PREFILTERED PISARENKO FREQUENCY ESTIMATOR FOR MULTIPLE REAL-VALUED SINUSOIDS

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ABSTRACT

An Infinite Impulse Response (IIR) notch filter based method is proposed for multiple real sinusoids frequency estimation. The estimator basically involves two steps. An initial frequency estimate is first obtained by solving the Least Squares (LS) equation based on the transversal part of the IIR filter. Based on the initial estimate, we introduce the recursive part and a LS cost function is then constructed from which the final estimate is acquired. Computer simulations show that the performance of the estimator approaches the Cramer-Rao Lower Bound for sufficiently high signal-to-noise ratios and/or data lengths.

1. INTRODUCTION

Detection and estimation of sinusoidal components frequencies, in the presence of broadband noise, are some of the most common problems in signal processing [1]. Numerous techniques have been developed for their treatment [2], including notch filtering [3], linear prediction, Yule-Walker methods [4] and subspace-based approaches [2].

Among the subspace-based methods, the Pisarenko Harmonic Decomposer (PHD) [5] is of historical interest because it was the first to exploit the eigenstructure of the covariance matrix, and its performance has been extensively studied [6, 7]. Interestingly, the PHD frequency estimator for a single real sinusoid can be implemented in a very simple way [8].

Although the PHD method constitutes a simple approach to frequency estimation, a number of statistical analysis have shown its inefficiency [6, 8]. Some attempts to improve the performance of the PHD estimator can be found in the literature. In [9], a variant termed *Reformed* PHD (RPHD) for single tone frequency estimation was proposed. Its performance is superior to that of the original PHD, although the statistical analysis in [9] revealed its inefficiency.

A different approach to frequency estimation is the use of online (i.e. adaptive) notch filters. However, high noise rejection and sharp cutoff bandpass characteristics are desirable traits which can only be obtained with very high order FIR structures, and thus Infinite Impulse Response (IIR) notch filters have become a popular choice. Several algorithms have been developed for the adaptation of these systems, essentially seeking the minimum point of some cost function; see e.g. [10, Ch. 10] and the references therein.

In earlier work [11], we have shown that prefiltering the incoming data with the recursive part of an IIR filter, improves the performance of PHD estimator for the single sinusoid case. In this paper, we extend the principle of this approach to the multiple sinusoids case. In a first step, we extend the idea of the RPHD estimator to the case of frequency estimation of multiple real-valued sinusoids. This solution is used as an initial estimate of the frequencies. In a second step, to improve the accuracy of the estimator, we prefilter the input signal with a purely recursive IIR filter and then an improved estimator, which we refer to as Prefiltered PHD (PPHD), is obtained by minimizing a Least Squares (LS) cost function. The prefilter is a function of the frequency parameters which are obtained in an iterative manner. For the case of a single real tone, a closed-form frequency estimator is also derived with performance approaching the Cramer-Rao Lower Bound (CRLB) for a range of Signal to Noise Ratio (SNR) values, provided that the data record length is sufficiently large.

The rest of the paper is organized as follows. The problem of multiple frequencies estimation is formulated in Section 2. The PPHD estimator is introduced in Section 3. Recursive implementation is also presented. In Section 4, closed-form single-tone frequency estimation is investigated. Simulation results are included to evaluate the performance of the estimator under different conditions in Section 5. Finally, conclusions are drawn in Section 6.

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2. PROBLEM FORMULATION

The problem of multiple real sinusoidal frequency estimation is formulated as follows. We are interested in the estimation of the unknown frequencies w_m, of pure sine waves $s_m(n)$, immersed in a noise $u(n)$. The model can be expressed as

$$\begin{aligned} y(n) &= s(n)+u(n) \\ &= \sum_{m=1}^{M} s_m(n)+u(n) \\ &= \sum_{m=1}^{M} \alpha_m \sin(w_m n+\varphi_m)+u(n), \qquad 1 \le n \le N, \end{aligned} \tag{1}$$

where N corresponds to the number of observations, M is the number of sinusoids, α_m are their amplitudes, φ_m are their random phases and $u(n)$ is a zero mean additive white noise, with variance σ_u^2, which is assumed to be independent of $s_m(n)$. The SNR for the m-th sinusoid is defined as $\mathrm{SNR}_m \stackrel{\triangle}{=} \alpha_m^2/(2\sigma_u^2)$.

