v/1113–v/1116.

[11] T. Cover and J. Thomas, *Elements of Information Theory*. Address: John Wiley and Sons, New York, 1991

[12] R. Hu, and J. Li, "Practical Compress-Forward in User Cooperation: Wyner-Ziv Cooperation," in *2006 IEEE International Symposium on Information Theory*, Seattle, WA, July 2006, pp. 489–493.

[13] C. T. Ng, N. Jindal, A. J. Goldsmith and U. Mitra, "Capacity gain from two-transmitter and two-receiver cooperation," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3822–3833, Oct. 2007.

[14] C. T. K. Ng, I. Maric, A. J. Goldsmith, S. Shamai, and R. D. Yates, "Iterative and one-shot conferencing in relay channels," in *Proc. IEEE Inform. Theory Workshop*, Uruguay, Mar. 2006, pp. 193–197.

[15] A. Gersho, R. M. Gray, *Vector quantization and. signal compression*. Address: Klower Academic Publisher, Boston, 1992.

[16] R. M. Gray and D. Neuhoff, "Quantization," *IEEE Transactions on Information Theory*, Vol. 44, No 6, pp. 2325–2383, Oct. 1998.