AN IMPROVED SPHERE DECODING SCHEME FOR MIMO SYSTEMS USING AN ADAPTIVE STATISTICAL THRESHOLD

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ABSTRACT

The fixed-complexity sphere decoder (FSD) has been previously proposed for multiple input-multiple output (MIMO) detection to overcome the two main drawbacks of the original sphere decoder (SD), namely the variable complexity and sequential structure. However, one issue for the FSD is that many redundant computations are introduced resulting in high power consumption, which will become more evident when many antennas are involved and/or higher-order constellations are utilised. In this paper, a statistical threshold based scheme (ST-FSD) is proposed in order to speedup the algorithm by eliminating its unnecessary search paths. The optimum threshold of the proposed scheme has been derived through analysis of the statistical distributions of the correct and erroneous estimate. Further, a tight lower bound on the threshold has been obtained by using the singular value decomposition (SVD) method and applied to the FSD. From simulation results, the proposed scheme is shown to be able to achieve a significant reduction in computational complexity with almost no performance degradation compared to the original FSD algorithm.

1. INTRODUCTION

During the last decade, multiple-input multiple-output (MIMO) technology has become one of the most popular approaches to meet the demands for high data rate communication. Various MIMO detectors based on different perspectives and methodologies have been proposed in the literature. The sphere detector (SD) [1-2] is an important, computationally efficient implementation of the maximum likelihood (ML) detector. However, the SD still has an exponential expected complexity regardless of the signal-to-noise ratio (SNR) for high numbers of antennas and large constellation sizes [3]. Therefore, a so-called fixed-complexity sphere decoder (FSD) has been proposed recently in [4-6], which combines a novel channel matrix preprocessing with a search through a fixed subset of the complete receive constellation. The FSD shows only a very small bit error ratio (BER) degradation compared to the original SD. Nevertheless, one drawback of the FSD is that many redundant computations are introduced resulting in high power consumption. This will become more evident when many antennas are involved and/or higher-order constellations are utilised.

In order to solve the above problem, the main idea in this work is to design a threshold scheme which is able to measure the "goodness" of estimates from different search paths traversed by the FSD and thus discard the unnecessary search paths that do not seem to lead to the final solution. Unlike other pruning approaches reported in literature [7], our work focuses on the FSD instead of the conventional SD or ML because the FSD is much more efficient from an implementation viewpoint than the alternatives in terms of achievable throughput [6]. Moreover, since the derived threshold only depends on the channel matrix, the need for tuning many parameters, often encountered in previous pruning approaches, is thus eliminated. This makes the algorithm very advantageous for practical implementation in real scenarios.

The remainder of the paper is organised as follows. Section 2 describes the system model and Section 3 briefly reviews the conventional SD and FSD algorithms. Details of the proposed statistical threshold based scheme are described in Section 4. Computer simulation results are presented in Section 5. Finally, Section 6 concludes the paper.

2. SYSTEM DESCRIPTION

Consider an MIMO system with N_T transmit antennas and N_R receive antennas ($N_R \geq N_T$) signalling through flat fading channels as shown in Figure 1.

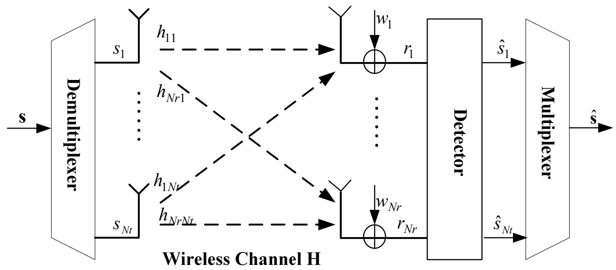

Figure 1 – A linear MIMO channel with N_T inputs and N_R outputs.

The input-output relationship of this system is given by

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{1}$$

where the transmitted vector can be denoted as $\mathbf{s} = [s_1, \cdots, s_{N_T}]^T$ and the superscript $(\cdot)^T$ stands for the transpose, $s_i \in \Delta$ where Δ is a digital constellation alphabet with size *P*, $\mathbf{r} = [r_1, \cdots, r_{N_R}]^T$ denotes the received vector and $\mathbf{w} = [w_1, \cdots, w_{N_R}]^T$ is a vector of independent zero-mean complex Gaussian noise samples with variance

σ^2 per real or imaginary component. For simplicity, it is assumed that $N_R= N_T=N$, thus the subscripts of transmit and receive antennas are omitted in the sequel. However, the algorithm can be applied for any choice of N_T and N_R, subject to $N_R \geq N_T$. Also, we assume that the channel matrix **H** is constant over a block of consecutive time instants and the detector has perfect knowledge about of the entries of **H**.