To estimate the sinusoidal components frequencies, an M-order IIR filter with constrained parameterization is used for a notch-based-estimation technique. The following filter is typically used in practice [3]

$$H(z) = \frac{A\left(z^{-1}\right)}{A\left(rz^{-1}\right)}, \tag{2}$$

where $A\left(z^{-1}\right) = a_0 + a_1 z^{-1} + \dots + a_M z^{-M} + \dots + a_{2M-1} z^{-2M+1} + a_{2M} z^{-2M}$, is a polynomial with symmetric real coefficients $a_m =$

$a_{2M-m}, m = 0,...,M$ and $a_0 = 1$. The parameter r $(0 \leq r < 1)$ is known as the pole contraction factor. The roots of the polynomial $A(z^{-1})$ lie on the unit circle in the complex *z-plane* in complex conjugate pairs. The polar angles of these roots are defined by the frequencies w_m, where $m = 1, ..., .M$. The relationship between w_m and $A\left(z^{-1}\right)$ is then given by

$$\sum_{k=0}^{2M} a_k \exp\left(-jw_m k\right) = 0 \quad \text{for} \quad m = 1,...,M. \tag{3}$$

The basic idea underlying notch-filter-based estimation techniques is the minimization, with respect to $\{a_m\}$, of the power of the notch filter output when the input is the observed signal $y(n)$. Due to the presence of $\{a_m\}$ in both the numerator and the denominator of (2), this output power is a nonquadratic function of $\{a_m\}$. Moreover, the presence of the noise term $u(n)$ in (1) will alter the location of this minimum, unless the pole contraction factor is sufficiently close to one [12].

For these reasons, we consider the minimization of the output power but fixing the denominator in (2), i.e. the notch filter transfer function becomes now

$$\bar{H}(z^{-1}) = \frac{A(z^{-1})}{B(rz^{-1})}, \tag{4}$$

where $B\left(z^{-1}\right) = 1 + b_1 z^{-1} + ... + b_M z^{-M} + ... + b_{2M-1} z^{-2M+1} + z^{-2M}$ is a polynomial with fixed symmetric real coefficients $b_m = b_{2M-m}, m = 1,...,M-1$. A typical choice for $\{b_m\}$ are the true parameters $\{a_m\}$; since $\{a_m\}$ are not available, later on we will present a means to select $\{b_m\}$.

3. PPHD ESTIMATOR

Based on the reformulation presented in previous section, the IIR notch filter output $e(n)$ is defined as

$$e(n) = \sum_{m=0}^{M-1} \tilde{a}_m \left[\tilde{y}(n-m) + \tilde{y}(n-2M+m)\right] + \tilde{a}_M \tilde{y}(n-M), \tag{5}$$

where $\{\tilde{a}_m\}$ are the estimates of $\{a_m\}$ up to a scalar since $\tilde{a}_0$ is not fixed to be unity and $\tilde{y}(n)$ is the result of prefiltering $y(n)$ with the recursive part $1/B(rz^{-1})$. $\tilde{y}(n)$ consists of a sinusoidal component $\tilde{s}(n)$ and a colored noise component $\tilde{u}(n)$.

Defining the $(M+1) \times 1$ vectors

$$\begin{aligned}
\tilde{A} &\triangleq [\tilde{a}_0, \tilde{a}_1, ..., \tilde{a}_M]^T, \\
\tilde{Y}_n &\triangleq [\tilde{y}(n) + \tilde{y}(n-2M), \tilde{y}(n-1) + \tilde{y}(n-2M+1), ..., \tilde{y}(n-M)]^T, \\
\tilde{S}_n &\triangleq [\tilde{s}(n) + \tilde{s}(n-2M), \tilde{s}(n-1) + \tilde{s}(n-2M+1), ..., \tilde{s}(n-M)]^T, \\
\tilde{U}_n &\triangleq [\tilde{u}(n) + \tilde{u}(n-2M), \tilde{u}(n-1) + \tilde{u}(n-2M+1), ..., \tilde{u}(n-M)]^T,
\end{aligned}$$

and the autocorrelation matrices of $\tilde{Y}_n$, $\tilde{S}_n$ and $\tilde{U}_n$

$$\begin{aligned}
R_{\tilde{Y}\tilde{Y}} &\triangleq \frac{1}{N-2M} \sum_{n=2M+1}^{N} \tilde{Y}_n \tilde{Y}_n^T, \\
R_{\tilde{S}\tilde{S}} &\triangleq \frac{1}{N-2M} \sum_{n=2M+1}^{N} \tilde{S}_n \tilde{S}_n^T, \\
R_{\tilde{U}\tilde{U}} &\triangleq \frac{1}{N-2M} \sum_{n=2M+1}^{N} \tilde{U}_n \tilde{U}_n^T.
\end{aligned}$$

