H_∞ FILTERING FOR AUTOREGRESSIVE MODELING BASED SPACE-TIME ADAPTIVE PROCESSING

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ABSTRACT

Space-Time Adaptive Processing (STAP) is now commonly used in radar engineering to detect the targets by using a phased array antenna system. However, the computational cost of the standard version and the memory storage are high. In addition, the detection could be more robust against interfering targets. To solve the above problems, autoregressive (AR) modelling of the disturbances, namely the sea clutter and the additive thermal noise, leads to a variant of the STAP. In that case, the key issue is the estimations of the multichannel AR process from the secondary data, i.e. the data received when analyzing the "cells" in the neighbourhood of the area under study. Off-line methods have been proposed, but they require a large number of secondary data. To reduce it, on-line method can be considered. Nevertheless, since the clutter has a K-distributed amplitude distribution, the Gaussian assumptions necessary to use Kalman filtering do not hold. To relax them, we suggest investigating the relevance of H_∞ *algorithm in this paper.*

1. INTRODUCTION

The purpose of Radar (RAdio Detection And Ranging) is mainly to detect and to locate targets. When a coherent pulse radar is used, target velocity can be also estimated. Nevertheless, two main problems occur when dealing with radar processing. Firstly, the long-range detection may be difficult due to the $1/R^4$ decay of the reflected power. Secondly, target detection is disturbed by the sea or ground clutter, the jamming and the additive thermal noise.

When phased array antenna system is used, multiple elements of the antenna array provide spatial information whereas pulse repetition periods supply temporal information. The Space-Time Adaptive Processing (STAP), which takes advantage of both domains [1] is now one of the key tools in modern radar processing. For the last years, a great deal of interest has been paid to variants of the STAP to reduce both the computational cost and the memory storage and to make the detection more robust against interfering targets. A few months ago, it should be noted that a special issue was dedicated to "new trends and findings in antenna array processing for radar" in [10].

Thus, by modelling the clutter-plus-noise as a multichannel autoregressive (AR) process [6], [3], [8] and [7], this leads to variants of the STAP known as the Parametric Adaptive Matched Filter (PAMF), the detector proposed by Lombardo [6], the Normalized Parametric Adaptive Matched Filter (NPAMF) and the space-time autoregressive filter (STAR).

In any case, the estimations of multichannel-AR matrices are required. In [6], an off-line maximum likelihood estimation is proposed, but it requires a large number of data. In [9], Schuman uses a Kalman filter to reduce the number of data to be processed and to characterize the clutter-plus-noise. Nevertheless, since sea and ground clutters have a *K*-distributed amplitude distribution [12], Kalman filtering should not be considered since Gaussian assumptions are not satisfied.

In this paper, we propose an H_∞-filtering based approach which avoids any statistical assumption on the model noise in the state space representation. In addition, we suggest distinguishing the influence of the clutter and the additive thermal noise. Indeed, the variance of the additive thermal noise, assumed to be white and Gaussian in the temporal and spatial domains, is known in practical case. Therefore, the clutter alone is modelled as a p^{th} order multichannel autoregressive process whereas the thermal noise is assumed to be an additive zero-mean white noise. This model has hence the advantage of designing a more "suited" clutter rejecter filter. Nevertheless, one has to compensate for the influence of the additive noise when estimating the multichannel-AR parameters. Therefore, the process and its parameters must be jointly estimated. This leads to a non-linear state space representation of the system. To solve this problem, we suggest linearizing the system around a nominal value [2]. More specifically, as this is done in the extended Kalman filter (EKF), the linearization is done around the last available estimate of the state vector. Our on-line method has the advantage of not requiring a large number of secondary data, unlike the maximum likelihood approach in [6].

The remainder of the paper is organized as follows. Section 2 makes it possible to recall the theory behind the STAP and its variants. In section 3, we introduce the model of the secondary data we consider and the way to estimate the model parameters by means of H_∞ filters. Finally, a comparative study with the maximum likelihood in Lombardo's method [6] is carried out and points out the relevance of H_∞ approach when dealing with real data.

2. PROBLEM STATEMENT

Let the radar antenna be a uniformly spaced linear array with N active elements, the separation distance of which is denoted as d. The radar transmits a coherent burst of M

pulses at a constant pulse-repetition frequency f_{pr} .The radar wavelength is denoted as λ . In addition, K range cells are studied. See figure 1.

