ROBUST BEAMFORMING WITH COMBINED WORST-CASE PERFORMANCE OPTIMIZATION AND SOFT CONSTRAINTS IN A MULTIPATH ENVIRONMENT

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ABSTRACT

A novel robust beamforming scheme for the processing of coherent signals in a multipath environment is proposed, where the worst-case performance optimization and the soft response constraint are combined together to suppress the multipath signals as well as the uncorrelated interferences. The proposed scheme is also robust against the direction-of-arrival (DOA) estimation errors. Simulation results verify its effectiveness.

1. INTRODUCTION

In multipath environments, signals travelling along different paths can be considered as coherent with the original source signal if their relative delays are much smaller than the reciprocal of the signal bandwidth. Conventional adaptive beamforming methods, which assume uncorrelated signal sources, suffer from signal cancellation in the presence of coherent signals [1].

Recently some robust adaptive beamformers [2][3][4][5] have been proposed to improve the robustness against steering vector mismatch. In these methods it is assumed that the actual steering vector of the signal of interest (SOI) belongs to a sphere or ellipsoid uncertainty set while the steering vectors of the interferences do not belong to the same set. Then by forcing the magnitude responses of all the steering vectors in such an uncertainty set to exceed unity while minimizing the output power, the gain of the SOI is kept at a certain level while the interferences and noises are suppressed. In [6], the worst-case performance optimization approach is extended to the scenario of a general multipath environment, where a steering vector set is constructed to include all the possible steering vectors of the SOI and its multipath signals. The magnitude responses toward all the steering vectors within this set are forced to be greater than unity when minimizing the output power of the beamformer. Since the beamformer has a relatively fixed response to each of the multipath signals, the SOI will not be cancelled completely.

Soft response constraint is another approach for robust beamforming. Based on the idea of minimizing the mean-squared error between the desired response and the actual response in the signal direction area, the width of the main beam can be specified and better interference cancellation can be obtained [7][8].

In this paper, we propose a novel robust beamformer for the multipath environment by combining the worst-case performance optimization and the method of soft constraints. In this approach, a spherical uncertainty set is constructed to include all the possible steering vectors in the main SOI direction, and the magnitude response at the worst-case within this set is constrained to be no less than unity. On the other hand, the multipath signals are nulled by using soft constraints to force their magnitude responses to be less than a very small level. In this way, signal cancellation caused by coherent signals can be prevented and robustness against DOA estimation error can be achieved.

The rest of this paper is organized as follows. In Section 2, the signal model for the multipath environment is introduced and the robust beamforming method in [6] is briefly reviewed. The proposed robust beamformer with a combination of both worst-case performance optimization and soft constraints is introduced in Section 3. Simulation results and performance comparisons are given in Section 4. Conclusions are drown in Section 5.

2. BACKGROUND

Consider a narrowband beamformer with a uniform linear array with M omnidirectional sensors as shown in Fig. 1. P uncorrelated narrowband input signals, denoted as $s_p(t)$, impinge on the array from DOAs θ_p $(p = 0, \ldots, P-1)$. Assume $s_0(t)$ is the SOI which has Q multipath signals from directions $\tilde{\theta}_q$ $(q = 1, \ldots, Q)$. Each multipath signal is modeled as a scaled and phase shifted version of the SOI, i.e.,

$$r_q(t) = \rho_q e^{-j\phi_q} s_0(t) \tag{1}$$

where $r_q(t)$, ρ_q, and ϕ_q denote the qth multipath signal, its magnitude attenuation factor and phase shift caused by multipath delays, respectively.

The beamformer output is given by

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{2}$$

where $\mathbf{w}$ is an $M \times 1$ complex weight vector, $\{\}^H$ stands for the Hermitian transpose, and $\mathbf{x}(t)$ is the observed array signal vector given by

$$\mathbf{x}(t) = \mathbf{a}(\theta_0)s_0(t) + \sum_{q=1}^{Q} \mathbf{a}(\tilde{\theta}_q) r_q(t) + \sum_{p=1}^{P-1} \mathbf{a}(\theta_p) s_p(t) + \mathbf{n}(t) \tag{3}$$

where $\mathbf{n}(t)$ is the spatially white noise vector uncorrelated with the SOI and interferences, and $\mathbf{a}(\theta)$ is the steering vector for the signal from DOA θ.