Using this notation, the IIR notch filter output $e(n)$ can be written as

$$e(n) = \tilde{A}^T \tilde{Y}_n = \tilde{A}^T \tilde{S}_n + \tilde{A}^T \tilde{U}_n. \tag{6}$$

To estimate the filter coefficients $\{\tilde{a}_m\}$, we minimize the following LS criterion

$$\begin{aligned}
J(N) &\triangleq \frac{1}{N-2M} \sum_{i=2M+1}^{N} e^2(i), \\
&= \tilde{A}^T R_{\tilde{Y}\tilde{Y}} \tilde{A}, \\
&= \underbrace{\tilde{A}^T R_{\tilde{S}\tilde{S}} \tilde{A}}_{(1)} + \underbrace{\tilde{A}^T R_{\tilde{U}\tilde{U}} \tilde{A}}_{(2)}.
\end{aligned} \tag{7}$$

Since the second term of (7) depends on $\tilde{A}$, the minimum of $J(N)$ does not correspond to the desired vector. Therefore, a biased estimate will result when minimizing $J(N)$ in the presence of noise.

To avoid this dependence, we propose to minimize the following cost function $\bar{J}(N)$

$$\bar{J}(N) \triangleq \frac{J(N)}{\tilde{A}^T R_{\tilde{U}\tilde{U}} \tilde{A}} = \frac{\tilde{A}^T R_{\tilde{Y}\tilde{Y}} \tilde{A}}{\tilde{A}^T R_{\tilde{U}\tilde{U}} \tilde{A}}. \tag{8}$$

This ratio is known as generalized Rayleigh quotient [13]. The vector $\tilde{A}$ minimizing $\bar{J}(N)$ satisfies

$$R_{\tilde{Y}\tilde{Y}} \tilde{A} = \bar{J}(N) R_{\tilde{U}\tilde{U}} \tilde{A} = \sigma_u^2 \bar{J}(N) \bar{R}_{\tilde{U}\tilde{U}} \tilde{A}, \tag{9}$$

where $\bar{R}_{\tilde{U}\tilde{U}}$ is the normalized autocorrelation matrix of $\tilde{U}_n$. Thus, the vector $\tilde{A}$ is the generalized eigenvector corresponding to the minimum generalized eigenvalue of $\left(R_{\tilde{Y}\tilde{Y}}, \bar{R}_{\tilde{U}\tilde{U}}\right)$.

An estimate of $\bar{R}_{\tilde{U}\tilde{U}}$ can be evaluated for sufficiently large N as

$$\lim_{N\to\infty} \bar{R}_{\tilde{U}\tilde{U}} = \begin{pmatrix} 2(\rho_0 + \rho_{2M}) & 2(\rho_1 + \rho_{2M-1}) & \cdots & 2\rho_M \\ 2(\rho_1 + \rho_{2M-1}) & 2(\rho_0 + \rho_{2M-2}) & \dots & 2\rho_{M-1} \\ \vdots & \vdots & \vdots & \vdots \\ 2\rho_M & 2\rho_{M-1} & \dots & \rho_0 \end{pmatrix}, \tag{10}$$

where the autocorrelation sequence ρ_p, for $p = 0, 1, ..., 2M$, is

$$\rho_p \triangleq \frac{1}{\sigma_u^2} \lim_{N\to\infty} \left[\frac{1}{N-2M} \sum_{i=2M+1}^{N} \tilde{u}(i)\tilde{u}(i-p)\right]. \tag{11}$$

Setting $c_m = r^m b_m$, the sequence ρ_p can be evaluated given that

$$\rho_p = \delta_p - c_1 \rho_{p-1} - ... - c_{2M} \rho_{p-2M}, \tag{12}$$

where $\delta_p = \begin{cases} 1 & \text{if } p = 0 \\ 0 & \text{otherwise} \end{cases}$.