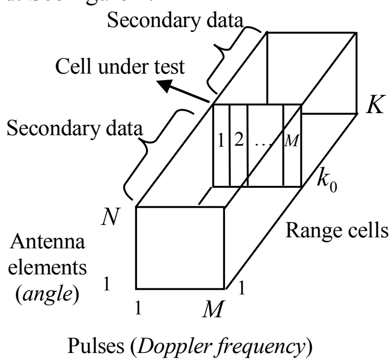

Figure 1: Radar data cube.

In the following, our purpose is to study whether there is a target in the k_0 th direction range or not. For this purpose, let us denote $x_{m,n}^{k_0}$ the radar echo received by the n^{th} antenna element corresponding to the m^{th} pulse.

Let us introduce the $MN \times 1$ input signal, named *snapshot* and denoted as $\underline{x}^{k_0}$. It is defined by:

$$\underline{x}^{k_0} = \left[\underline{x}^{k_0}(1)^T \quad \underline{x}^{k_0}(2)^T \quad \cdots \quad \underline{x}^{k_0}(M)^T \right]^T \qquad (1)$$

where $\underline{x}^{k_0}(m) = \left[x_{m,1}^{k_0} \quad x_{m,2}^{k_0} \quad \cdots \quad x_{m,N}^{k_0} \right]^T$ for $m = 1,...,M$.

The snapshot corresponds [11] to the returns of a possible target (Its Doppler frequency is $f_t^{k_0}$ and its azimuth's angle is $\varphi_t^{k_0}$) and the interferences[1], namely the clutter and the additive thermal noise. Thus, one has:

$$\underline{x}^{k_0} = \alpha^{k_0} \underline{s}(w^{k_0}, v^{k_0}) + \underline{c}^{k_0} + \underline{n} \qquad (2)$$

where α^{k_0} denotes the complex target attenuation factor, $\underline{c}^{k_0}$ denotes the clutter returns, $\underline{n}$ is the additive noise, which is spatially and temporally white with covariance matrix Σ_n and $\underline{s}(w^{k_0}, v^{k_0})$ is the target steering vector, associated with the spatial and Doppler parameters, denoted $w^{k_0} = d / \lambda \sin(\varphi_t^{k_0})$ and $v^{k_0} = f_t^{k_0} / f_{pr}$ respectively.

It should be noted that the target steering vector satisfies:

$$\underline{s}(w^{k_0}, v^{k_0}) = \underline{b}(w^{k_0}) \otimes \underline{a}(v^{k_0}) = \underline{b}(w^{k_0}) \otimes \left[\underline{a}_1 \ \underline{a}_2 \ \ldots \ \underline{a}_M \right]$$
$$= \left[1 \ e^{j2\pi w^{k_0}} \ \cdots \ e^{j2\pi(N-1)w^{k_0}} \right]^T \otimes \left[1 \ e^{j2\pi v^{k_0}} \ \cdots \ e^{j2\pi(M-1)v^{k_0}} \right]^T \qquad (3)$$

where $\otimes$ denotes the Kronecker product.

To maximize the signal-to-interference-plus-noise ratio (SINR), the space time filter weights are defined by:

$$\underline{w}_{opt}^{k_0} = \kappa R^{-1} \underline{s}(w^{k_0}, v^{k_0}) \qquad (4)$$

where κ denotes a constant gain and R is the $NM \times NM$ interference covariance matrix, which is unknown and hence needs to be estimated.

The sample matrix inversion (SMI) [1] consists in estimating R by using the secondary data, i.e. $y^k = \left\{ \underline{x}^k \right\}_{k \in [1,K], k \neq k_0}$. The higher the number of secondary data is, better the estimation of the matrix R should be, provided that the clutter is stationary[2]. Nevertheless, there are three main drawbacks:

1. the computational cost and the memory storage are prohibitive;
2. the risk of potential targets in the secondary data increases when the number of secondary data increases.
3. when there is a large-dynamic range signal, numerical instability may happen when R is inverted.

