ANALYZING RATE-CONSTRAINED BEAMFORMING SCHEMES IN WIRELESS BINAURAL HEARING AIDS

Sriram Srinivasan and Albertus C. den Brinker

Philips Research
High Tech Campus 36, Eindhoven, The Netherlands
{sriram.srinivasan,bert.den.brinker}@philips.com

ABSTRACT

A binaural hearing aid set-up where the left and right ear devices are connected by a rate-constrained wireless link is considered, and the performance gain resulting from beamforming is quantified. Each device is assumed to have two or more microphones. The transmitting device sends a signal at a rate R to the receiving device where it is combined with the locally available signals to obtain a minimum mean-squared error estimate of the desired signal. Different practically realizable choices for the signal to be transmitted are considered: an estimate of the desired signal, an estimate of the interfering signal (relevant for multi-microphone interference cancellation), and the unprocessed signal. It is not obvious which scheme provides the best rate-gain trade-off. In fact, it is shown that this varies depending on the configuration of the desired and interfering sources. This paper provides a framework to quantify and thus compare the performance of the above-mentioned schemes in terms of the rate R and the resulting beamforming gain.

1. INTRODUCTION

Improving the intelligibility of speech in the presence of interfering talkers and noise is an important goal in the design of hearing aids [1]. Modern hearing aids typically contain multiple microphones to enable directional processing, whereby the speech of a desired talker generally assumed to be located in front of the user is preserved, and interfering talkers in the rear half plane are suppressed. The distance between microphones on a single device is usually less than one cm due to the small size of such devices, which limits the beamforming gain that can be achieved. A higher gain may be obtained by using the microphone signal from both the left and right hearing aids. The larger spacing between the microphones in such a binaural array is especially beneficial in the low frequencies, and when an interfering talker is located in the front half plane.

A wired connection between the left and right hearing aids is undesirable for aesthetic reasons, and a wireless connection is necessary. Wireless transmission of data, however, consumes a high amount of power, which is a scarce resource in hearing aids. It is thus of interest to limit the communication bit-rate. One device, e.g., the right ear device, transmits a signal at a rate R to the other device, in this case the left ear device. At the receiving end, a minimum mean-squared error (MMSE) estimate of the desired signal is obtained using the received signal, and the locally observed microphone signals. The processing is symmetric, and an estimate is obtained at the right ear using the signal transmitted by the left device, and the local right ear microphone signals. Due to symmetry, only one case is considered in the remainder of this paper, where the right ear device is the transmitting device.

There are different choices for the signal that the right ear device should transmit, each of which introduces a different rate-gain trade-off. This problem can be seen as a remote Wyner-Ziv problem [2, 3], where the right device encodes its signals such that the left device obtains the MMSE estimate of the desired signal, with the locally observed signals at the left device as side information. The above solution provides the best rate-gain trade-off, i.e., given a certain communication bit-rate, the beamforming gain achieved by this scheme is the highest. However such a scheme is not practically realizable as it requires knowledge of the joint statistics of left and right observations, which are not available in the hearing aid scenario.

A sub-optimal but practical solution presented in [4] is shown in Fig. 1, where the right device first obtains an estimate of the desired signal using the right ear microphone signals, and then transmits the estimate at rate R. An advantage of this scheme is that as only information regarding the desired signal is transmitted, the required bit-rate can be expected to be lower than when transmitting the unprocessed signal. While this scheme is optimal when the noise signals at the different microphones are spatially uncorrelated, it is sub-optimal in the presence of a localized (point source) interferer, where the noise signals at the different microphones are correlated. From an information point of view, transmitting only an estimate of the desired signal does not convey all the information that could be used in reconstructing the desired signal at the left device. Specifically, lack of information about the interferer in the transmitted signal results in an inability to exploit the larger left-right microphone spacing (provided by the binaural set-up) to cancel the interferer. For example, consider a desired source located at 0° (front of the user) and an interferer located at 45°. The closely spaced endfire array on the hearing aid cannot efficiently cancel an interferer at 45°, whereas the binaural array is more effective in this case. Transmitting only an estimate of the desired signal precludes this advantage. This paper proposes and investigates two practical alternatives to circumvent this problem.