Equation (12) induces the following system of equations

$$Q\Omega = \Gamma, \tag{13}$$

where $\Omega = [\rho_0, \rho_1, ..., \rho_{2M}]^T$, $\Gamma = [1, 0, ..., 0]^T$ and Q is given by

$$\begin{pmatrix} 1 & c_1 & \dots & c_M & c_{M+1} & \dots & c_{2M} \\ c_1 & 1 + c_2 & \dots & c_{M+1} & c_{M+2} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_M & c_{M-1} + c_{M+1} & \dots & 1 + c_{2M} & 0 & \dots & 0 \\ c_{M+1} & c_M + c_{M+2} & \dots & c_1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{2M} & c_{2M-1} & \dots & c_M & c_{M-1} & \dots & 1 \end{pmatrix},$$

Hence, Ω is evaluated as

$$\Omega = Q^{-1}\Gamma. \tag{14}$$

In the following, we refer the estimator based on the generalized eigenproblem (9) as Prefiltered PHD (PPHD) estimator.

3.1. Particular Case: $r = 0$

In the particular case, when $r = 0$, the IIR notch filter is equivalent to the transversal part $A\left(z^{-1}\right)$. The estimate of $\tilde{A}$ in this case is given by the generalized eigenvector corresponding to the minimum generalized eigenvalue of

$$\left(R_{YY}, D\right), \tag{15}$$

where D is an $(M+1 \times M+1)$ matrix defined as: $D = diag(2,2,...,2,1)$ and R_{YY} is the autocorrelation matrix of the vector $Y = [y(n)+y(n-2M), y(n-1)+y(n-2M+1), ..., y(n-M)]^T$.

For the particular case of a single real-valued sinusoid, it is possible to show that a closed-form estimator can be obtained from the generalized eigenvector problem (15). This estimator is equivalent to the RPHD estimator presented in [9]

$$\hat{w} = \arccos\left(\frac{\gamma_N + \sqrt{\gamma_N^2 + 8\beta_N^2}}{4\beta_N}\right), \tag{16}$$

where

$$\gamma_N \triangleq \sum_{i=3}^{N}\left([y(i)+y(i-2)]^2 - 2y^2(i-1)\right), \tag{17}$$

$$\beta_N \triangleq \sum_{i=3}^{N}[y(i)+y(i-2)]\,y(i-1). \tag{18}$$

Therefore, the generalized eigenvalue approach (15) is an extension of the RPHD estimator to the case of multiple sinusoids.

3.2. Proposed algorithm

Observe that, if properly chosen, the data prefilter $1/B(rz^{-1})$ has the potential to enhance the desired frequencies over the noise, thus improving the effective SNR_m. So far, we have assumed that the parameters $\{b_m\}$ of the prefilter are fixed *a priori.* Note that these parameters determine the angular position of the poles of the data prefilter, and thus ideally one would like to choose $\{b_m\} = \{a_m\}$. Since $\{a_m\}$ are of course not available, it seems natural to use an iterative scheme in which the prefilter is designed at each iteration based on the frequency estimate obtained in the previous step. In the first iteration, $r = 0$ can be used (thus, the extension of the RPHD estimator to the multiple sinusoid case, presented in the previous section, is used as starting point), after which a suitable value $r > 0$ is set. The iterative procedure is summarized as follows

1. Find an initial estimate $\{\tilde{a}_m^{(0)}\}$ given by the generalized RPHD by computing the generalized eigenvector corresponding to the minimum generalized eigenvalue of (R_{YY}, D).

Set r to a suitable value $r \in (0,1)$. For $k = 1, 2, 3, \ldots$

2. Set $\left\{\tilde{b}_m^{(k)}\right\} = \left\{\tilde{a}_m^{(k-1)}\right\}$, where $\left\{\tilde{b}_m^{(k)}\right\}$ are the estimates of the coefficients $\left\{b_m^{(k)}\right\}$, and obtain the prefiltered data $\tilde{y}(i) = [1/B(rz^{-1})]y(i)$.
3. Evaluate the estimate of $\bar{R}_{\tilde{U}\tilde{U}}$ using (10) and (14).
4. Compute the new estimate $\{\tilde{a}_m^{(k)}\}$ corresponding to the generalized eigenvector corresponding to the minimum generalized eigenvalue of $\left(R_{\tilde{Y}\tilde{Y}}, \bar{R}_{\tilde{U}\tilde{U}}\right)$.
5. Repeat steps 2, 3 and 4 until convergence. Obtain the frequency estimates $\tilde{\omega}_m$ via (3).

