SIMILARITIES AND DIFFERENCES AMONG FILTERED MULTITONE MODULATION REALIZATIONS AND ORTHOGONAL FILTER BANK DESIGN

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ABSTRACT

This paper addresses the efficient realization of a filtered multitone (FMT) modulation system and the orthogonal filter bank (FB) design problem. We describe three possible realizations. Although they appear similar, we aim at highlighting the analogies and differences. We then consider the design of an orthogonal FB and we propose a simplified method that is derived from the exploitation of the structure of the first efficient implementation.

1. INTRODUCTION

Filter bank (FB) modulation systems deploy a transmission technique where a high rate information signal is transmitted through a wide band channel by the simultaneous modulation of a set of parallel signals at low rate. Two popular FB modulation architectures are orthogonal frequency division multiplexing (OFDM) and filtered multitone modulation (FMT) [1]. While the former uses a rectangular prototype pulse, the latter uses a confined frequency response pulse. In FMT two open problems are the efficient realization and the system design such that the FB is orthogonal. Three efficient realizations have been recently proposed in [2]-[5]. They are all based on the deployment of a discrete Fourier transform (DFT) and a polyphase FB network. Although they appear similar, in this paper we describe the differences. Further, we show that the polyphase decomposition of the signals used in the first realization allows deriving the orthogonal FB equations and writing them in a number of uncoupled subsets having a small number of parameters. This simplifies the FB design and search for optimal pulses. Numerical examples are also reported.

2. FMT SCHEME AND NOTATION

We consider an FMT scheme as depicted in Fig. 1 where the discrete-time transmitted signal at the output of the synthesis FB, $x : Z(1) \to \mathbb{C}$, is obtained by the modulation of M data streams at low rate $a^{(k)} : Z(N) \to \mathbb{C}$, with $k \in \{0,1,\cdots,M-1\}$, that belong to the *QAM* signal set. $Z(N)$ denotes the set of integer numbers multiple of N. Using the operator notation, as summarized in Tab. 1, the transmitted signal can be written as

$$x(n) = \sum_{k=0}^{M-1} \sum_{l \in \mathbb{Z}} a^{(k)}(Nl) g^{(k)}(n-Nl) = \sum_{k=0}^{M-1} \left[\mathscr{I}_N[a^{(k)}] * g^{(k)} \right](n) \tag{1}$$

where M is the number of channels of the transmitter, and N is the sampling-interpolation factor. According to (1), the signals $a^{(k)}(Nn)$ are upsampled by a factor N and are filtered by the modulated pulses $g^{(k)}(n) = g(n) W_M^{-kn}$, with $g(n)$ being the prototype filter of the synthesis bank and $W_M^{kn} = e^{-j\frac{2\pi}{M}kn}$. Then, the sub-channel signals are summed and transmitted over the channel. After propagation through the channel and with the addition of background

The work of this paper has been partially supported by the European Communitys Seventh Framework Programme FP7/2007-2013 under grant agreement n. 213311, project OMEGA-Home Gigabit Networks.

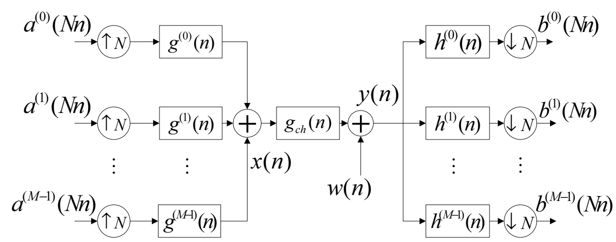

Figure 1: Modified FMT scheme.

OPERATOR NOTATION	
Convolution	$[x * h](n)$
Translation	$\tau^a[x](n) = x(n+a)$
Sampling	$\mathscr{C}_N[x](Nn) = x(Nn)$
Interpolation	$\mathscr{I}_N[y](n) = \begin{cases} y(n) & \text{if} \quad n \in Z(N) \\ 0 & \text{otherwise} \end{cases}$
SIGNALS AND CONSTANTS	
M	number of sub-channels
N	sampling - interpolation factor
M_1	$l.c.m.(M,N)$ (least common multiple)
M_0	M_1/N
N_0	M_1/M
L_f	prototype filter length
L_M	L_f/N
L_N	L_f/M
W_M	$e^{-j\frac{2\pi}{M}}$
$a^{(k)}(Nn)$	data input at the $k-th$ sub-channel
$b^{(k)}(Nn)$	output at the $k-th$ sub-channel
$g(n)$	synthesis bank prototype filter
$h(n)$	analysis bank prototype filter
$\tilde{a}^{(k)}(Nn)$	$a^{(k)}(Nn)\, W_M^{kNn}$
$\tilde{b}^{(k)}(Nn)$	$b^{(k)}(Nn)\, W_M^{-kNn}$
$g^{(k)}(n)$	$g(n)\, W_M^{-kn}$
$h^{(k)}(n)$	$h(n)\, W_M^{-kn}$
$div[A,B]$	$floor(A/B)$
$mod[A,B]$	$A - div[A,B]\,B$
$y_a = \sum_{b=0}^{N_0-1} x_{a+Mb}$ $a \in \{0,..,M-1\}$	$M-$periodic repetition of x_i $i \in \{0,..,N_0M-1\}$
$x_a = y_{mod[a,M]}$ $a \in \{0,..,N_0M-1\}$	N_0-cyclic extension of y_i $i \in \{0,..,M-1\}$