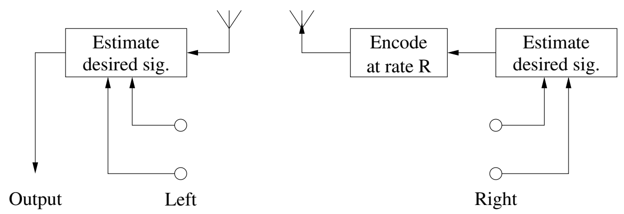

Fig. 1. The scheme of [4]. The desired signal is first estimated from the right ear microphone signals, and then transmitted at a rate R. At the left ear, the desired signal is estimated from the received signal and the local microphone signals.

The first approach is to obtain an estimate of the interference-plus-noise at the right hearing aid using the right ear microphone signals, and transmit this estimate to the left device. This scheme is similar to the one in Fig. 1, except that the signal being estimated at the right ear is the undesired signal. Intuitively, this would enable better performance in the presence of a localized interferer as both the locally available microphone signals and the received signal can be used in the interference cancellation process, and this is indeed observed for certain situations in the simulations described later in the paper. Again, as only information regarding the interferer is transmitted, at a given bit-rate, a better description of the transmitted signal is possible than when transmitting the unprocessed signal.

Following the information point of view one step further leads to the second scheme proposed in this paper, which is to just transmit one of the right ear microphone signals at rate R, as shown in Fig. 2. This signal conveys the most information about both the desired and the undesired signal. However, a potential disadvantage of such a scheme is that the required bit-rate might be higher than when either transmitting an estimate of the desired or interfering signal.

When analyzing the different schemes mentioned above, the following trade-off arises. On the one hand, transmitting an estimate of the desired or interfering signal allows a better description of the transmitted signal using fewer bits compared to transmitting the unprocessed microphone signal where the information content is higher. However, from the information point of view discussed previously, the latter case allows for improved estimation of the desired signal. Deciding which scheme provides the best gain-rate trade-off is not obvious and thus merits further study. This paper provides a framework to quantify the performance of the above-mentioned schemes in terms of the rate R, the location of the desired source and interferer, and the signal-to-interference ratio.

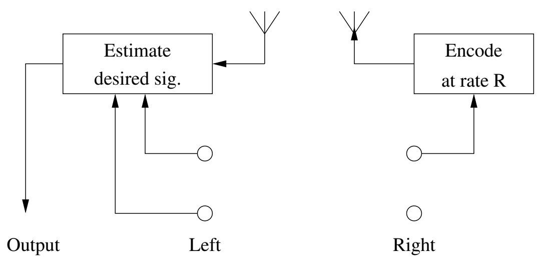

Fig. 2. One right ear microphone signal is transmitted at a rate R. At the left ear, the desired signal is estimated from the received signal and the local microphone signals.

2. SIGNAL MODEL

Consider a desired source $s(n)$ in the presence of an interfering source $i(n)$, and uncorrelated noise. The left ear signal model can be written as

$$x_l^k(n) = h_l^k(n) \star s(n) + g_l^k(n) \star i(n) + u_l^k(n), \quad k = 1 \dots K, \quad (1)$$

where $h_l^k(n)$ and $g_l^k(n)$ are the transfer functions between the k^{th} microphone on the left hearing aid and the desired and interfering sources respectively, $u_l^k(n)$ is uncorrelated zero-mean white Gaussian noise at the k^{th} microphone on the left hearing aid (e.g., sensor noise), K is the number of microphones on the left hearing aid, n is the time index, and the operator $\star$ denotes convolution. The sources are assumed to be zero-mean independent and jointly Gaussian random processes. For the MMSE estimation, it is convenient to view the signal model in the frequency domain:

$$X_l^k(\omega) = H_l^k(\omega)S(\omega) + G_l^k(\omega)I(\omega) + U_l^k(\omega), \quad (2)$$

where the upper case entities in (2) are obtained by applying the discrete Fourier transform to the corresponding lower case entities in (1). A similar right ear model follows:

$$X_r^k(\omega) = H_r^k(\omega)S(\omega) + G_r^k(\omega)I(\omega) + U_r^k(\omega), \quad (3)$$