Simulation results in Section 5 demonstrate that using few iterations in the estimation procedure, the algorithm is able to achieve global convergence with performance approaching the CRLB for sufficiently large SNRs and/or data lengths. The major computational requirement of the algorithm is to determine the generalized eigendecomposition vectors, which requires $O((M+1)^3)$ multiplications in addition to $O(N(M+1)^2)$ multiplications for the estimation of the autocorrelation matrix. Comparing with high-resolution methods such as MUSIC and ESPRIT, which require $O(Np^2)$ multiplications for the estimation of the data correlation matrix, where $p > 2M+1$ is the size of this matrix, in addition to $O(p^3)$ multiplications in the eigendecomposition of this estimated matrix, the proposed estimator has thus the lower complexity implementation. Moreover, when compared to the constrained weighted LS frequency estimators, developed in the Pisarenko framework in [14] from a generalized eigenvalue problem, the proposed estimator has the lower complexity implementation since the above estimator, in addition to $O((M+1)^3)$ multiplications required for the generalized eigendecomposition problem, $O(N^3)$ multiplications are required to determine the inverse of the weighting matrix in this algorithm.

4. CLOSED-FORM ESTIMATOR FOR SINGLE-TONE FREQUENCY ESTIMATION

For single-tone frequency estimation, it is possible to derive a closed-form estimator as follows. An approximate of $\bar{R}_{\tilde{U}\tilde{U}}$, which is a 2×2 matrix for $M = 2$, can be evaluated as in (10), where we can easily show, using (14), that ρ_p for $p = 0,1,2$ is given by

$$\rho_0 = \left(1+r^2\right)\xi_{r,b}, \tag{19}$$

$$\rho_1 = -rb\xi_{r,b}, \tag{20}$$

$$\rho_2 = -r^2\left(r^2 - b^2 + 1\right)\xi_{r,b}, \tag{21}$$

with

$$\xi_{r,b} \doteq \frac{1}{\left(1-r^2\right)\left(\left(1+r^2\right)^2 - r^2b^2\right)}. \tag{22}$$

The matrix $R_{\tilde{Y}\tilde{Y}}$ is evaluated as

$$R_{\tilde{Y}\tilde{Y}} = \begin{pmatrix} \beta_0 & \beta_1 \\ \beta_1 & \beta_2 \end{pmatrix}, \tag{23}$$

where

$$\beta_0 = \frac{1}{N-2M}\sum_{i=2M+1}^{N}(\tilde{y}(i)+\tilde{y}(i-2))^2, \tag{24}$$

$$\beta_1 = \frac{1}{N-2M}\sum_{i=2M+1}^{N}\tilde{y}(i-1)(\tilde{y}(i)+\tilde{y}(i-2)), \tag{25}$$

$$\beta_2 = \frac{1}{N-2M}\sum_{i=2M+1}^{N}\tilde{y}^2(i-1). \tag{26}$$

The generalized eigenvalue problem of (9) can now be expressed as

$$\begin{pmatrix} \tilde{a}_0\beta_0 + \tilde{a}_1\beta_1 \\ \tilde{a}_0\beta_1 + \tilde{a}_1\beta_2 \end{pmatrix} = \lambda \begin{pmatrix} 2\tilde{a}_0(\rho_0+\rho_2) + 2\tilde{a}_1\rho_1 \\ 2\tilde{a}_0\rho_1 + \tilde{a}_1\rho_0 \end{pmatrix}. \tag{27}$$

Eliminating λ using (27) and setting $\tilde{a}_0 = 1$, yields to

$$\upsilon_N\tilde{a}_1^2 + \eta_N\tilde{a}_1 - 2\varpi_N = 0, \tag{28}$$

where

$$\upsilon_N = (1+r^2)\beta_1 + 2rb\beta_2, \quad (29)$$
$$\eta_N = (1+r^2)\beta_0 - 2(1-r^4+r^2b^2)\beta_2, \quad (30)$$
$$\varpi_N = rb\beta_0 + (1-r^4+r^2b^2)\beta_1. \quad (31)$$

The root which corresponds to the frequency estimate is

$$\tilde{a}_1 = -\frac{\eta_N + \sqrt{\eta_N^2 + 8\upsilon_N\varpi_N}}{2\upsilon_N}. \quad (32)$$