Table 1: Operator notation and useful signals and constants.

noise, the received signal $y(n)$ is processed by the analysis FB whose outputs are

$$b^{(k)}(Nn) = \sum_{m \in \mathbb{Z}} y(m) h^{(k)}(Nn-m) = \mathscr{C}_N \left[y * h^{(k)} \right](Nn) \tag{2}$$

In Tab. 1 we summarize the notation related to the operators, constants and signals used in this paper.

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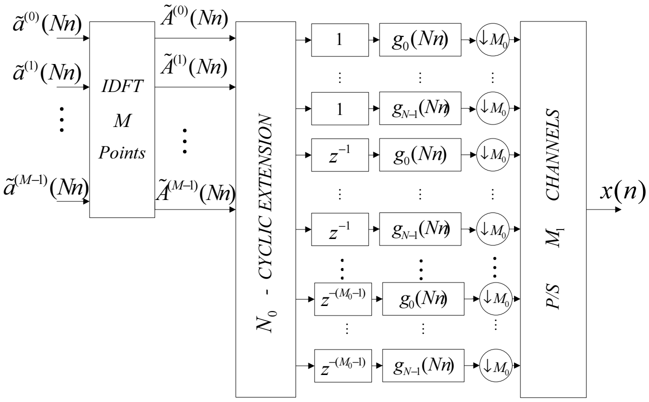

Figure 2: Syntesis bank (method A).

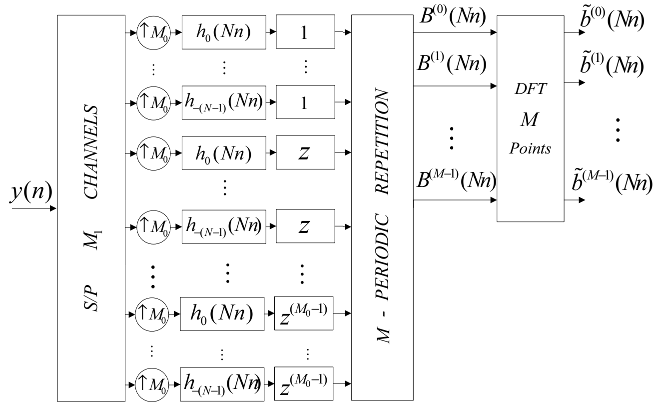

Figure 3: Analysis bank (method A).

3. REALIZATION A: M_1-ORDER POLYPHASE DECOMPOSITION OF THE SIGNALS

3.1 Synthesis bank (method Tonello)

A first efficient realization of the synthesis bank is derived if we perform a polyphase decomposition of order $M_1 = M_0 N = N_0 M = l.c.m.[M,N]$ of the signal x. Following [2], the polyphase components $x_i : \mathbb{Z}(M_1) \to \mathbb{C}$, of x, with $i \in \{0,1,\cdots,M_1-1\}$, can be written using the operator notation as follows (details are omitted for space limitations)

$$x_i(M_1 n) = \mathscr{C}_{M_0}\left[A^{(mod[i,M])} * \tau^{-div[i,N]}\left[g_{mod[i,N]}\right]\right](M_1 n) \quad (3)$$

where $A^{(l)}$, with $l \in \{0,1,\cdots,M-1\}$, is the M-point IDFT of the data symbols modulated by W_M^{kNn}, $\tilde{a}^{(k)}(Nn) = a^{(k)}(Nn)W_M^{kNn}$, i.e., $A^{(l)}(Nn) = \sum_{k=0}^{M-1} \tilde{a}^{(k)}(Nn)\, W_M^{-kl}$. The N-order polyphase components of the prototype filter g are denoted with $g_p(Nn) = \mathscr{C}_N\left[\tau^p[g]\right](Nn)$ with $p \in \{0,1,\cdots,N-1\}$.

Therefore, as shown in Fig. 2, the synthesis FB realization comprises the following operations: the data streams $\tilde{a}^{(k)}$ are processed by an M-point IDFT block, the output block is cyclically extended to a block of size M_1, and filtered, after a delay, with the N-order polyphase components of the prototype pulse. Finally, the filter outputs are sampled by a factor M_0 and parallel-to-serial converted.