To compensate for the above problems, alternative approaches have been proposed, such as Knowledge-aided STAP, the higher-order methods and deterministic (single shot) techniques [3]. In this paper, we focus our attention on minimal sample support methods. This latter includes three kinds of approaches:

1. element-space/beam-space, pre/post-Doppler STAP [11].
2. subspace techniques such as those proposed in [4]: as the interference covariance matrix is usually rank deficient, the eigenvectors corresponding to the predominant eigenvalues span the interference subspace, whereas the remaining eigenvectors span the noise subspace. Then, a projection into the interference-free subspace can be done. This leads to the eigencanceller method (EC), whose weight vector is orthogonal to the interference subspace.
3. approaches based on a multichannel autoregressive modelling of the disturbances (clutter+noise). Thus, in [9], the AR parameters are obtained with a Kalman filter. At that stage, they are used in the NPAMF [7]. In a Ground Moving Target Indication simulation, the authors notice that the secondary data required is much smaller than $N \times M$.

In [6], for a given range cell k , Lombardo proposes to model the clutter+noise returns of the m^{th} pulse as follows:

$$\underline{y}^k(m) = -\sum_{i=1}^{p} A^i \underline{y}^k(m-i) + \underline{u}(m) \qquad (5)$$

where $\left\{ A^i \right\}_{i=1,..,p}$ are the AR matrix parameters of size $N \times N$ and $\underline{u}(m)$ is a $N \times 1$ zero-mean white noise vector. Its correlation matrix is denoted Σ_u and satisfies:

$$\Sigma_u = diag\left(\left[\sigma_{u,1}^2 \quad \cdots \quad \sigma_{u,N}^2 \right] \right) \qquad (6)$$

Then, the adaptive weight matrix w_{AR} is defined by:

$$w_{AR} = H^{T*} \Sigma_u^{-1} H S(w^{k_0}, v^{k_0}) \qquad (7)$$

where $H = \left[A^p \ A^{p-1} \ \ldots \ A^1 \ I_N \right]$ and S is the steering matrix deduced from the steering vector $\underline{s}(w^{k_0}, v^{k_0})$:

$$S = \begin{bmatrix} \underline{s}(\underline{b}, \underline{a}_1) & \underline{s}(\underline{b}, \underline{a}_2) & \cdots & \underline{s}(\underline{b}, \underline{a}_{M-p}) \\ \underline{s}(\underline{b}, \underline{a}_2) & \underline{s}(\underline{b}, \underline{a}_3) & \cdots & \underline{s}(\underline{b}, \underline{a}_{M-p+1}) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{s}(\underline{b}, \underline{a}_{p+1}) & \underline{s}(\underline{b}, \underline{a}_{p+2}) & \cdots & \underline{s}(\underline{b}, \underline{a}_M) \end{bmatrix}$$

1 In this paper, it should be noted that the jammer is not studied.

2 When a small number of secondary data is considered, one has to design robust detector that takes into account the influence of potential interfering targets in the secondary data. In [6], Lombardo suggests various strategies such as the median test output (MTO).

At that stage, target detection consists in:

1/ evaluating $q(X^{k_0}) = \sum_{m=1}^{M-p} w_{AR}^{T*}(m) X^{k_0}(m)$

$$= \sum_{m=1}^{M-p} e^{-j2\pi\nu^{k_0}(m-1)} w_{AR}^{T*}(1) X^{k_0}(m)$$

$$\text{with } X^{k_0} = \begin{bmatrix} \underline{x}^{k_0}(1) & \underline{x}^{k_0}(2) & \cdots & \underline{x}^{k_0}(M-p) \\ \underline{x}^{k_0}(2) & \underline{x}^{k_0}(3) & \cdots & \underline{x}^{k_0}(M-p+1) \\ \vdots & \vdots & \ddots & \vdots \\ \underline{x}^{k_0}(p+1) & \underline{x}^{k_0}(p+2) & \cdots & \underline{x}^{k_0}(M) \end{bmatrix} \quad (8)$$

$w_{AR}^{T*}(m)$ and $X^{k_0}(n)$ denote the m^{th} column of w_{AR}^{T*} and the n^{th} column of X^{k_0}.

2/ comparing $q(X^{k_0})$ to a threshold η [6].