where the relevant terms are defined analogously to the left ear. Let

$$\begin{aligned} \mathrm{E}\{S(\omega)S^\dagger(\omega)\} &= \Phi_s(\omega), \\ \mathrm{E}\{I(\omega)I^\dagger(\omega)\} &= \Phi_i(\omega), \\ \mathrm{E}\{U_r^m(\omega)U_r^{n\dagger}(\omega)\} &= \mathrm{E}\{U_l^m(\omega)U_l^{n\dagger}(\omega)\} \\ &= \delta(m-n)\Phi_u(\omega)\ \forall m, n, \end{aligned}$$

where $\dagger$ indicates complex conjugate transpose. Define the vectors

$$\begin{aligned} \mathbf{X}_l(\omega) &= [X_l^1(\omega), \dots, X_l^K(\omega)]^T, \\ \mathbf{X}_r(\omega) &= [X_r^1(\omega), \dots, X_r^K(\omega)]^T. \end{aligned}$$

Let $\Phi_{\mathbf{X}_l}(\omega) = \mathrm{E}\{\mathbf{X}_l(\omega)\mathbf{X}_l^\dagger(\omega\}$ and $\Phi_{\mathbf{X}_r}(\omega) = \mathrm{E}\{\mathbf{X}_r(\omega)\mathbf{X}_r^\dagger(\omega\}$. For any $X(\omega)$ and $Y(\omega)$, define $\Phi_{XY}(\omega) = \mathrm{E}\{X(\omega)Y^\dagger(\omega)\}$.

3. RATE-CONSTRAINED BEAMFORMING

Denote the signal transmitted by the right device as $x_t(n)$, and its power spectral density (PSD) by $\Phi_t(\omega)$. The following parametric rate distortion relation holds [5]:

$$\begin{aligned} R(\lambda) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left(0, \log_2 \frac{\Phi_t(\omega)}{\lambda}\right) d\omega \\ D(\lambda) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \min(\lambda, \Phi_t(\omega)) d\omega, \end{aligned} \quad (4)$$

where the rate is expressed in bits per sample. The distortion here is the mean-squared error (MSE) between $x_t(n)$ and its reconstruction. Each value of λ corresponds to an R-D pair. Let the right device compress $x_t(n)$ at a rate R_0 bits per sample, which corresponds to a certain λ_0, and a distortion D_0. The signal received at the left ear can be written in the frequency domain as [5]

$$\tilde{X}_t(\omega) = \eta(\omega)X_t(\omega) + Z(\omega), \quad (5)$$

where

$$\begin{aligned} \eta(\omega) &= \max(0, \beta(\omega)), \\ \beta(\omega) &= \frac{\Phi_t(\omega) - \lambda_0}{\Phi_t(\omega)}, \text{ and} \\ \mathrm{E}\{Z(\omega)Z^\dagger(\omega)\} &= \max(0, \lambda_0\beta(\omega)). \end{aligned} \quad (6)$$

At the left (receiving) ear, estimation is performed using the signal

$$\mathbf{X}(\omega) = [\mathbf{X}_l^T(\omega) \quad \tilde{X}_t(\omega)]^T. \quad (7)$$

Let $\Phi_{\mathbf{X}}(\omega) = \mathrm{E}\{\mathbf{X}(\omega)\mathbf{X}^\dagger(\omega)\}$. An MMSE estimate $\hat{S}_l(\omega)$ of the desired signal $S_l(\omega) = H_l^1(\omega)S(\omega)$ is obtained at the left ear as

$$\hat{S}_l(\omega) = \mathbf{W}(\omega)\mathbf{X}(\omega), \quad (8)$$

where $\mathbf{W}(\omega)$ is the multi-channel Wiener filter given by

$$\mathbf{W}(\omega) = \Phi_{S_l\mathbf{X}}(\omega)\Phi_{\mathbf{X}}^{-1}(\omega), \quad (9)$$

where $\Phi_{S_l\mathbf{X}}(\omega) = \mathrm{E}\{S_l(\omega)\mathbf{X}^\dagger(\omega)\}$. The resulting MSE is given by

$$\xi(\omega) = \Phi_{S_l}(\omega) - \Phi_{S_l\mathbf{X}}(\omega)\Phi_{\mathbf{X}}^{-1}(\omega)\Phi_{\mathbf{X}S_l}(\omega), \quad (10)$$

where $\Phi_{S_l}(\omega) = \mathrm{E}\{S_l(\omega)S_l^\dagger(\omega)\}$. The MSE $\xi(\omega)$ can be rewritten in an intuitively appealing form in terms of the MSE resulting when estimation is performed using only $\mathbf{X}_l$ and a reduction term due to the availability of the innovation process $\tilde{X} = \tilde{X}_t - \mathrm{E}\{\tilde{X}_t|\mathbf{X}_l\}$. The following theorem follows by applying results from linear estimation theory [6, ch. 4]:

Theorem 3.1 *Let $\tilde{X} = \tilde{X}_t - \mathrm{E}\{\tilde{X}_t|\mathbf{X}_l\}$. $\tilde{X}$ represents the innovation or the 'new' information provided by the wireless link. Then, the MSE $\xi(\omega)$ can be written as*

$$\xi(\omega) = \xi_l(\omega) - (\Phi_{S_l}(\omega) - \xi_{lr}(\omega)), \tag{11}$$

where

$$\xi_l(\omega) = \Phi_{S_l}(\omega) - \Phi_{S_l\mathbf{X}_l}(\omega)\Phi_{\mathbf{X}_l}^{-1}(\omega)\Phi_{S_l\mathbf{X}_l}^{\dagger}(\omega) \tag{12}$$

is the error in estimating $S_l(\omega)$ from $\mathbf{X}_l(\omega)$ alone, and

$$\xi_{lr}(\omega) = \Phi_{S_l}(\omega) - \Phi_{S_l\tilde{X}}(\omega)\Phi_{\tilde{X}}^{-1}(\omega)\Phi_{S_l\tilde{X}}^{\dagger}(\omega) \tag{13}$$

is the error in estimating $S_l(\omega)$ from $\tilde{X}(\omega)$.

Theorem 3.1 provides an intuitive understanding of the wireless beamforming scheme. In the absence of a link between the two ears, i.e., when $R = 0$, $\xi_{lr}(\omega) = \Phi_{S_l}(\omega)$, and thus the MSE $\xi(\omega)$ equals $\xi_l(\omega)$ as only the left ear signals are available. For positive rates, $\xi_{lr}(\omega) \leq \Phi_{S_l}(\omega)$ and there is thus a reduction in MSE as seen from (11).

There are two choices for the signal $X_t(\omega)$ corresponding to the two proposed schemes discussed in Section 1, and they are considered in the following subsections.

3.1. Transmitting an estimate of the undesired signal

The first choice for $X_t(\omega)$ corresponds to an estimate of the undesired interference-plus-noise signal, which is given by[1]

$$\begin{aligned} X_t(\omega) &= X_r^1(\omega) - \Phi_{S_l\mathbf{X}_r}(\omega)\Phi_{\mathbf{X}_r}^{-1}(\omega)\mathbf{X}_r(\omega) \\ &= \mathbf{u}^T\mathbf{X}_r(\omega) - \Phi_{S_l\mathbf{X}_r}(\omega)\Phi_{\mathbf{X}_r}^{-1}(\omega)\mathbf{X}_r(\omega) \\ &= A_{\mathrm{int}}(\omega)\mathbf{X}_r(\omega), \end{aligned} \tag{14}$$

where

$$\mathbf{u} = [1, 0, \cdots, 0]_{1\times K}^T, \text{ and}$$
$$A_{\mathrm{int}}(\omega) = \mathbf{u}^T - \Phi_{S_l\mathbf{X}_r}(\omega)\Phi_{\mathbf{X}_r}^{-1}(\omega).$$

The relation between the bit-rate and the resulting MSE in estimating the desired signal at the left ear follows from (4) and (10):

$$\begin{aligned} R_{\mathrm{int}}(\lambda) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left(0, \log_2 \frac{A_{\mathrm{int}}(\omega)\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{int}}^{\dagger}(\omega)}{\lambda}\right) d\omega, \\ \xi_{\mathrm{int}}(\lambda) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_l}(\omega) - \Phi_{S_l\mathbf{X}}(\omega)\Phi_{\mathbf{X}}^{-1}(\omega)\Phi_{\mathbf{X}S_l}(\omega)\, d\omega, \end{aligned} \tag{15}$$