An iterative implementation of this closed-form single-tone frequency estimator can be derived as follows

1. Obtain an initial estimate $\tilde{a}_1^{(0)}$ based on RPHD (i.e. (32) with $r = 0$).

Set r to a suitable value $r \in (0,1)$. For $k = 1, 2, 3, \ldots$

2. Set $\tilde{b}_1^{(k)} = \tilde{a}_1^{(k-1)}$ and obtain the prefiltered data $\tilde{y}(i) = [1/B(rz^{-1})]y(i)$.
3. Compute the new estimate $\tilde{a}_1^{(k)}$ using (32).
4. Repeat Steps 2 and 3 until convergence. Obtain the frequency estimate $\tilde{\omega}_1$ via $\tilde{\omega}_1 = \arccos(\tilde{a}_1^{(k)})$.

5. NUMERICAL RESULTS

Computer simulations were carried out to validate the performance of the the proposed algorithm.

In the algorithm implementation, we adopt a different pole contraction factor $r^{(k)}$ at different iterations. In practice, no *a priori* information is available on the input sine wave so that its frequency may fall outside of the prefilter passband, especially for small SNR and/or short data lengths. Therefore, it makes sense to use a wider passband at the first iteration that becomes narrower as iterations evolve. This 'bandwidth thinning' strategy is common in the design of adaptive notch filters [10]. A simple way to do this is to let r grow exponentially from $r^{(1)}$ to a final value $r^{(\infty)}$ according to

$$r^{(k+1)} = \lambda r^{(k)} + (1-\lambda)r^{(\infty)}, \qquad 0 < \lambda < 1. \quad (33)$$

The parameter λ determines the change rate of $r^{(k)}$, which should be larger for large data lengths in order to speed up convergence. On the other hand, with short data records a slower variation of r should help increase sensitivity to the presence of the sine wave. Thus, λ should somehow be inversely proportional to N.

In our simulation, we apply the algorithm with the numerical values $r^{(1)} = 0.75$; $r^{(\infty)} = 0.995$ and

$$\lambda(N) = 0.93/(1+(N/200)^2). \quad (34)$$

In Fig. 1 and Fig. 2, we consider a single sinusoid with $\omega_1 = 0.4\pi$. Fig. 1 shows the MSE along the iterations setting SNR $= 10$ dB, and for $N = 20$, 200 and 300. Similarly, in Fig. 2 we fix $N = 200$ and consider SNR $= 0$, 10 and 20 dB. Also shown is the CRLB for this frequency estimation problem [15].

A noticeable improvement can be observed with just one iteration; furthermore, convergence is achieved in about four iterations, and the achieved MSE is very close to the CRLB.

In the following simulation, we consider 3 sinusoids with frequencies $[w_1, w_2, w_3] = [0.3, 0.34, 0.7]\pi$. The performance of the PPHD estimator (using four iterations) is compared to the RPHD estimator, the LS version of R-ESPRIT developed in [16], root-MUSIC [17] as well as the CRLB.

In Fig. 3, Fig. 4 and Fig. 5, we depict the MSE of the 3 frequencies as function of SNR_3. We consider that SNR_1=SNR_2=10 dB and

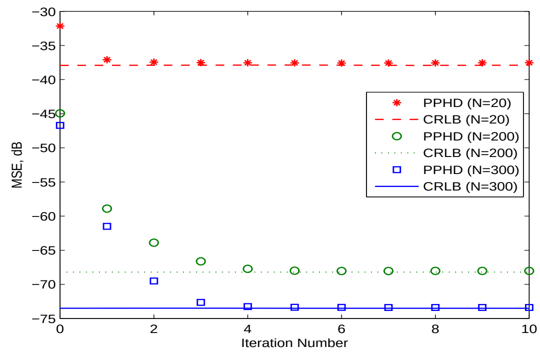

Fig. 1. Mean squared frequency errors versus iteration number k.

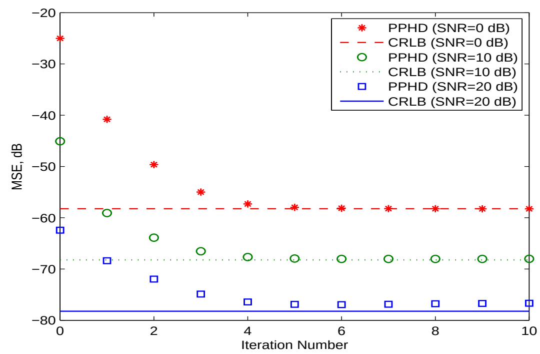

Fig. 2. Mean squared frequency errors versus iteration number k.