3.2 Analysis bank (method Tonello)

According to [2], the efficient realization of the analysis FB is obtained with a M_1-order polyphase decomposition of the signal $y(n)\, W_M^{-in}$. The output signals $\tilde{b}^{(k)}$, with $k \in \{0,1,\cdots,M-1\}$, can then be written as follows

$$\tilde{b}^{(k)}(Nn) = b^{(k)}(Nn)W_M^{-kNn} = \sum_{p=0}^{M-1}\left(\sum_{m=0}^{N_0-1}\left[\mathscr{I}_{M_0}[y_{p+Mm}] * \tau^{div[p+Mm,N]}[h_{-mod[p+Mm,N]}]\right](Nn)\right)W_M^{-kp}$$

where $y_k : \mathbb{Z}(M_1) \to \mathbb{C}$, are the M_1-order polyphase components of the received signal y that are defined as $y_k(M_1 n) = \mathscr{C}_{M_1}\left[\tau^k[y]\right](M_1 n)$ with $k \in \{0,1,\cdots,M_1-1\}$, and $h_{-l} : \mathbb{Z}(N) \to \mathbb{C}$ are the N-order polyphase components of the prototype filter h defined as $h_{-l}(Nn) = \mathscr{C}_N\left[\tau^{-l}[h]\right](Nn)$ with $l \in \{0,1,\cdots,N-1\}$.

Therefore, as shown in Fig. 3, the analysis FB realization comprises the following operations: the received signal is serial-to-parallel converted with a converter of size M_1, the output signals are upsampled by a factor M_0, filtered with the N-order polyphase components of the prototype pulse. Then, after a delay, the periodic repetition with period M of the block of coefficients of size M_1 is applied. Finally, the M-point DFT is performed.

4. REALIZATION B: M_1-ORDER POLYPHASE DECOMPOSITION OF THE PULSES

4.1 Syntesis bank (method Siclet et al.)

In a second method [3]-[4], the efficient realization of the synthesis FB is obtained by performing an M_1-order polyphase decomposition of the filter $g^{(k)}$. It can be shown that this yields the components $x_\alpha : \mathbb{Z}(N) \to \mathbb{C}$, with $\alpha \in \{0,1,\cdots,N-1\}$,

$$x_\alpha(Nn) = \sum_{\beta=0}^{M_0-1}\left[\bar{A}^{(mod[\alpha+N\beta,M])} * \tau^{-\beta}\left[\mathscr{I}_{M_0}[g_{\alpha+N\beta}]\right]\right](Nn) \quad (4)$$

In (4), $\bar{A}^{(l)} : Z(N) \to \mathbb{C}$, with $l \in \{0,1,\cdots,M-1\}$, is the M-point IDFT of $a^{(k)}$, and $g_i : \mathbb{Z}(M_1) \to \mathbb{C}$, are the M_1-order polyphase components of the prototype filter g that are defined as $g_i(M_1 n) = \mathscr{C}_{M_1}\left[\tau^i[g]\right](M_1 n)$ with $i \in \{0,1,\cdots,M_1-1\}$.

Therefore, the sythesis FB realization comprises the following operations: the data signals $a^{(k)}$ are processed with an M-IDFT, the output block is cyclically extended to a block of size M_1, and filtered, after a delay, with the M_1-order polyphase components of the prototype pulse. The output blocks of size M_1 are periodically repeated with period N and parallel-to-serial converted by a converter of size N.

4.2 Analysis bank (method Siclet et al.)

In this second method, the efficient realization of the analysis FB is obtained by exploiting the M_1-order polyphase decomposition of the pulses $h^{(k)}$ with $k \in \{0,1,\cdots,M-1\}$. Following [3], the FB outputs can be written as

$$b^{(k)}(Nn) = \sum_{p=0}^{M-1}\left(\sum_{m=0}^{N_0-1}\left[\tau^{div[p+Mm,N]}\left[y_{mod[p+Mm,N]} * \mathscr{I}_{M_0}[h_{-p-Mm}]\right]\right](Nn)\right)W_M^{kp}$$

where $y_i : \mathbb{Z}(N) \to \mathbb{C}$, are the N-order polyphase components of the received signal y that are defined as $y_i(Nn) = \mathscr{C}_N\left[\tau^i[y]\right](Nn)$ with $i \in \{0,1,\cdots,N-1\}$. The M_1-order polyphase components of the prototype filter h are defined as $h_{-l}(M_1 n) = \mathscr{C}_{M_1}\left[\tau^{-l}[h]\right](M_1 n)$ with $l \in \{0,1,\cdots,M_1-1\}$. Therefore, this realization comprises the following operations: the received signal is serial-to-parallel converted by a size N converter, it is filtered with the M_1-order polyphase components of the prototype pulse after appropriate delays. Finally, a periodic repetition with period M on the output blocks is computed, and an M-DFT is performed.