where

$$\begin{aligned} \mathbf{X}(\omega) &= \begin{bmatrix} \mathbf{X}_l(\omega) \\ \max(0, \beta_{\mathrm{int}}(\omega))\, A_{\mathrm{int}}(\omega)\mathbf{X}_r(\omega) + Z_{\mathrm{int}}(\omega) \end{bmatrix}, \\ \beta_{\mathrm{int}}(\omega) &= \frac{A_{\mathrm{int}}(\omega)\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{int}}^{\dagger}(\omega) - \lambda}{A_{\mathrm{int}}(\omega)\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{int}}^{\dagger}(\omega)}, \\ \mathrm{E}\{Z_{\mathrm{int}}(\omega)Z_{\mathrm{int}}^{\dagger}(\omega)\} &= \max(0, \lambda\beta_{\mathrm{int}}(\omega)). \end{aligned} \tag{16}$$

[1]For linear estimation in the additive noise model, estimating the undesired signal is equivalent to subtracting an estimate of the desired signal from the observation.

3.2. Transmitting the unprocessed microphone signal

The second choice for $X_t(\omega)$ corresponds to transmitting one raw unprocessed right ear microphone signal, without loss of generality chosen to be $X_r^1(\omega)$. The rate-MSE equations in this case can be obtained as in the previous case. First let $X_r^1(\omega) = A_{\mathrm{raw}}\mathbf{X}_r(\omega)$ where $A_{\mathrm{raw}} = [1, 0, \cdots, 0]_{1\times K}$. Then,

$$\begin{aligned} R_{\mathrm{raw}}(\lambda) &= \frac{1}{4\pi}\int_{-\infty}^{\infty} \max\left(0, \log_2 \frac{A_{\mathrm{raw}}\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{raw}}^{\dagger}}{\lambda}\right) d\omega, \\ \xi_{\mathrm{raw}}(\lambda) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{S_l}(\omega) - \Phi_{S_l\mathbf{X}}(\omega)\Phi_{\mathbf{X}}^{-1}(\omega)\Phi_{\mathbf{X}S_l}(\omega)\, d\omega, \end{aligned} \tag{17}$$

where

$$\begin{aligned} \mathbf{X}(\omega) &= \begin{bmatrix} \mathbf{X}_l(\omega) \\ \max(0, \beta_{\mathrm{raw}}(\omega))\, A_{\mathrm{raw}}\mathbf{X}_r(\omega) + Z_{\mathrm{raw}}(\omega) \end{bmatrix}, \\ \beta_{\mathrm{raw}}(\omega) &= \frac{A_{\mathrm{raw}}\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{raw}}^{\dagger}(\omega) - \lambda}{A_{\mathrm{raw}}\Phi_{\mathbf{X}_r}(\omega)A_{\mathrm{raw}}^{\dagger}(\omega)}, \\ \mathrm{E}\{Z_{\mathrm{raw}}(\omega)Z_{\mathrm{raw}}^{\dagger}(\omega)\} &= \max(0, \lambda\beta_{\mathrm{raw}}(\omega)). \end{aligned} \tag{18}$$

4. PERFORMANCE ANALYSIS

First, the performance measure used to evaluate the different schemes is introduced. The experimental set-up used for the performance analysis is then described. Two cases are then considered: one where the desired signal is observed in the presence of uncorrelated (e.g., sensor) noise, and the other where the desired signal is observed in the presence of a localized interferer in addition to uncorrelated noise.

4.1. Performance measure

As in [3, 4], the performance gain is defined as the ratio between MSE at rate 0 and MSE at rate R. For example, for the case when an estimate of the undesired signal is transmitted as in Section 3.1, the gain is defined as

$$G_{\mathrm{int}}(R) = 10\log_{10}\frac{\xi_{\mathrm{int}}(0)}{\xi_{\mathrm{int}}(R)}, \tag{19}$$

where $\xi_{\mathrm{int}}(R)$ is the MSE incurred in estimating the desired signal at the left ear when an estimate of the undesired signal is transmitted from the right ear at rate R. $\xi_{\mathrm{int}}(R)$ is obtained from the parametric relation in (15). $G_{\mathrm{int}}(R)$ represents the gain in dB due to the availability of the wireless link. $G_{\mathrm{raw}}(R)$ is computed similarly. The quantities $G_{\mathrm{opt}}(R)$, which denotes the gain from the optimal scheme, and $G_{\mathrm{sig}}(R)$,which denotes the gain when an estimate of the desired signal is transmitted, are obtained for comparisons as in [4].