3. SPHERE DECODER AND FIXED-COMPLEXITY SPHERE DECODER

The conventional SD approach to the problem of obtaining the estimate, $\hat{\mathbf{s}}$, is to find a candidate that minimises the squared Euclidean distance (ED) metric within a hypersphere of radius D^2 around the received signal, which can be mathematically represented by

$$\hat{\mathbf{s}}_{\mathrm{SD}} = \arg\min_{\hat{\mathbf{s}}\in\Phi\subset\Delta} \|\mathbf{r}-\mathbf{H}\hat{\mathbf{s}}\|^2 \leq D^2 \quad (2)$$

where $\| \ \|^2$ stands for the Euclidean norm and Φ refers to some subset of Δ defined by the radius constraint.

After structuring the channel using the QR decomposition, i.e. **H=QR**, where **Q** is an orthogonal matrix and **R** is an upper triangular matrix, the sphere constraint in Eq. (2) can be rewritten as

$$\|\mathbf{y}-\mathbf{R}\hat{\mathbf{s}}\|^2 \leq D^2 \quad (3)$$

where $\mathbf{y}=\mathbf{Q}^H\mathbf{r}$ and the superscript $(\cdot)^H$ stands for the Hermitian transpose. Due to the triangular structure of **R**, the solution of Eq. (3) can be obtained recursively starting from the top layer k=1 to the bottom layer k=N.

It is known that the computational complexity of SD algorithm increases significantly as the number of antennas increases or when the SNR is low. Therefore, the FSD is proposed to overcome the disadvantages of SD by searching over only a fixed number of lattice points $\mathbf{H}\hat{\mathbf{s}}$ around the received signal **r,** independent of the noise level. Actually, the FSD is based on the observation of the fact [5] that the diagonal entries of **R** satisfy

$$E[r_{NN}^2] < E[r_{N-1N-1}^2] < \cdots < E[r_{11}^2] \quad (4)$$

Consequently, the number of candidates at layer k which denoted as n_k should satisfy

$$E[n_1] \geq E[n_2] \geq \cdots \geq E[n_N] \quad (5)$$

Therefore, the main idea of FSD is to assign a fixed but distinct number of candidates to be searched per layer independent of the initial radius. It is found that in order to achieve asymptotical ML performance and the same diversity performance as the ML detector, the distribution of candidates ($\mathbf{n}_s$) searched by FSD should follow [4]

$$n_k = \begin{cases} P, & \text{for } k=1, \cdots, L \\ 1, & \text{for } k=L+1, \cdots, N \end{cases} \quad (6)$$

where P is the constellation size and L is minimum number of layers whose candidates' distribution should be set as P. As an example, L=1 is found by [4] for an N=4 MIMO system.

4. PROPOSED STATISTICAL THRESHOLD APPROACH

The basic motivation for the proposed statistical threshold based FSD (ST-FSD) scheme is the fact that all the search paths of FSD have different probabilities of finding the final solution and the total number of those paths can be predetermined. Hence, instead of performing a search over all of these paths blindly in parallel as in the FSD, the proposed ST-FSD treats all the FSD search paths as its potential candidates but searches them in sequence. The main hypothesis here is that when applying a suitable threshold test (whose threshold value is denoted as T_{th}) to judge the "goodness" of ED metrics found by different paths, it would hopefully eliminate unnecessary searches and obtain a complexity advantage over the FSD algorithm. This reduction could be exploited to increase the number of received signal vectors ***r*** that can be processed per second or to reduce the power consumption of the FSD hardware circuit. We will show later that with high probability only a small fraction of FSD search paths are typically needed, especially when SNR is high enough, thus the proposed ST-FSD is able to offer a desirable performance-complexity trade-off.

Clearly, for the proposed scheme, it is vital to guarantee that the algorithm is able to find the candidate solutions that lie close to the final solution as early as possible and thus expedite the tree search. This can be achieved by adopting a Schnorr-Euchner (SE) enumeration strategy [8], where the child nodes of a parent node are generated in ascending order of their ED metrics and thus the path with the highest probability of finding the best solution can be searched first. Also, an optimum threshold value T_{th} must be selected in order to achieve a good trade-off between performance and complexity. In following subsections, we will discuss how to determine the optimum threshold by analysing the statistical properties of the ED metrics of the estimates.