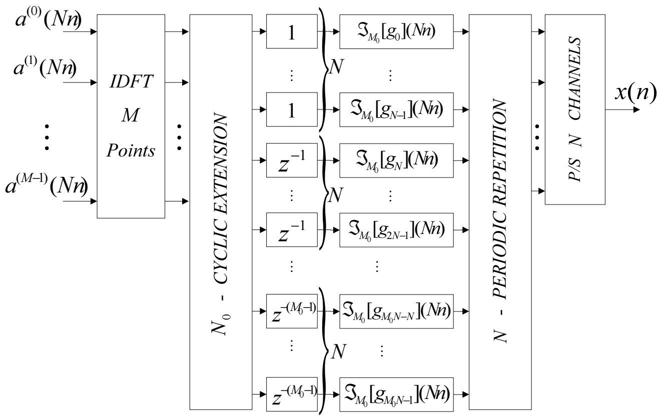

Figure 4: Synthesis bank (method B).

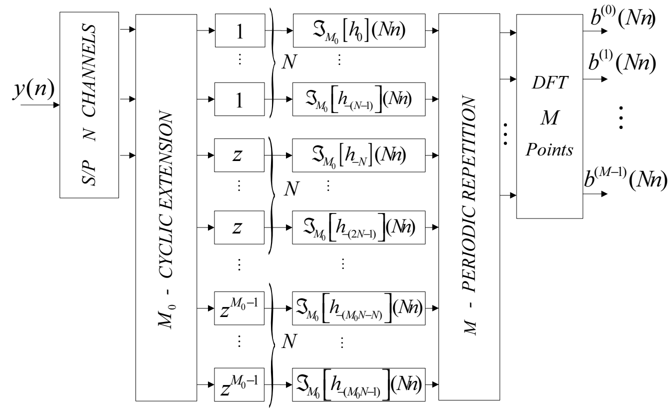

Figure 5: Analysis bank (method B).

5. REALIZATION C: L_F-ORDER POLYPHASE DECOMPOSITION OF THE PULSES

5.1 Synthesis bank (method Weiss et al.)

The third method of realizing the synthesis FB is described in [5]. It starts from the assumption that the prototype pulse $g = [p_0, p_1, \cdots, p_{L_f-1}]$ has length $L_f = L_M N = L_N M$, i.e., without loss of generality, a multiple of both M and N. Then, if we exploit the L_f-order polyphase decomposition of the filter $g^{(k)}$, each having a single coefficient, the α-th N-order polyphase component of the signal x can be written as

$$x_\alpha(Nn) = \sum_{\beta=0}^{M_0-1} \tau^{-\beta} \left[\bar{A}^{(mod[\alpha+N\beta,M])} \times p_{\alpha+N\beta} \right](Nn)$$

Therefore, the realization comprises the following operations: the data signals $a^{(k)}$ are processed with an M-IDFT, the output blocks are cyclically extended, to form a block of size $L_N M$. Then, the outputs after a proper delay are multiplied by the polyphase coefficients of the prototype filter. Each output block is periodically repeated with period N, and parallel-to-serial converted with a converter of size N.

5.2 Analysis bank (method Weiss et al.)

According to [5], the efficient realization of the analysis FB is obtained exploiting the L_f-order polyphase decomposition of the filter $h^{(k)}(n) = g^{(k)*}(-n)$ with $k \in \{0, 1, \cdots, M-1\}$. It can be shown that the FB outputs are obtained as follows

$$b^{(k)}(Nn) = \sum_{j=0}^{M-1} \left(\sum_{m=0}^{L_N-1} \tau^{div[j+Mm,N]} \left[y_{mod[j+Mm,N]} \times p_{j+Mm} \right](Nn) \right) W_M^{kj}$$

where $y_l : \mathbb{Z}(N) \to \mathbb{C}$ are the N-order polyphase components of the received signal y defined as $y_l(Nn) = \mathscr{C}_N \left[\tau^l [y] \right](Nn)$ with $l \in \{0, 1, \cdots, N-1\}$ and p_k is the k-th coefficient of the filter g.

Therefore, the received signal is serial-to-parallel converted with a converter of size N. The outputs are delayed and multiplied with the coefficients of the prototype filter. The resulting block is periodically repeated with period M, and finally, an M-DFT is applied.

6. DIFFERENCES AMONG FMT REALIZATIONS

All three realizations deploy an M-point DFT, and essentially differ in the MIMO polyphase FB network which has size $M_1 x M_1$ in the first and second realization, while it has size $L_f x L_f$ in the third realization. Furthermore, the polyphase components of the pulses have different length as summarized in Tab. 2. When $L_f = M_1$ the implementations B and C are equivalent. In Tab. 2 we also report the complexity of the three structures in terms of number of complex operations per unit sampling time. They have essentially identical complexity, although the hardware realization may change because of the different way polyphase filtering is done.