4.2. Experimental set-up

In the analysis, the number of microphones on each hearing aid was set to two, i.e. $K = 2$. The spherical head shadow model described in [7] was used to obtain the transfer functions $H_l^k(\omega)$, $H_r^k(\omega)$, $G_l^k(\omega)$, and $G_r^k(\omega)$, for $k = 1 \ldots K$. The distance between microphones on a single hearing aid was assumed to be 0.01m. The radius of the sphere was set to 0.0875m. The desired, interfering and noise sources were assumed to have flat PSDs Φ_s, Φ_i, and Φ_u respectively, in the band $[-\Omega, \Omega]$, where $\Omega = 2\pi F$, and $F = 8000$ Hz. Note that $\Phi_t(\omega)$ is not flat due to the non-flat transfer functions.

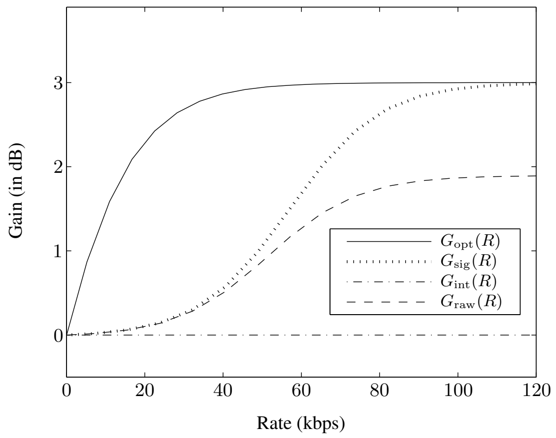

Fig. 3. Performance gain for the three schemes when a desired signal is observed in the presence of uncorrelated noise (i.e. SIR $=\infty$).

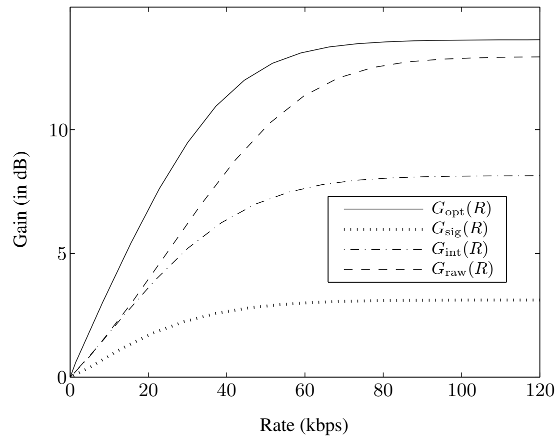

Fig. 4. Performance gain for the different schemes when a desired signal is observed in the presence of uncorrelated noise at 20 dB SNR, and an interfering source at 0 dB SIR located at -30°.

4.3. Desired source in uncorrelated noise

The desired source is assumed to be located at 0° in front of the hearing aid user. This is a common assumption in hearing aids [1]. The signal-to-noise ratio (SNR), computed as $10\log_{10}\Phi_s/\Phi_u$, is assumed to be 20 dB. The signal-to-interference ratio (SIR), computed as $10\log_{10}\Phi_s/\Phi_i$, is assumed to be infinity, i.e., $\Phi_i = 0$.

Fig. 3 plots the gain due to the availability of the wireless link for this case. At high rate, both $G_{\mathrm{opt}}(R)$ and $G_{\mathrm{sig}}(R)$ approach 3 dB as this situation corresponds to a doubling of microphones from two to four, and thus the familiar 3 dB gain in uncorrelated noise. As expected, $G_{\mathrm{opt}}(R)$ provides the upper bound on performance and reaches its limit at a lower rate than $G_{\mathrm{sig}}(R)$. $G_{\mathrm{raw}}(R)$ saturates at a lower value as there are only three microphone signals available for the estimation. Finally, transmitting an estimate of the undesired signal leads to zero gain in this case as the noise is spatially uncorrelated and thus the transmitted signal does not contribute to the estimation of the desired signal at the left ear.