4.1 Optimum Statistical Threshold

In statistics, there are two hypotheses about the estimate $\hat{\mathbf{s}}$ consisting of the null hypothesis $\mathcal{H}_0(\hat{\mathbf{s}}=\mathbf{s})$ and the alternative hypothesis $\mathcal{H}_1$ $(\hat{\mathbf{s}}\neq\mathbf{s})$, which represent the correct and wrong estimates, respectively. Thus we denote the normalized ED metric as

$$\rho = \frac{\|\mathbf{r}-\mathbf{H}\hat{\mathbf{s}}\|^2}{\sigma^2} \quad (7)$$

Consequently, under the above two hypotheses, Eq. (7) can be formulated as

$$\rho = \begin{cases} \dfrac{\|\mathbf{r}-\mathbf{H}\hat{\mathbf{s}}\|^2}{\sigma^2} = \dfrac{\|\mathbf{w}\|^2}{\sigma^2}, & \text{if } \mathcal{H}_0(\hat{\mathbf{s}}=\mathbf{s}) \\ \dfrac{\|\mathbf{r}-\mathbf{H}\hat{\mathbf{s}}\|^2}{\sigma^2} = \dfrac{\|\mathbf{H}(\mathbf{s}-\hat{\mathbf{s}})+\mathbf{w}\|^2}{\sigma^2}, & \text{if } \mathcal{H}_1(\hat{\mathbf{s}}\neq\mathbf{s}) \end{cases} \quad (8)$$

Therefore, in case of the null hypothesis, ρ is the sum of 2N squared zero mean Gaussian distributions, hence is a scaled central chi-square distribution (CSD) random variable with 2N degrees of freedom and mean 2N, denoted as χ^2_{2N}. The probability density function (PDF) of ρ is given by

$$f_{\chi^2_{2N}}(\rho)=\begin{cases}\dfrac{1}{2^N\Gamma(N)}e^{-\frac{\rho}{2}}\rho^{N-1}, & \rho>0\\ 0, & \rho\le 0\end{cases} \tag{9}$$

In case of the alternative hypothesis, ρ is the sum of 2N squared non-zero mean Gaussian distributions, thus is a non-central chi square distribution (Non-CSD) random variable with 2N degrees of freedom and mean $2N+\gamma$, denoted as $\chi^2_{2N,\gamma}$, where γ is the non-central parameter and is defined as

$$\gamma=\left\|\mathbf{H}(\mathbf{s}-\hat{\mathbf{s}})\right\|^2\big/\sigma^2 \tag{10}$$

The PDF of ρ is given by

$$f_{\chi^2_{2N,\gamma}}(\rho)=\begin{cases}\displaystyle\sum_{k=0}^{\infty}\frac{(\frac{\gamma}{2})^k}{k!\Gamma(N+k)}\frac{\rho^{N+k-1}}{2^{N+k}}e^{-\frac{\gamma+\rho}{2}}, & \rho>0\\ 0, & \rho\le 0\end{cases} \tag{11}$$

To this end, for every estimate $\hat{\mathbf{s}}$, we present the following threshold check criterion

$$\begin{cases}\text{if } \rho\le T_{th}, \text{ then assume } \hat{\mathbf{s}} \text{ is the correct estimate}\\ \text{if } \rho> T_{th}, \text{ then assume } \hat{\mathbf{s}} \text{ is the wrong estimate}\end{cases} \tag{12}$$

Based on statistical signal detection theory in Chapter 3 of [9], we have a lemma as follows.

Lemma: For a particular error pattern with a specific non-central chi square distribution (γ), the intersection point of the PDF curves of χ^2_{2N} and $\chi^2_{2N,\gamma}$, is the optimum threshold T_{th}.

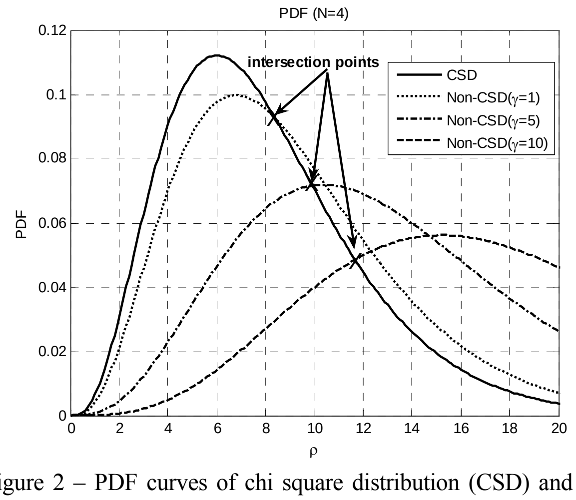

Figure 2 – PDF curves of chi square distribution (CSD) and non-central chi square distribution (Non-CSD) with N=4.