Another difference is that when we derive the perfect reconstruction (orthogonality) conditions in matrix form from the efficient realization, we obtain a different structure that can be exploited in the design and search of optimal orthogonal pulses.

Polyphase Filter Length	Transmitter Complexity	Receiver Complexity
Method A		
L_f/N	$\frac{\alpha M \log M + M_0 N\, 2 \left\lceil \frac{L_f}{M_0 N} \right\rceil}{N}$	$\frac{\alpha M \log M + M_0 N\, 2 \left\lceil \frac{L_f}{M_0 N} \right\rceil - 1}{N}$
Method B		
L_f/M_1	$\frac{\alpha M \log M + M_0 N\, 2 \left\lceil \frac{L_f}{M_0 N} \right\rceil - 1}{N}$	$\frac{\alpha M \log M + M_0 N\, 2 \left\lceil \frac{L_f}{M_0 N} \right\rceil - 1}{N}$
Method C		
1	$\frac{(\alpha M \log M + 2\, L_f - 1)}{N}$	$\frac{(\alpha M \log M + 2\, L_f - 1)}{N}$

Table 2: Differences among FMT implementations.

7. PERFECT RECONSTRUCTION AND ORTHOGONALITY (EXPLOITING REALIZATION A)

To derive perfect reconstruction conditions we can exploit the realization A of Fig. 2 and 3. Perfect reconstruction is achieved if the M-IDFT output coefficients at the transmitter $A^{(k)}$ are identical (despite a delay) to the input block $B^{(k)}$ of coefficients to the M-DFT at the receiver. In the Appendix A we show that if we perform a further polyphase decomposition of the filters g_i and h_i of order M_0 to remove the sampler and the interpolator in Fig. 2 and 3, the relation between the polyphase components of the output $B^{(k)}$ and the input $A^{(k)}$ becomes

$$B_\beta^{(a)}(M_1 n) = \sum_{\alpha=0}^{M_0-1} \left[A_\alpha^{(a)} * \sum_{b=0}^{N_0-1} g'_{Mb+a-N\alpha} * h'_{N\beta-a-Mb} \right](M_1 n) \quad (5)$$

where the signal $A_\alpha^{(a)} : Z(M_1) \to \mathbb{C}$ and $B_\beta^{(a)} : Z(M_1) \to \mathbb{C}$ with $\alpha, \beta \in \{0, 1, \cdots, M_0 - 1\}$ are the M_0 order polyphase component of $A^{(a)}$ and $B^{(a)}$:

$$A_\alpha^{(a)}(M_1 n) = \mathscr{C}_{M_0} \left[\tau^\alpha A^{(a)} \right](M_1 n) \qquad \alpha \in \{0, 1, \cdots, M_0 - 1\}$$

$$B_\beta^{(a)}(M_1 n) = \mathscr{C}_{M_0} \left[\tau^\beta B^{(a)} \right](M_1 n) \qquad \beta \in \{0, 1, \cdots, M_0 - 1\}$$

and $g'_i : \mathbb{Z}(M_1) \to \mathbb{C}$, $h'_i : \mathbb{Z}(M_1) \to \mathbb{C}$ with $i \in \{0, 1, \cdots, M_1 - 1\}$ are the i-th component of the M_0 order polyphase component of g_i and h_i. As it is shown in the Appendix A they are equal to the M_1-order polyphase components of the prototype pulse g and h, i.e.,

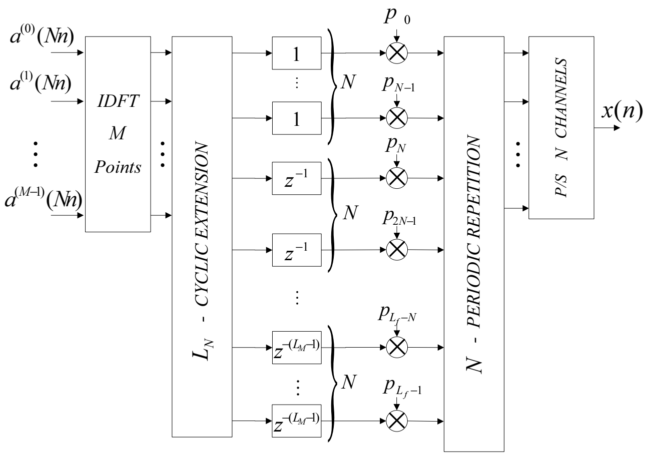

Figure 6: Synthesis bank (method C).