It is to be noted that the performance measure considered in this paper indicates the benefit provided by the wireless link and not the absolute performance gain. In the binaural set-up considered here, where each hearing aid has two microphones, the absolute SNR gain at infinite bit-rate for the optimal scheme is 6 dB. A 3 dB gain in uncorrelated noise is provided by the two-microphone system on the left device. An additional 3 dB gain is provided by the availability of the two right ear signals due to the presence of a wireless link.

4.4. Desired and interfering sources in uncorrelated noise

The behavior of the different schemes in the presence of a localized interferer is of interest in the hearing aid scenario. As before a desired source is assumed to be located at 0°, and the SNR is set to 20 dB. In addition, an interferer is assumed to be located at -30°, and the SIR is set to 0 dB. Fig. 4 compares the four schemes for this case. It is evident from the dotted curve that transmitting an estimate of the desired signal leads to poor performance. Transmitting an estimate of the interferer, interestingly, results in a higher gain as seen from the dash-dot curve, and can be explained as follows. At high rates, the interferer is well preserved in the transmitted signal. Better interference suppression is now possible using the binaural array (larger spacing) than with the closely spaced monaural array, and thus the improved performance.

Transmitting the unprocessed signal results in an even higher gain, $G_{\mathrm{raw}}(R)$, that approaches the gain resulting from the optimal scheme. In this case, not only is better interference rejection possible, but also better estimation of the desired signal as the transmitted signal contains both the desired and undesired signals. $G_{\mathrm{raw}}(R)$ is higher than $G_{\mathrm{sig}}(R)$ and $G_{\mathrm{int}}(R)$ even at low bit-rates. This is significant because as mentioned in Section 1, the gain-rate trade-off achieved by the different schemes is not obvious. A disadvantage when transmitting the unprocessed signal is that bits are used to describe the interferer and the uncorrelated noise at the cost of a better description of the desired signal. However, this pays off as the benefits arising from binaural interference cancellation more than offset the disadvantage.

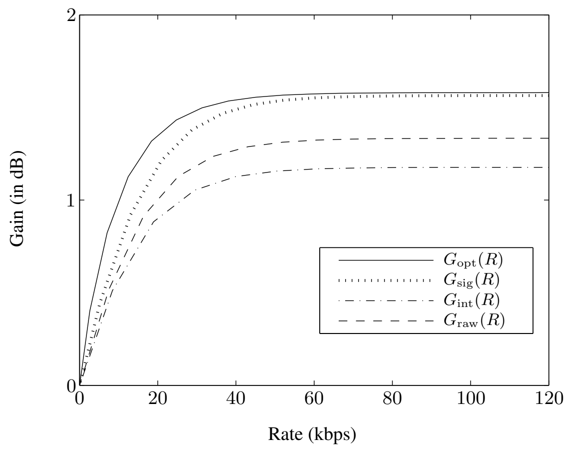

Fig. 5. Performance gain for the different schemes when a desired signal is observed in the presence of uncorrelated noise at 0 dB SNR, and an interfering source at 0 dB SIR located at -30°.

It is to be noted that the level of uncorrrelated noise plays an important role. Here, an SNR of 20 dB has been considered. For this amount of uncorrelated noise, good interference suppression is still possible, and so $G_{\mathrm{raw}}(R)$ is high. At higher SNRs, the difference between $G_{\mathrm{raw}}(R)$ and the other two sub-optimal schemes will be even higher. At low SNRs, the overhead of having to spend bits to describe the uncorrelated noise results in low values of $G_{\mathrm{raw}}(R)$ as seen from Fig. 5. The simulation parameters to obtain Fig. 5 are identical to those of Fig. 4 except that the SNR is 0 dB. However,

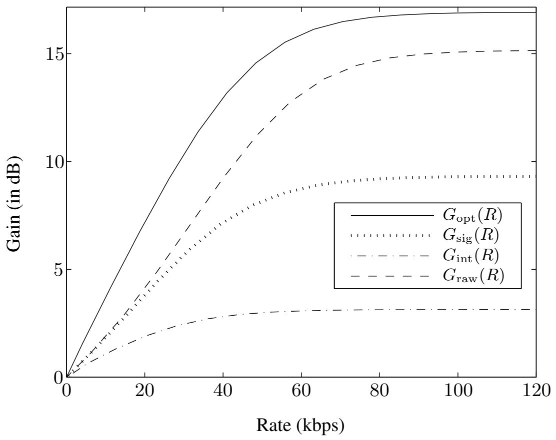

Fig. 6. Performance gain for the different schemes when a desired signal is observed in the presence of uncorrelated noise at 20 dB SNR, and an interfering source at 0 dB SIR located at 30°.