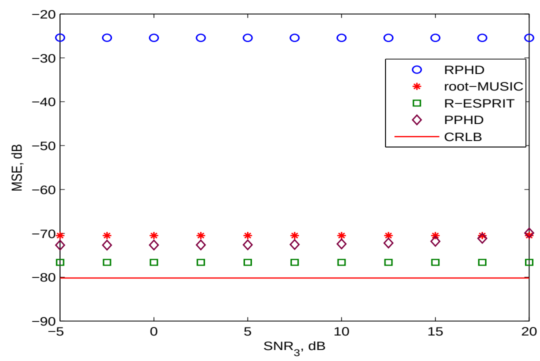

Fig. 3. Mean Squared frequency errors versus SNR_3 for w_1.

SNR_3 varies from -5 to 20 dB. The sequence length $N = 500$ and the size of the snapshot vectors for root-MUSIC and R-ESPRIT were fixed to 30. This value is chosen in order to achieve a compromise between good performance and lower complexity computation. As seen, the PPHD has comparable performance as root-MUSIC and R-ESPRIT.

In Fig. 6, the 3 sinusoids have the same SNR=10 dB. We plot the MSE as function of N. In addition to outperforming RPHD, the MSE achieved by the proposed estimate is close to the CRLB.

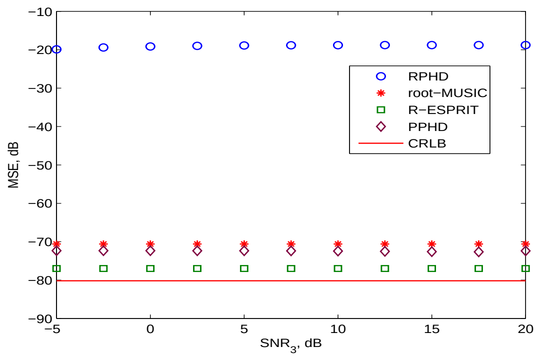

Fig. 4. Mean Squared frequency errors versus SNR_3 for w_2.

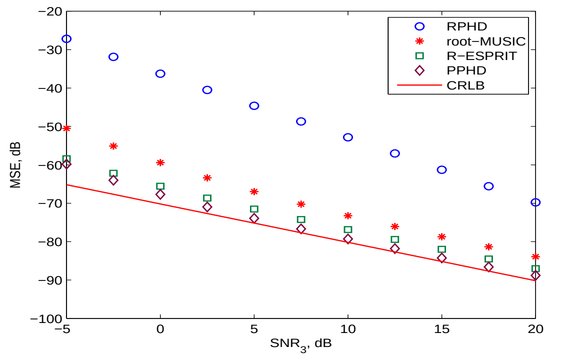

Fig. 5. Mean Squared frequency errors versus SNR_3 for w_3.

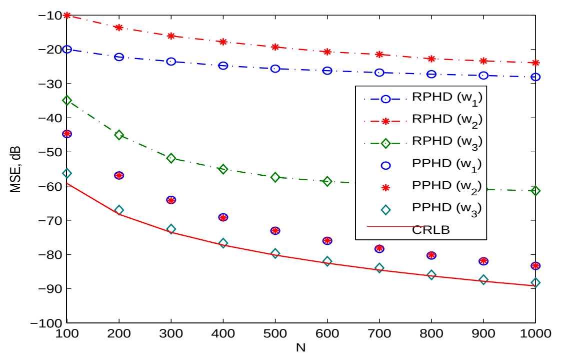

Fig. 6. Mean Squared frequency errors versus N.

6. CONCLUSION

In this paper, a frequency estimation algorithm for multiple real sinusoids in white noise based on IIR notch filter was presented. The estimator basically involves two steps. An initial frequency estimate is first obtained by solving the Least Squares equation based on the transversal part of the IIR filter. Based on the initial estimate, we introduce the recursive part and an optimally Least Squares cost function is then constructed from which the final estimate is acquired, referred to as PPHD estimator. For a single tone, a closed-form frequency estimate was obtained. Simulation results show that PPHD performance is close to the CRLB.

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