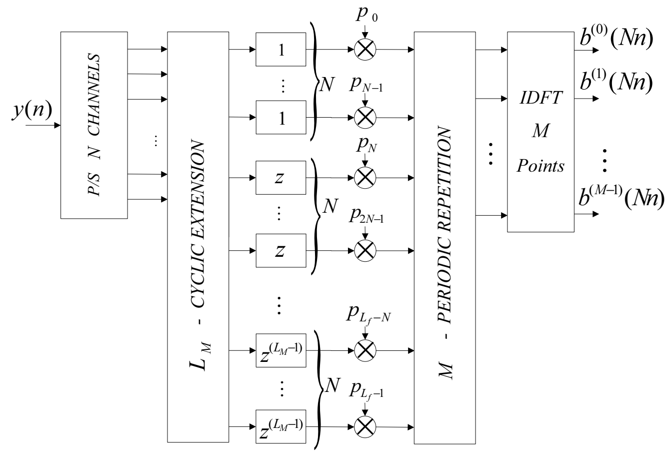

Figure 7: Analysis bank (method C).

$$g_i'(M_1 n) = \mathscr{C}_{M_1}\left[\tau^i[g]\right](M_1 n) \qquad i \in \{0,1,\cdots,M_1-1\}$$

$$h_i'(M_1 n) = \mathscr{C}_{M_1}\left[\tau^i[h]\right](M_1 n)$$

Therefore, from (5) the *perfect reconstruction* condition becomes

$$\sum_{b=0}^{N_0-1}\left[g'_{Mb+a-N\alpha} * h'_{N\beta-a-Mb}\right](M_1 n) = \delta_m\,\delta_{\alpha-\beta} \qquad (6)$$

with δ_k being the Kronecker delta. Applying the Z-transform to (6) the relation becomes

$$\sum_{b=0}^{N_0-1} G'_{Mb+a-N\alpha}(z)\, H'_{N\beta-a-Mb}(z) = \delta_{\alpha-\beta}$$

Therefore, if we define the following $M_0 \times N_0$ matrix:

$$\mathbf{G}_a(z) = \begin{bmatrix} G'_a(z) & \cdots & G'_{(N_0-1)M+a}(z) \\ G'_{a-N}(z) & \cdots & G'_{(N_0-1)M+a-N}(z) \\ \vdots & \vdots & \vdots \\ G'_{a-(M_0-1)N}(z) & \cdots & G'_{(N_0-1)M+a-(M_0-1)N}(z) \end{bmatrix}^T$$

and we assume a matched analysis FB, i.e., $\mathbf{H}_{-a}(z) = \mathbf{G}_a^{T*}(1/z^*)$ the orthogonality conditions can be written in matrix form as

$$\mathbf{H}_{-a}(z)\,\mathbf{G}_a(z) = \mathbf{I}_{M_0} \qquad a \in \{0,1,\cdots,M-1\} \qquad (7)$$

It is interesting to note that the FB is orthogonal when every submatrix in (7) is orthogonal. Further, for certain prototype pulse lengths, each sub-matrix contains pulse coefficients that are distinct from those in another sub-matrix. In these cases, every sub-matrix is uncoupled and the orthogonal design is simplified since we obtain sets of uncoupled equations. For example, if we assume $M = 1024$ sub-channels, and we choose $M_0 = 2$ and $N_0 = 3$ (which implies $N = 1536$), and a pulse with length $M_1 = M_0 N = 3072$, the orthogonality relations yield 512 matrices with 2 variables each. In turn, this implies that we need to independently solve 512 subsystems with only 2 variables each.

Orthogonal matrices can be constructed via the parametrization with angles as proposed in [3]. For every choice of angles the system is orthogonal. Then, a cost function can be defined to obtain optimal pulses. For instance, to build a FB with optimal sub-channel spectral containment we can search for pulses that maximize the in-band-to-out-of-band energy ratio. In the next section we describe a specific example.

7.1 Example of prototype pulse design

Let us assume a transmission system with M being a power of 2, $M_0 = 2$, $N_0 = 3$, and $L_f = 3M$. Hence, $N = 3M/2$. The submatrices in (7) have the following structure

$$\mathbf{G}_a(z) = \begin{bmatrix} p_a & p_{M+a} & p_{2M+a} \\ z^{-1}\,p_{M_1+(a-N)} & z^{-1}\,p_{M_1+(M+a-N)} & p_{2M+a-N} \end{bmatrix}^T$$

with $a \in \{0,1,\cdots,M/M_0-1\}$. If a filter coefficient is present in the submatrix $\mathbf{G}_i$, it cannot be present in any other submatrix $\mathbf{G}_j$ with $j \neq i$. This implies that the subsystems are uncoupled.