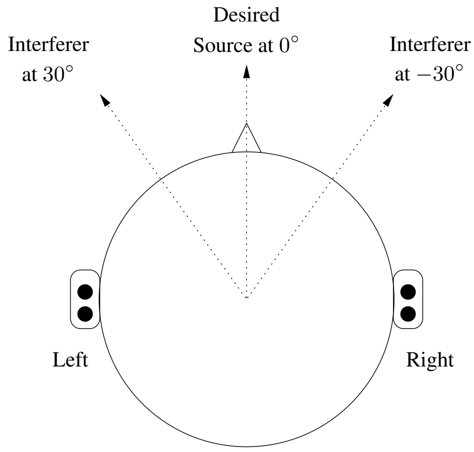

Fig. 7. For an interferer located at 30°, the SIR at the left ear is lower than at the right ear due to head shadow.

the overall gain that can be achieved at such low SNRs is limited, and thus the difference between the different systems becomes less significant. In fact, the maximum gain is lower than the 3 dB gain attained in Fig. 3, where only uncorrelated noise is present, without an interferer.

Fig. 6 considers the case when the interferer is located at 30° instead of -30°, which leads to an interesting result. The behavior of $G_{\mathrm{opt}}(R)$ and $G_{\mathrm{raw}}(R)$ is similar to Fig. 4, but the curves $G_{\mathrm{sig}}(R)$ and $G_{\mathrm{int}}(R)$ appear to be almost inter-changed with respect to Fig. 4. This reversal in performance can be intuitively explained by the head shadow effect. Note that the performance gain is measured at the left ear. When the interferer is located at 30°, the SIR at the left ear is lower than the SIR at the right ear as the interferer is closer to the left ear, and shadowed by the head at the right ear, see Fig. 7. Thus at the right ear, it is possible to obtain a good estimate of the desired signal but not of the interferer. Thus, transmitting an estimate of the desired signal leads to better performance than transmitting an estimate of the interferer. For an interferer located at -30°, the interference-to-signal ratio is higher at the right ear, and thus it is possible to obtain a better estimate of the interferer than possible at the left ear. Transmitting this estimate to the left ear provides information that can be exploited for interference cancellation.

4.5. Discussion

From the above analysis, it can be concluded that a decision on which signal to transmit needs to be made depending on the SIR. At high SIRs (SIR$=\infty$ in the limit, thus only uncorrelated noise) transmitting an estimate of the desired signal is better than transmitting the raw microphone signal. At low SIRs, the converse holds. A simple rule of thumb is to always transmit the unprocessed microphone signal as the penalty at high SIRs is negligible (see Fig. 3) compared to the potential gains at low SIRs (see Figs. 4 and 6). Moreover, a good performance gain is more critical at low SIRs than at high SIRs. In addition, since transmitting the unprocessed signal does not involve signal estimation at the transmitting ear, such a scheme results in a lower computational load and reduced delays.

5. CONCLUSIONS

The theoretical performance of different practical rate-constrained transmission strategies for noise reduction in binaural hearing aids has been studied. The schemes differ in which signal is transmitted from one ear to the other: the first transmits a local estimate of the desired signal, the second transmits a local estimate of the undesired signal, and the third transmits one unprocessed local microphone signal. At the receiving ear, the signal received over the wireless link is combined with the local microphone signals to obtain an MMSE estimate of a desired signal. From the analysis, it is clear that at a given bit-rate R, transmitting the unprocessed microphone signal is a useful practical transmission strategy at practical signal-to-interference ratios. In the presence of a localized interferer, such a scheme provides valuable gains over alternative schemes that include transmitting an estimate of either the desired or undesired signal obtained at the transmitting ear. Furthermore, not having to obtain an estimate before transmission results in a lower computational load, and reduced delay, both of which are critical in hearing aid applications.

6. REFERENCES

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