The PDF curves of χ^2_{2N} and $\chi^2_{2N,\gamma}$ with N=4 are shown in Figure 2. It can be seen that the intersection points of χ^2_{2N} and $\chi^2_{2N,\gamma}$ follow a linear function of N and γ, denoted as $J(N,\gamma)$, which can be obtained by the function approximation method as follows

$$J(N,\gamma)=\alpha*\gamma+2N \tag{13}$$

where α is the coefficient depending on the system parameter N. As an example, for a system with N=4, it is found that $\alpha\approx 0.3465$. Based on the above lemma and monotonically increasing property of the $J(N,\gamma)$, we have following corollary.

Corollary: For a given system with various error patterns, the intersection point of the PDF χ^2_{2N} and $\chi^2_{2N,\gamma_{\min}}$, i.e. $J(N,\gamma_{\min})$, is the optimum threshold T_{th}, where $\gamma_{\min}$ is the minimum value of γ.

However, we noticed that the calculation of $\gamma_{\min}$ is very complicated because it needs to consider every possible error pattern in Eq. (10). Therefore, in the next subsection, we will derive a lower bound on the threshold by approximating the value of $\gamma_{\min}$ using the singular value decomposition (SVD) method, which is then used as the threshold value T_{th} for hypothesis test in Eq. (12).

4.2 Lower Bound on the Threshold

From [10], the SVD technique decomposes the channel matrix **H** into the following factored form

$$\mathbf{H}=\mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^H \tag{14}$$

where **U** and **V** are unitary matrices, and $\mathbf{\Lambda}$ is the diagonal matrix of singular values of **H** as follows

$$\mathbf{\Lambda}=\begin{bmatrix}\lambda_1 & 0 & \cdots & 0\\ 0 & \lambda_2 & \cdots & 0\\ \vdots & & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_N\end{bmatrix} \tag{15}$$

where $\lambda_1\ge\lambda_2\ge\cdots\ge\lambda_N$. Note that the smallest singular value is $\lambda_{\min}=\lambda_N$.

From matrix algebra, it is known that for any vector **x**

$$\left\|\mathbf{Hx}\right\|^2\ge\lambda^2_{\min}\left\|\mathbf{x}\right\|^2 \tag{16}$$

Therefore, based on the Eq. (10) and Eq. (16), the lower bound of $\gamma_{\min}$ can be calculated as

$$\begin{aligned}\gamma_{\min}&=\min(\left\|\mathbf{H}(\mathbf{s}-\hat{\mathbf{s}})\right\|^2\big/\sigma^2)\\ &\ge\lambda^2_{\min}\left\|\mathbf{s}-\hat{\mathbf{s}}\right\|^2\big/\sigma^2\ge\lambda^2_{\min}d^2_{\min}\big/\sigma^2\end{aligned} \tag{17}$$

where $d^2_{\min}=\min_{i\ne j}\left\|(s_i-s_j)\right\|^2$ is the minimum squared distance of two constellation points. Thus, given a specific modulation scheme and channel matrix, the lower bound of $\gamma_{\min}$ is calculated from Eq. (17) and the approximate intersection point $J(N, \gamma_{\min})$ is computed from Eq. (13) that then is used as the practical threshold T_{th}, which is clearly adaptive to the channel conditions.

5. SIMULATION RESULTS AND DISCUSSIONS

Figure 3 shows the BER performance of the SD, FSD and ST-FSD decoders using uncoded 4-QAM, 16-QAM and 64-QAM modulation for a 4×4 system. The distribution of candidates adopted by the FSD is $\mathbf{n}_s = \{P, 1, 1, 1\}$, where P=4, 16, 64, respectively, , which has been proven to achieve quasi-ML performance [5]. As expected, the FSD shows very small BER degradation compared to the original SD and the proposed ST-FSD scheme has almost same performance as the FSD for all considered modulation types. However, there is a considerable reduction in computational complexity for the ST-FSD compared with its FSD counterpart, which is clearly shown in Figure 4. The relative computational complexity is defined as the average number of nodes visited by the ST-FSD divided by that of the FSD. We can observe that the proposed scheme saves at least 42%, 60%, and 68% computational load of the FSD for 4-QAM, 16-QAM and 64-QAM, respectively. The computational complexity gain of the proposed scheme is also quite robust to SNR variations. Also, the complexity reduction of higher-order modulation, e.g., 64-QAM, is bigger than that of lower order ones, e.g., 4-QAM, which makes the ST-FSD scheme even more helpful for realising high data rates over MIMO systems.