We assume the DOAs of the SOI and its multipath signals are estimated in advance [9, 10, 11]. The pre-estimated DOAs are denoted as $\bar{\theta}_i$ $(i = 0, 1, \cdots, Q)$. The DOA estimation errors are assumed to be less than μ°. Therefore, the actual DOAs of the SOI and its multipath signals can be modeled as belonging to a group of clusters, i.e., DOA regions, centred at $\bar{\theta}_i$ with a radius μ°. A brief geometry of 3 clusters is shown in Fig. 2.

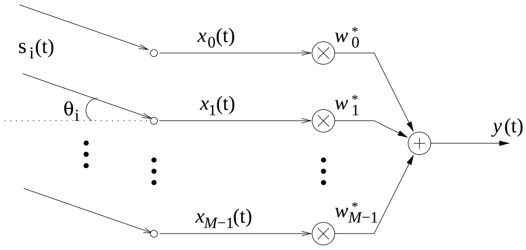

Figure 1: Beamformer with an M-sensor linear array.

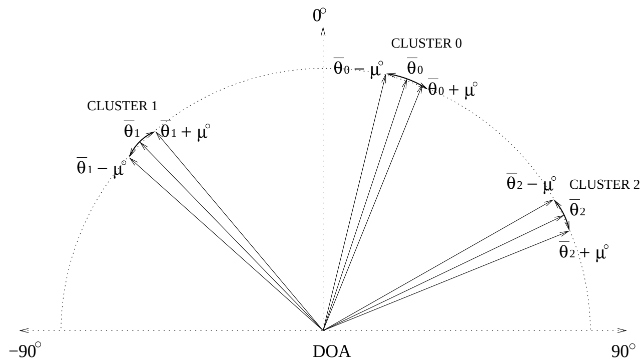

Figure 2: Geometry of the clusters.

In the method in [6], the actual steering vectors of the SOI and its multipath signals are assumed to belong to the following steering vector set

$$\mathscr{A} = \{\tilde{\mathbf{a}} \,|\, \tilde{\mathbf{a}} = \mathbf{a}(\bar{\theta}) + \mathbf{e}, \|\mathbf{e}\| \le \varepsilon, \bar{\theta} \in [\bar{\theta}_0, \cdots, \bar{\theta}_Q]\} \quad (4)$$

where $\tilde{\mathbf{a}}$ is the actual steering vector, $\mathbf{e}$ is a complex vector that describes the effect of steering vector distortion, $\|\ \|$ denotes the Euclidean norm operation and ε is the bound of $\mathbf{e}$.

The formulation of the robust beamformer is written as the following constrained minimization problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad |\mathbf{w}^H \tilde{\mathbf{a}}| \ge 1, \ \forall \tilde{\mathbf{a}} \in \mathscr{A} \quad (5)$$

where $\mathbf{R}$ is the sample covariance matrix of $\mathbf{x}$, which is given by

$$\mathbf{R} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{x}[n] \mathbf{x}^H[n] \quad (6)$$

with N being the number of training snapshots.

In this approach, the magnitude response of the SOI and its multipath signals is constrained to be greater than unity. As a result, the response of the beamformer to all of these coherent signals, i.e. the SOI and its multipath signals, will be relatively fixed and it is very unlikely that they will cancel each other completely at the output. In this way, when the beamformer output variance is minimized, the interferences will be suppressed, and the SOI will be preserved to some degree.

3. THE PROPOSED ROBUST BEAMFORMER

In this section, we propose a new robust beamforming scheme for the multipath environment. In the main SOI cluster, the worst-case performance optimization is used to keep the SOI undistorted; in the remaining clusters, soft constraints are employed to null the multipath signals.