Now, the orthogonality conditions for the a-th subsystem are given by the following equations

$$\begin{cases} p_a^2 + p_{M+a}^2 + p_{2M+a}^2 = 1 \\ p_{M_1+(a-N)}^2 + p_{M_1+(M+a-N)}^2 + p_{2M+a-N}^2 = 1 \\ p_a\, p_{M_1+(a-N)} + p_{M+a}\, p_{M_1+(M+a-N)} = 0 \\ p_{2M+a}\, p_{2M+a-N} = 0 \end{cases}$$

with $a \in \{0,1,\cdots,M/M_0-1\}$.

In order to solve the system using a minimal set of variables, we can parameterize the pulse coefficients with angles [6]-[7]. Thus, choosing $p_{2M+a} = 0$ the system solution yields

$$\begin{cases} p_a = cos(\theta_{a,1}) \\ p_{M+a} = sin(\theta_{a,1}) \\ p_{2M+a} = 0 \\ p_{M_1+(a-N)} = -sin(\theta_{a,1})\, sin(\theta_{a,2}) \\ p_{M_1+(M+a-N)} = cos(\theta_{a,1})\, sin(\theta_{a,2}) \\ p_{2M+a-N} = cos(\theta_{a,2}) \end{cases}$$

For every choice of $(\theta_{a,1}, \theta_{a,2})$ with $a \in \{0,1,\cdots,M/M_0-1\}$ the FMT scheme is orthogonal. We then define the vectors $\boldsymbol{\theta}_1 = [\theta_{0,1}, \theta_{1,1}, \cdots, \theta_{M/M_0,1}]$ and $\boldsymbol{\theta}_2 = [\theta_{0,2}, \theta_{1,2}, \cdots, \theta_{M/M_0,2}]$ and we search for pulses that maximize the in-band energy, i.e., the maximize the following metric

$$\max_{(\boldsymbol{\theta}_1,\boldsymbol{\theta}_2)} \frac{\int_{-1/MT}^{1/MT} |G(f,\boldsymbol{\theta}_1,\boldsymbol{\theta}_2)|^2\, df}{\int_{-\infty}^{\infty} |G(f,\boldsymbol{\theta}_1,\boldsymbol{\theta}_2)|^2\, df}$$

The search can be done using optimization methods. In Fig. 8 we show the obtained pulses for $M = 64, 256, 1024$.

7.2 Performance in a wireless fading channel

In order to evaluate the robustness of the scheme, we consider transmission over a wireless dispersive fading channel having impulse response $g_{ch}(n) = \sum_{p=0}^{N_p-1} \alpha_p\, \delta_{n-p}$ where α_p are independent Gaussian variables with power $\Omega_p = \Omega_0\, e^{-pT/\gamma T}$, and γ is the normalized delay spread. The channel is truncated at $-20dB$. This channel introduces a loss of system orthogonality that we evaluate in terms of Signal-to-Interference Power Ratio $SIR = M_S/M_I$ versus delay spread γ, where M_S is the average power of the received signal, and M_I is the average interferences power. The simulation has been done for the case $M = 64, 256, 1024$, $N = 3/2M$ and $L_f = 3M$ with the pulses in Fig. 8. As a baseline, we have also considered a conventional root-raised cosine pulse with roll-of factor $R = 0.2$ and length $L_{RRC} = L_f$. As shown in Fig. 9 the orthogonal filter bank has considerable better performance especially for low values of delay spread γ. Further, with the orthogonal FMT FB the SIR performance improves as the number of sub-channels increases.

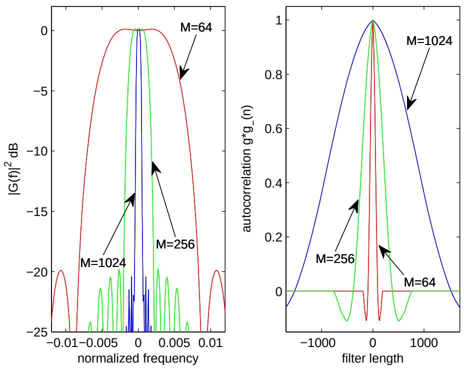

Figure 8: Pulse frequency response and autocorrelation.

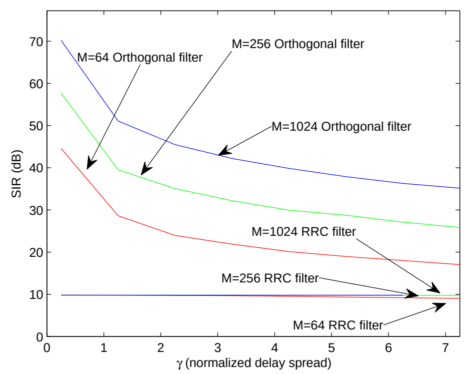

Figure 9: SIR in a dispersive fading channel.