As the matrix $\left\{A^i\right\}_{i=1,..,p}$ and Σ_u are unknown, they have to be estimated. In [3], this is done by using a generalized version of the Square-Root Normalized Maximum Entropy Method (SRN-MEM). In [6], the authors suggest using the maximum likelihood estimates which satisfies:

$$\hat{A} = \left[\hat{A}^1 \; \hat{A}^2 \; ... \; \hat{A}^p\right] = -\hat{R}_{Y,01}^{*} \hat{R}_{Y,00}^{-1} \quad (9)$$

$$\hat{\Sigma}_u = \frac{1}{K(M-p)} (\hat{R}_{Y,11} - \hat{R}_{Y,01}^{*} \hat{R}_{Y,00}^{-1} \hat{R}_{Y,01}) \quad (10)$$

For this purpose, the secondary data are stored as follows:

$$Y = \left[Y^1 \; Y^2 \cdots Y^{k_0-1} \; Y^{k_0+1} \cdots Y^K\right] \quad (11)$$

and the estimation of interference covariance matrix $\hat{R}_Y$ is partitioned as follows:

$$\hat{R}_Y = YY' = \begin{bmatrix} \overbrace{\hat{R}_{Y,00}}^{Np} & \overbrace{\hat{R}_{Y,01}}^{N} \\ \hat{R}_{Y,10}^{*} & \hat{R}_{Y,11} \end{bmatrix} \begin{matrix} \} Np \\ \} N \end{matrix} \quad (12)$$

Nevertheless, this method requires a large number of secondary data. Therefore, we propose an alternative approach.

3. ON-LINE NOISE COMPENSATED METHODS

A. Problem statement

As mentioned in the introduction, we suggest modelling the clutter+noise as a noisy *N*-AR process. Thus, one has:

$$\underline{c}^k(m) = -\sum_{i=1}^{p} A_c^i \underline{c}^k(m-i) + \underline{u}_c(m) \quad (13)$$

where $\underline{u}_c(m)$ is a $N \times 1$ zero-mean white noise vector. Its correlation matrix is denoted Σ_{u_c} and satisfies:

$$\Sigma_{u_c} = diag\left(\left[\sigma_{u_c,1}^2 \quad \cdots \quad \sigma_{u_c,N}^2\right]\right) \quad (14)$$

Without target, the secondary data $\underline{y}^{k_0}(m)$ are hence modelled as follows:

$$\underline{y}^k(m) = \underline{c}^k(m) + \underline{n}(m) \quad (15)$$

If the thermal noise covariance matrix Σ_n is available, we can use on-line noise compensated methods to estimate $\left\{A_c^i\right\}_{i=1,..,p}$ and then to apply STAP based AR-filter [6].

When starting the on-line estimation with the first cell, any initial condition can be considered. Then, the last estimates of the model parameters obtained with this cell serve as initial condition for the on-line estimation based on the next cell, and so on.

B. State space representation of the system

Let us first have a look on the state space representation of the system (13)(15).For this purpose, let us consider the column vector $\underline{c}_p(m)$ of size Np defined by:

$$\underline{c}_p(m) = \left[\underline{c}^k(m)^T \quad \cdots \quad \underline{c}^k(m-p+1)^T\right]^T \quad (16)$$

Equation (13) can hence be rewritten as follows:

$$\underline{c}_p(m) = F(m,m-1)\underline{c}_p(m-1) + \left[\underline{u}_c(m)^T \quad 0_{1\times N} \quad \cdots \quad 0_{1\times N}\right]^T$$
$$= F(m,m-1)\underline{c}_p(m-1) + \underline{u}_{c0pad}(m) \quad (17)$$

$$\text{with } F(m,m-1) = \begin{bmatrix} -A_c^1 & \cdots & \cdots & -A_c^{p-1} & -A_c^p \\ I_N & 0_N & \cdots & 0_N & 0_N \\ 0_N & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0_N & \vdots \\ 0_N & \cdots & 0_N & I_N & 0_N \end{bmatrix} \quad (18)$$

In addition, the observation equation (15) can be expressed in a matrix form, as follows:

$$\underline{y}^k(m) = G\,\underline{c}_p(m) + \underline{n}(m) \quad (19)$$

with $G = [I_N \; \underbrace{0_N \;\; \cdots \;\; 0_N}_{p-1}]$, The covariance matrix of $\underline{n}(m)$ is assumed to be equal to Σ_n.