3.1 Worst-Case Constraint on the Main SOI Cluster

We denote the actual steering vector of the SOI in the main cluster as $\mathbf{b}$, which belongs to a spherical set $\mathscr{B}$. Then we force the magnitude of the array response for this particular steering vector $\mathbf{b}$ that corresponds to the smallest value of $\left|\mathbf{w}^H \mathbf{b}\right|$, i.e., the worst-case, to be greater than one,

$$\begin{aligned} &\min_{\mathbf{b} \in \mathscr{B}} \left|\mathbf{w}^H \mathbf{b}\right| \ge 1 \\ &\mathscr{B} = \left\{ \mathbf{b} \,\middle|\, \mathbf{b} = \mathbf{a}(\bar{\theta}_0) + \mathbf{e}, \ \|\mathbf{e}\| \le \varepsilon \right\}. \end{aligned} \quad (7)$$

With $\mathbf{b}$ defined as above and by applying the triangle and Cauchy-Schwarz inequalities, we have

$$\begin{aligned} \left|\mathbf{w}^H \mathbf{b}\right| &= \left|\mathbf{w}^H \left[\mathbf{a}(\bar{\theta}_0) + \mathbf{e}\right]\right| \\ &\ge \left|\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)\right| - \left|\mathbf{w}^H \mathbf{e}\right| \\ &\ge \left|\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)\right| - \varepsilon \|\mathbf{w}\|. \end{aligned} \quad (8)$$

The worst-case constraint in (7) is equal to

$$\left|\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)\right| - \varepsilon \|\mathbf{w}\| \ge 1. \quad (9)$$

The nonconvex problem (9) can be transformed into a convex form by forcing the imaginary component of $\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)$ to be zero [2],

$$\begin{aligned} \varepsilon \|\mathbf{w}\| &\le \mathbf{w}^H \mathbf{a}(\bar{\theta}_0) - 1 \\ \mathrm{Im}\left\{\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)\right\} &= 0 \end{aligned} \quad (10)$$

where $\mathrm{Im}\{\}$ denotes the imaginary component.

3.2 Soft Constraints on the Multipath Signal Clusters

The actual steering vector of the multipath signal in the qth ($q = 1, \ldots, Q$) cluster is denoted as $\mathbf{c}_q$ and it belongs to the steering vector set of $\mathscr{C}_q$. We use soft constraints to keep the magnitude of the array response for $\mathbf{c}_q$ to be smaller than a predefined very small value δ,

$$\begin{aligned} &\left|\mathbf{w}^H \mathbf{c}_q\right| \le \delta, \ \forall \mathbf{c}_q \in \mathscr{C}_q \\ &\mathscr{C}_q = \left\{ \mathbf{c}_q \,\middle|\, \mathbf{c}_q = \mathbf{a}(\bar{\theta}_q + \Delta), \ -\mu^\circ \le \Delta \le \mu^\circ \right\} \end{aligned} \quad (11)$$

where Δ is the DOA estimation error. By choosing a small enough value of δ, the multipath signals can almost be nulled out completely, and their cancellation to the SOI is prevented.

The constraint in (11) is a semi-infinite constraint, which can be approximated in a straight forward way by sampling the DOAs in the clusters. We choose a set of K uniformly spaced angles $[\hat{\theta}_{q1}, \cdots, \hat{\theta}_{qK}]$ in the qth cluster. The problem (11) is then replaced by a set of ordinary inequality constraints,

$$\begin{aligned} &\left|\mathbf{w}^H \mathbf{a}(\hat{\theta}_{qk})\right| \le \delta \\ &\mathbf{a}(\hat{\theta}_{qk}) \in \mathscr{C}_q \\ &q = 1, \ldots, Q \\ &k = 1, \ldots, K. \end{aligned} \quad (12)$$

When K is sufficiently large, (12) provides a good approximation to (11).

3.3 The Robust Beamformer

Our robust beamformer is constructed by minimizing the output power under the worst-cast constraint (7) and the soft constraint (11), which is expressed as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \begin{aligned} &\min_{\mathbf{b} \in \mathscr{B}} \left| \mathbf{w}^H \mathbf{b} \right| \geq 1 \\ &\left| \mathbf{w}^H \mathbf{c}_q \right| \leq \delta, \; \forall \mathbf{c}_q \in \mathscr{C}_q. \end{aligned} \tag{13}$$

In (13), the worst-case constraint guarantees the distortionless response for the SOI with steering vector mismatches. On the other hand, the multipath signals with DOA mismatches are well suppressed by the soft constraints. As a result, signal cancellation caused by correlation is prevented and robustness against DOA estimation errors is achieved.