8. SUMMARY AND CONCLUSION

In this paper we have compared three efficient realizations of a filtered multitone (FMT) modulation system. We have shown that these implementations have the same complexity in terms of complex operations, but they are different in terms of hardware implementation and matrix representation due to different polyphase filters structure. We have then considered the design of an orthogonal FB exploiting the matrix structure of the first realization (method Tonello [2]) that allows deriving a method that considerable simplifies the design of orthogonal filter banks for a subset of cases.

A. DEMONSTRATION OF PERFECT RECONSTRUCTION CONDITION FOR METHOD A

We consider the implementation A in Fig. 2 and 3. The signal of sub-channel a at the input of the receiver DFT is given by

$$B^{(a)}(Nn) = \sum_{b=0}^{N_0-1} [\mathscr{I}_{M_0}[x_{a+Mb}] * h_{-a-Mb}](Nn) \qquad (8)$$

The analysis sub-channel pulse has been obtained by the the N-order polyphase decomposition of the prototype filter h, i.e., $h_l = \mathscr{C}_N[\tau^{-l}[h]]$ with $l \in \{0,1,\cdots,M_1-1\}$. We now perform a further M_0-order polyphase decomposition of the sub-channel pulse and we obtain that it equals the M_1-order polyphase component of h. This is shown in what follows

$$\mathscr{C}_{M_0}[\tau^{\beta}[h_{-a-Mb}]] = \mathscr{C}_{M_0}[\tau^{\beta}[\mathscr{C}_N[\tau^{-a-Mb}[h]]] =$$
$$= \mathscr{C}_{M_0}[\mathscr{C}_N[\tau^{N\beta}[\tau^{-a-Mb}[h]]]] = \mathscr{C}_{M_1}[\tau^{N\beta-a-Mb}[h]] =$$
$$= h'_{N\beta-a-Mb}$$

Now, (8) can be rewritten as

$$B^{(a)}(Nn) = \sum_{b=0}^{N_0-1}\left[\mathscr{I}_{M_0}[x_{a+Mb}] * \sum_{\beta=0}^{M_0-1}\tau^{-\beta}[\mathscr{I}_{M_0}[h'_{N\beta-a-Mb}]]\right](Nn)$$
$$= \sum_{\beta=0}^{M_0-1}\tau^{-\beta}[\mathscr{I}_{M_0}[\sum_{b=0}^{N_0-1} x_{a+Mb} * h'_{N\beta-a-Mb}](Nn)$$
$$= \sum_{\beta=0}^{M_0-1}\tau^{-\beta}[\mathscr{I}_{M_0}[B_{\beta}^{(a)}]](Nn)$$

where we have defined

$$B_{\beta}^{(a)}(M_1 n) = \sum_{b=0}^{N_0-1}\left[x_{a+Mb} * h'_{N\beta-a-Mb}\right](M_1 n)$$
$$= \sum_{b=0}^{N_0-1}\left[\mathscr{C}_{M_0}[A^{(a+Mb)} * g_{a+Mb}] * h'_{N\beta-a-Mb}\right](M_1 n) \qquad (9)$$

Similarly to what has been done for the pulses h_l, we can perform an M_0-order polyphase decomposition of the pulses g_l, and obtain that it equals the M_1-order polyphase decomposition of the prototype pulse g, i.e.,

$$\mathscr{C}_{M_0}[\tau^{-\alpha}[g_{a+Mb}]] = \mathscr{C}_{M_1}[\tau^{a+Mb-N\alpha}[h]] = g'_{a+Mb-N\alpha}$$

It follows that (9) can be written as

$$B_{\beta}^{(a)}(M_1 n) =$$
$$= \sum_{b=0}^{N_0-1}\left[\mathscr{C}_{M_0}[A^{(a)} * \sum_{\alpha=0}^{M_0-1}\tau^{\alpha}[\mathscr{I}_{M_0}[g'_{a+Mb-N\alpha}]]] * h'_{N\beta-a-Mb}\right](M_1 n)$$
$$= \sum_{\alpha=0}^{M_0-1}\left[\mathscr{C}_{M_0}[\tau^{\alpha}[A^{(a)}]] * \sum_{b=0}^{N_0-1} g'_{a+Mb-N\alpha} * h'_{N\beta-a-Mb}\right](M_1 n)$$

Finally, if we define $A_{\alpha}^{(a)} = \mathscr{C}_{M_0}[\tau^{\alpha}[A^{(a)}]]$ the M_0-order polyphase component of $A^{(a)}$, we can rewrite

$$B_{\beta}^{(a)}(M_1 n) = \sum_{\alpha=0}^{M_0-1}\left[A_{\alpha}^{(a)} * \sum_{b=0}^{N_0-1} g'_{a+Mb-N\alpha} * h'_{N\beta-a-Mb}\right](M_1 n)$$

which corresponds to (8).

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