Nevertheless, the coefficients of the AR-parameter matrix A_c^l, namely $\left\{a_{n1,n2}^l\right\}_{\substack{l=1,...,p \\ n1=1,...,N \\ n2=1,...,N}}$, are unknown and hence need to be estimated. Therefore, let us assume that the column vector $\underline{\theta}(m)$ of size $N^2 p$ is defined by

$$\begin{aligned} \underline{\theta}(m) = [&a_{11}^1(m) \cdots a_{1N}^1(m) \cdots a_{11}^p(m) \cdots a_{1N}^p(m) \ldots \\ &a_{21}^1(m) \cdots a_{2N}^1(m) \cdots a_{21}^p(m) \cdots a_{2N}^p(m) \ldots \\ &a_{N1}^1(m) \cdots a_{NN}^1(m) \cdots a_{N1}^p(m) \cdots a_{NN}^p(m)]^T. \end{aligned} \quad (20)$$

and satisfies in the general case:

$$\underline{\theta}(m) = \underline{\theta}(m-1) + \underline{w}(m) \quad (21)$$

where $\underline{w}(m)$ is a $N^2 p \times 1$ zero-mean white noise vector whose autocorrelation matrix is denoted Σ_w.

To estimate both the process vector and the *N*-AR parameter matrices, the extended state vector $\underline{z}(m)$ has to be considered. It is defined by:

$$\underline{z}(m) = \left[\underline{\theta}(m)^T \quad \underline{c}_p(m)^T\right]^T \quad (22)$$

Given (17), (21) and (22), $\underline{z}(m)$ is updated as follows:

$$\underline{z}(m) = \begin{bmatrix} I_{N^2p} & 0_{N^2p\times Np} \\ 0_{Np\times N^2p} & F(m,m-1) \end{bmatrix} \underline{z}(m-1) + \underline{\Omega}(m)$$

$$\underline{z}(m) = B\underline{z}(m-1) + \sum_{n=1}^{N} \underline{H}_n \underline{z}(m-1)^T C_n \underline{z}(m-1) + \underline{\Omega}(m) = \varphi(\underline{z}(m-1); m, m-1) + \underline{\Omega}(m) \quad (23)$$

with $B = \begin{bmatrix} I_{N^2p} & 0_{N^2p\times N(p-1)} & 0_{N^2p\times N} \\ 0_{N\times N^2p} & 0_{N\times N(p-1)} & 0_N \\ 0_{N(p-1)\times N^2p} & I_{N(p-1)} & 0_{N(p-1)\times N} \end{bmatrix}$,

$\underline{\Omega}(m) = \begin{bmatrix} \underline{w}(m) \\ \underline{u}_{c0pad}(m) \end{bmatrix}$, $\underline{H}_n = [\underbrace{0\cdots0}_{N^2p+n-1}\ 1\underbrace{0\cdots0}_{Np-n}]^T$ and

$$C_n = \begin{bmatrix} 0_{pN} & \cdots & 0_{pN} & 0_{(n-1)pN\times pN} \\ \vdots & \ddots & \vdots & I_{pN} \\ 0_{pN} & \cdots & 0_{pN} & 0_{N^2p-(n-1)pN\times pN} \end{bmatrix}, \forall n \in [1, N].$$

In addition, given (15) and (22), the extended state vector and the observation vector are related by:

$$\underline{y}^k(m) = H\,\underline{z}(m) + \underline{n}(m) \quad (24)$$

with $H = \begin{bmatrix} 0_{N\times N^2p} & G \end{bmatrix}$.

The state-space representation (23)-(24) of the system (13),(15)where the AR parameters are unknown is hence non-linear. When dealing with H_∞ filter, we focus on a linear combination of the extended state vector $\underline{z}(m)$:

$$L\underline{z}(m) = \begin{bmatrix} 0_{1\times N^2p} & \underline{c}^k(m)^T & 0_{1\times N(p-1)} \end{bmatrix}^T \quad (25)$$

where $L = diag\begin{bmatrix} 0_{1\times N^2p} & \underbrace{1 \ldots 1}_{N} & 0_{1\times N(p-1)} \end{bmatrix}$.

Given the state-representation (23)-(25), the H_∞ filter aims at minimising the criterion J_∞ :

$$J_\infty = \sup_{\underline{\Omega}(m),\underline{n}(m),\underline{e}(0)} \frac{\sum_m \underline{e}(m)^{T*}\underline{e}(m)}{\sum_m \left[\underline{\Omega}(m)^{T*} Q^{-1} \underline{\Omega}(m) + \underline{n}(m)^{T*} \Sigma_n^{-1} \underline{n}(m)\right]} \quad (26)$$

where $\underline{e}(m) = L(\hat{\underline{z}}(m) - \underline{z}(m))$ and $\hat{\underline{z}}(m)$ is the estimate of $\underline{z}(m)$. The matrices Σ_n and Q are weighting matrices, tuned by the practitioner.