By substituting (10) and (12), the problem in (13) is then converted into the following formulation,

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \begin{aligned} &\varepsilon \|\mathbf{w}\| \leq \mathbf{w}^H \mathbf{a}(\bar{\theta}_0) - 1 \\ &\text{Im}\{\mathbf{w}^H \mathbf{a}(\bar{\theta}_0)\} = 0 \\ &\left| \mathbf{w}^H \mathbf{a}(\hat{\theta}_{qk}) \right| \leq \delta \\ &\mathbf{a}(\hat{\theta}_{qk}) \in \mathscr{C}_q \\ &q = 1, \ldots, Q \\ &k = 1, \ldots, K \end{aligned} \tag{14}$$

which can be solved efficiently using the second-order cone programming (SOCP) based approach [12, 13].

4. SIMULATIONS

In our simulations, a uniform linear array with 20 sensors spaced half a wavelength apart is used. We assume the SOI has two multipath signals, and there are four uncorrelated interferences with interference-to-noise ratio (INR) of 30 dB. The noise signals are spatially white Gaussian. The actual DOA of the SOI falls in the cluster with a pre-estimated centre $\bar{\theta}_0 = 10^\circ$, and the estimated cluster centres for the two multipath signals are $\bar{\theta}_1 = -15^\circ$ and $\bar{\theta}_2 = 40^\circ$ respectively. In all examples, the cluster radius is $\mu = 2^\circ$ except for the example of Fig. 5 where its value varies. For the soft constraint in the multipath clusters, 10 samples per degree are used and the value of δ is 0.01. In each simulation run, the actual DOA of the SOI and the multipath signals are randomly generated in their clusters; the multipath attenuation factor ρ_q is randomly generated in the region $[0.1, 1]$ and the phase shift ϕ_q in the region $[0, 2\pi]$. DOAs of the interferences are randomly generated in the region $[-90^\circ, 90^\circ]$ with at least 10 degrees apart from each other and the pre-estimated cluster centres, except for the example of Fig. 4 where the DOAs are specified. 1000 training snapshots are used for the received data. The SeDuMi [14] and YALMIP [15] MATLAB toolboxes are used to compute the weight vector of our robust beamformer. The simulation results are obtained by averaging 500 independent runs.

In the first example we compare the output signal-to-interference-plus-noise ratio (SINR) of the proposed method and the method in [6] with respect to the steering vector distortion bound coefficient ε. The input signal-to-noise ratio (SNR) of the SOI is 10 dB for both methods. It is shown in Fig. 3 that for all the values of ε the proposed method has

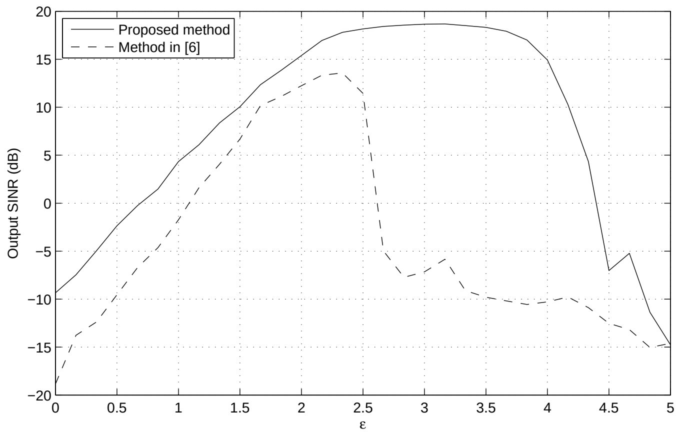

Figure 3: Output SINR versus ε.