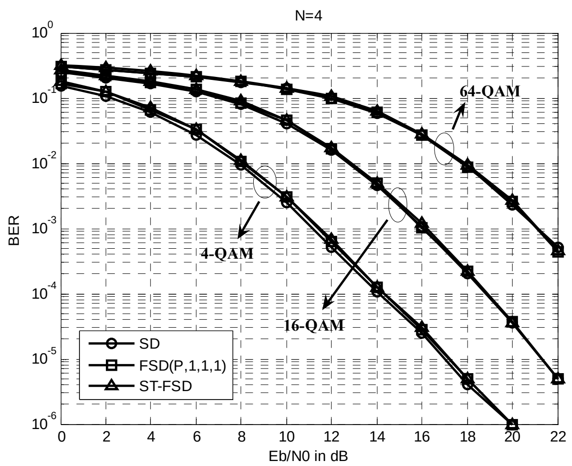

Figure 3 – BER performance of the SD, FSD and ST-FSD as a function of the Eb/N0 in a 4×4 MIMO system.

As an example, details of the number of paths chosen by ST-FSD for 16-QAM modulation are shown in Figure 5. For each Eb/N0, there are *P+1* bar columns which represent the number of paths traversed by the ST-FSD in ascending order. Specifically, for the first *P* columns, the pth column denotes the percentage of time that ST-FSD traversed p paths in total and the estimates from the first to the $(p-1)$th paths cannot satisfy the predesigned threshold, while the pth path is able to meet the threshold check and the ST-FSD terminates the tree search after that. The *P+1* column (unshaded column) denotes the likelihood that the ST-FSD traverses all *P* paths and none of those results satisfied the threshold check criterion. Then the ST-FSD chooses the path which gives the minimum ED metric as its final solution, which is obviously the same as the FSD. For instance, from Figure 5 it can be seen that at Eb/N0=20dB, the ST-FSD finishes the search after traversing the first path (the minimum SE path) 79% of the time while about 15% of the final results need full a 16 path search. This clearly shows that the original FSD algorithm processes many unnecessary paths and results in more redundant computation. It is evident that the ST-FSD is more efficient than the FSD due to the fact that ST-FSD usually completes after searching a reduced number of paths.

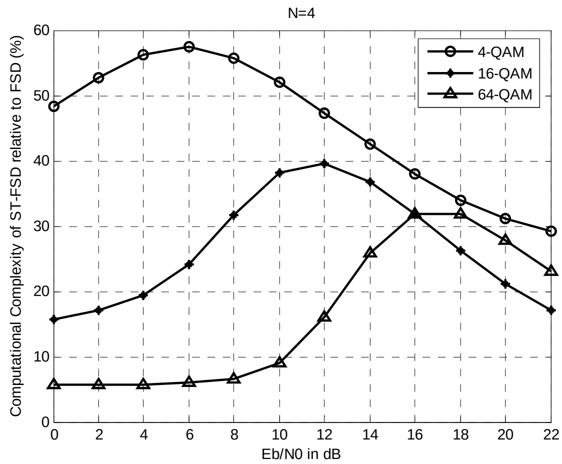

Figure 4 – Computational complexity of ST-FSD relative to FSD in a 4×4 MIMO system.

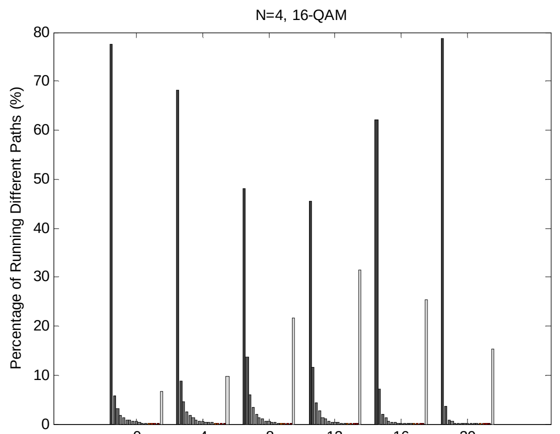

Figure 5 – Percentage of running different paths for the ST-FSD scheme in a 4×4 MIMO system with 16-QAM modulation.