According to Hassibi and al. [5], as a closed-form solution to the above optimal H_∞ estimation problem does not always exist, the following suboptimal design strategy is usually considered:

$$J_\infty < \gamma^2 \quad (27)$$

where $\gamma > 0$ is a prescribed level of disturbance attenuation.

In the following, let us denote $\hat{\underline{z}}(m/l)$ the estimation of the state vector at the time m based on l observations $\left\{\underline{y}^{(i)}\right\}_{i=1,\ldots,l}$.

Equation (23) is not linear and the standard H_∞ filter cannot be considered. To solve this problem, we use the same linearization used in the EKF, based on the 1st-order Taylor expansion of φ around the last available estimate of the state vector, namely $\hat{\underline{z}}(m-1/m-1)$.

C. Recursive solution of H_∞ filter.

When H_∞ filter is used, the *a priori* estimation of the state vector is obtained from the *a priori* estimate $\hat{\underline{z}}(m-1/m-1)$ by using (23):

$$\hat{\underline{z}}(m/m-1) = \varphi(\hat{\underline{z}}(m-1/m-1), m, m-1) \quad (28)$$

There is an H_∞ estimator for a given $\gamma > 0$ if there exists a stabilizing symmetric positive definite solution $P(m/m) > 0$ to the following Riccati-type equation:

$$P(m/m-1) = \Phi(\hat{\underline{z}}(m-1/m-1); m, m-1) P(m-1/m-1) \Phi^T(\hat{\underline{z}}(m-1/m-1); m, m-1) + Q \quad (29)$$

and

$$P(m/m) = P(m/m-1) - P(m/m-1)\begin{bmatrix} H^T & L^T \end{bmatrix} M^{-1} \begin{bmatrix} H \\ L \end{bmatrix} P(m/m-1) \quad (30)$$

where $M = \begin{bmatrix} \Sigma_n & 0 \\ 0 & -\gamma^2 I_{p(N^2+N)} \end{bmatrix} + \begin{bmatrix} H \\ L \end{bmatrix} P(m/m-1) \begin{bmatrix} H^T & L^T \end{bmatrix}$

$$\Phi(\hat{\underline{z}}(m-1/m-1); m, m-1) = \left.\frac{\partial \varphi(\underline{z}(m))}{\partial \underline{z}(m)}\right|_{\underline{z}(m)=\hat{\underline{z}}(m-1/m-1)} = B + \sum_{n=1}^{N} \underline{H}_n \underline{z}(m)^T (C_n + C_n^T)\Big|_{\underline{z}(m)=\hat{\underline{z}}(m-1/m-1)}$$

The gain $K(m)$ of the H_∞ filter is defined as follows:

$$K(m) = P(m/m-1)H^T\left[HP(m/m-1)H^T + \Sigma_n\right]^{-1} \quad (31)$$

It is the state vector is updated as follows:

$$\hat{\underline{z}}(m/m) = \hat{\underline{z}}(m/m-1) + K(m)\left[\underline{y}^k(m) - H\hat{\underline{z}}(m/m-1)\right] \quad (32)$$

4. SIMULATIONS

We have carried out various simulation tests. The radar works in X bandwidth and on sidelooking configuration. It has N=4 antennas and provides M=64 coherent pulses. K=410 range cells are available and there are at most three targets (angle $\varphi = 0$ deg) in the data cube, defined by:

- Target 1: range cell $k_1 = 216$, speed $v_1 = 4m/s$;
- Target 2: range cell $k_2 = 256$, speed $v_2 = 4m/s$;
- Target 3: range cell $k_3 = 296$, speed $v_3 = -4m/s$.

The signal to clutter ratio (SCR) is set to -5dB and the clutter noise ratio (CNR) is 20 dB. In all this section, figures described the output Z of each filter estimated as follows:

$$Z(w,v)=\frac{\left|Tr(S(w,v)^{T*}H^{T*}\Sigma_u HX)\right|^2}{Tr(S(w,v)^{T*}H^{T*}\Sigma_u HS(w,v))} \qquad (33)$$

In fig. 2, one provides the results obtained in the range cell k_2, with the identity SMI, i.e. $\hat{R}=I_{NM}$. The target and the clutter can be identified in the angle-Doppler domain. Thus, the clutter disturbs the detection and increases the probability of false alarms.