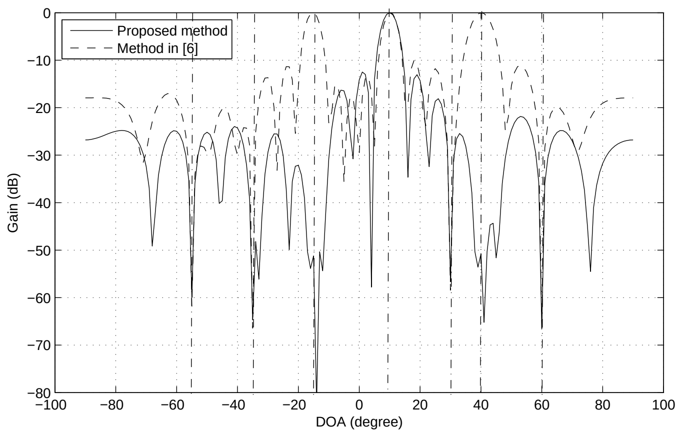

Figure 4: Beampatterns.

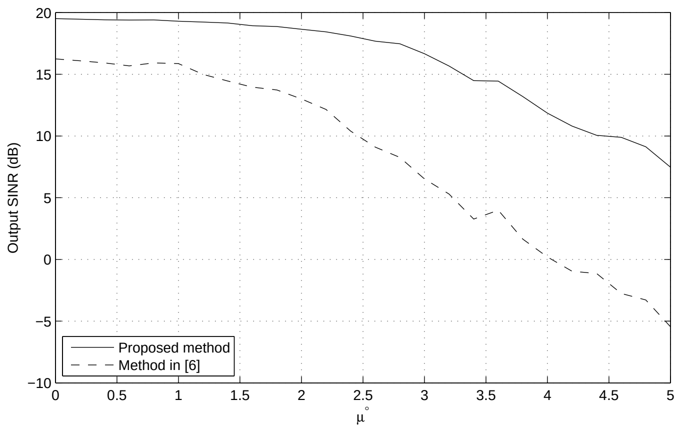

Figure 5: Output SINR versus μ.

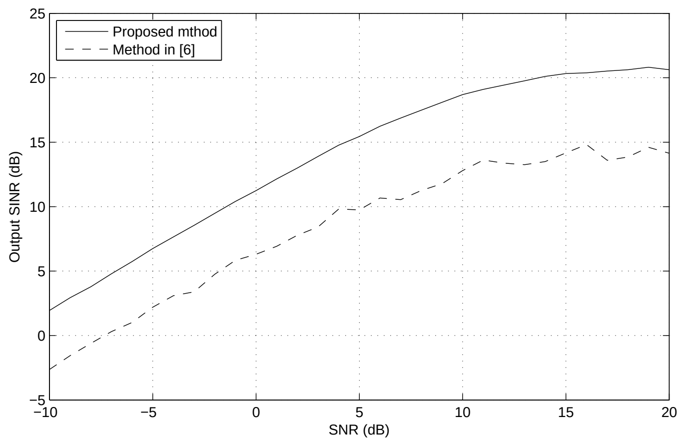

Figure 6: Output SINR versus SNR.

a much better performance than the method in [6], although their optimum performance is achieved at different ε.

The resultant beampatterns of the two methods are given in Fig. 4 with SNR $= 10$ dB, and $\varepsilon = 3$ for the proposed method and $\varepsilon = 2$ for the method in [6]. In this example, the DOAs of the four interferences are $-55°$, $-35°$, $30°$, and $60°$, respectively. The dashed vertical lines represent the directions of input signals. It is clear that the proposed method has deep nulls at the multipath clusters and the interference directions, while being distortionless at the SOI cluster.

In Fig. 5 and Fig. 6, the performances of the two methods are compared with various cluster radius μ and different input SNR respectively. The input SNR is 10 dB in the example of Fig. 5. Both examples are with $\varepsilon = 3$ for the proposed method and $\varepsilon = 2$ for the method in [6]. The proposed method outperforms the method in [6] significantly in both examples.

5. CONCLUSIONS

In this paper, a novel robust beamforming scheme with combined worst-case performance optimization and soft constraint has been proposed for effective reception of the SOI in a multipath environment. The distortionless response for the SOI with steering vector mismatch is guaranteed by the worst-case performance optimization, and the multipath signals with DOA mismatch are well suppressed by the soft constraints. As a result, signal cancellation caused by multipath correlation is prevented and robustness against DOA estimation errors is achieved.

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