Figure 6 shows the effect of SE path ordering on the proposed ST-FSD scheme in the case of 16QAM modulation. When switching off the SE ordering, the complexity gain deteriorates considerably and the largest complexity reduction shrinks from 60% to around 33%. It has thus been shown that it is favourable to combine our threshold strategy with the ordering scheme in order to expedite the algorithm tree search and achieve more complexity reduction.

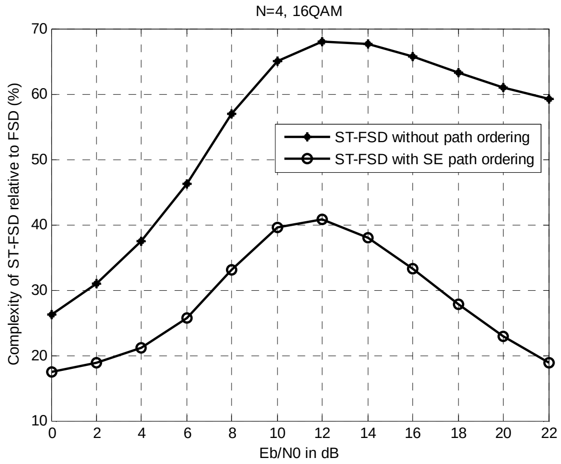

Figure 6 – The effect of the path ordering scheme on the complexity gain in 16-QAM with *N*=4.

In order to investigate the effects of varying the threshold on the BER and complexity performance, we deliberately changed the operating threshold by multiplying the threshold by a positive coefficient θ , which is chosen as 0.25, 0.5, 1, 2, and 4, respectively. As shown in Figure 7 and 8, when θ reduces below 1, the BER shows marginally enhanced performance while the computational complexity increases considerably. On the other hand, when θ increases above 1, the BER deteriorates significantly although the computational complexity decreases notably. Clearly, the proposed threshold offers a desirable trade-off between performance and complexity.

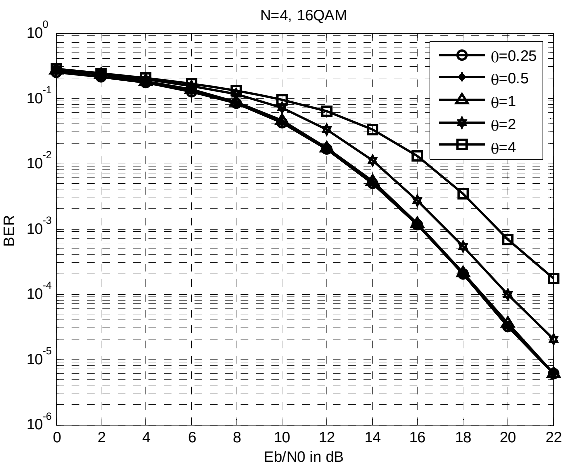

Figure 7 – The effects of varying the threshold on BER performance in 16-QAM with *N*=4.

6. CONCLUSIONS

In this paper, a novel statistical threshold assisted FSD algorithm was proposed. The conventional FSD has been restructured by employing the proposed threshold criterion as well as the SE path ordering, and it turns out that a good choice of the threshold results in an important reduction in the complexity while maintaining almost the same BER performance as the original FSD. Also of importance is the fact that the proposed scheme is effective at both low and high SNR regions and more efficient for high-order constellations, making it particularly desirable for realising high data rates over MIMO systems. Future work will include the analysis of the ST-FSD in larger MIMO systems and a real-time hardware implementation of this algorithm.

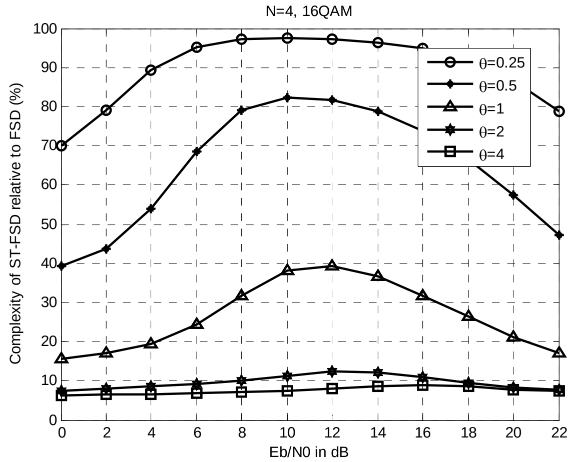

Figure 8 – The effects of varying the threshold on computational complexity in 16-QAM with *N*=4.

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