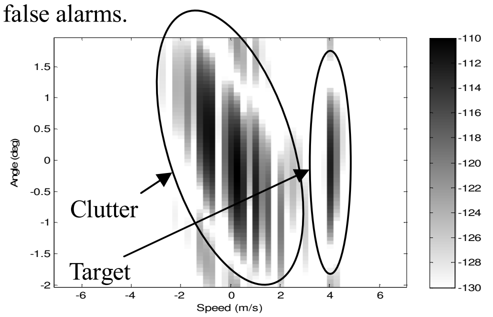

Figure 2: Output Z (dB) after SMI where $\hat{R}=I_{NM}$.

In the following, K=4 range cells are used around k_2. Fig. 3 is obtained with angle set to zero. We apply the method proposed by Lombardo [6] and use two methods to estimate the AR parameters: the maximum likelihood estimation proposed in [6] which is an off-line approach and the H_∞ filter.

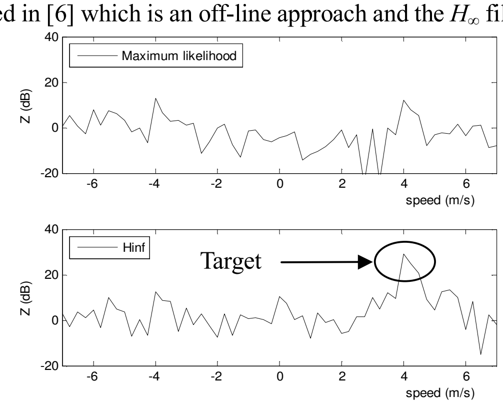

Figure 3: Output Z with Lombardo's method and H_∞ filter and $K=4$.

Unlike the maximum likelihood approach, with K =4, the H_∞ filter makes it possible to detect the target.

When K is chosen higher, targets 2 and 3 have more chance to be in the secondary data and hence may disturb the clutter rejection. To avoid it, Lombardo [6] uses the MTO. This technique consists in dividing secondary data in five groups and in searching the one which gives the best clutter rejection. Thus, a scaling factor $\mu(Y)$ is defined for each one:

$$\mu_i(Y)=\frac{5}{K}\sum_{k=1}^{K/5}\frac{\left|Tr(S^{T*}H^{T*}\Sigma_u HY^k)\right|^2}{Tr(S^{T*}H^{T*}\Sigma_u HS)},\ i\in[1,5]. \qquad (34)$$

In our case, the MTO is described in fig. 4, with $T_i(w,v)=\frac{Z(w,v)}{\mu_i(w,v)}$ and K=20 for each group. The median value of T_i is then conserved. In our simulation protocol, since T_3 provides the median value, we present the corresponding results. They are detailed in fig. 5.

5. CONCLUSIONS

Due to the K-distributed clutter, using algorithm such as H_∞ has the advantage of avoiding Gaussian assumptions. We show the relevance of this method with few secondary data. In addition, combining H_∞ with MTO makes the detection more robust against contamination of secondary data. We are currently investigating how to address CFAR properties in that kind of approaches.

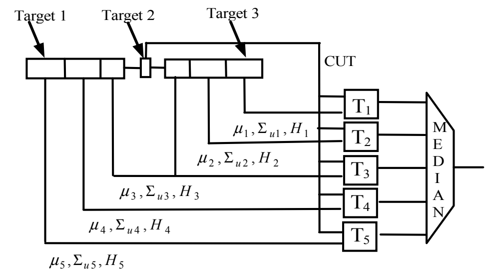

Figure 4: Principle of MTO

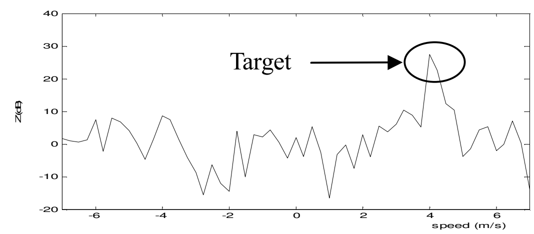

Figure 5: Output Z of MTO with H_∞ filter.

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