MAXIMIZATION OF USEFUL-TO-NULL SUBCARRIER ENERGY RATIO FOR BLIND MULTICARRIER SIMO CHANNEL SHORTENING

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ABSTRACT

We propose a blind adaptive channel shortening technique to design a time domain finite impulse response equalizer (TEQ) in multicarrier (MC) single-input, multiple-output systems. Exploiting the presence of null subcarriers in many practical MC transmission systems, the proposed approach attempts to maximize the ratio of the energy of useful subcarriers to that of the interference and noise present in null subcarrier bins. An adaptive formulation of this criterion is also developed, in order to allow the TEQ track possible channel variations. Numerical simulations are provided, illustrating the advantages of the proposed technique over recently developed blind channel shorteners.

1. INTRODUCTION

Multicarrier (MC) modulation techniques are widely used in high speed digital communications, as they provide an effective means to combat channel frequency selectivity as long as the channel impulse response is not longer than the cyclic prefix (CP) inserted by the transmitter [1, 2]. However, whenever the CP is not sufficiently long, subcarrier orthogonality is lost, resulting in inter-carrier (ICI) as well as inter-symbol (ISI) interferences. In this case, equalization at the receiver to shorten the effective channel to an appropriate length becomes attractive, since increasing the CP extension results in reduced bandwidth efficiency.

Channel shortening is accomplished by means of a time domain equalizer (TEQ), which is a finite impulse response (FIR) filter. The TEQ is placed in cascade with the channel to produce an effective shortened impulse response. Most channel shortening schemes in the literature have been designed in the trained, nonadaptive context for Digital Subscriber Line (DSL) systems and most of them have high complexity [3]-[7]. A number of these methods have given rise to adaptive algorithms for the TEQ; however, they require the insertion of training sequences, which reduce the data throughput. When training symbols are not available, blind adaptive shortening algorithms can replace supervised approaches. Blind adaptive TEQ algorithms exploit different signal properties, such as the presence of the CP in the MC signal [9] or its autocorrelation characteristic [8].

De Courville *et al.* [10] proposed a blind adaptive TEQ that relies on the presence of null subcarriers within the transmission bandwidth, which usually serve as buffer zones to limit adjacent channel interference, and are also useful for carrier frequency offset estimation [11]. The TEQ is then adapted by minimizing a quadratic criterion based on the minimization of the energy of those frequency bins corresponding to the null subcarriers. The resulting Carrier Nulling (CN) algorithm requires neither a CP nor any training data. However, it tends to shorten the channel impulse response to a single spike rather than to a prespecified length (the CP duration). Due to this fact, its performance is expected to be suboptimal in terms of subcarrier Signal-to-Noise Ratio (SNR). Romano and Barbarossa [12] propose a frequency-hopping approach for the null tones which is claimed to achieve a Maximum Shortening SNR (MSSNR) solution. However, in practice the position of the null tones is fixed, and moreover, it is well known that the MSSNR solution does not necessarily result in good subcarrier SNR performance [13]. A different approach is adopted in [14], by combining the CN cost with another blind criterion based on CP restoration.

All of the above CN-based methods focus on the null tones and do not consider the effect of the TEQ on the carriers of interest. We propose a channel shortener that addresses both issues by considering the maximization of the energy ratio between the useful and null tones in a single-input, multiple-output (SIMO) configuration.

The paper is organized as follows. Section 2 gives the MC SIMO system model. Sections 3 and 4 present the novel shortening criterion and a blind adaptive implementation respectively. Section 5 provides some numerical results in the context of wireless systems and Section 6 concludes.

2. MULTICARRIER SYSTEM MODEL

The idea behind MC modulation is based on the observation that overlapping subcarriers can be placed closely together without interfering with each other. An easy way to do this is to map the data to be transmitted onto complex valued symbols from a given constellation (BPSK, QPSK, QAM etc.) and then transform them into the time domain using the inverse discrete Fourier transform which is usually implemented by using the Inverse Fast Fourier Transform (IFFT).

The simplified baseband equivalent MC system model is shown in Figure 1. At the transmitter and after modulation, the data sequence is converted into N parallel sub sequences, $z_{k,n}$, where $n \in \{1,2,\cdots,N\}$ refers to the subcarrier number and k is the block index, before parallel to serial (P/S) and after serial to parallel (S/P) operations the notation $z_{k,n}$ is used to represent the sample z_{kN+n}. The k-th block $\mathbf{z}_k \doteq [z_{k,1}\, z_{k,2} \cdots z_{k,N}]^T$ is used to modulate the different subcarriers by means of an IFFT operation, the result block vector is denoted by $\mathbf{x}_k \doteq [x_{k,1}\, x_{k,2} \cdots x_{k,N}]^T$

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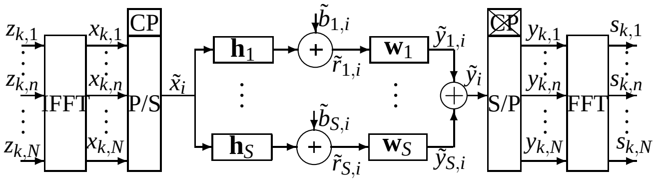

Figure 1: MC SIMO system model (N: MC block size; P: CP length; $M = N + P$, k: MC block index; i: time domain sample index; n: frequency bin index.)

and is expressed by $\mathbf{x}_k = \mathbf{F}^H \mathbf{z}_k$ where $\mathbf{F}$ is the FFT matrix, $\{\mathbf{F}\}_{k,l} \doteq \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}kl}$ and $k, l \in \{0, 1, \cdots, N-1\}$. To ensure that subcarriers remain orthogonal after propagation through the channel, the last P samples (corresponding to the cyclic prefix) of $\mathbf{x}_k$ are copied and added to the beginning of block $\mathbf{x}_k$ to form the k-th transmitted MC block (with length $M = N + P$), given by $\tilde{\mathbf{x}}_k \doteq [\tilde{x}_{kM+1}\, \tilde{x}_{kM+2} \cdots \tilde{x}_{kM+M}]^T = [x_{k,N-P+1}\, x_{k,N-P+2} \cdots x_{k,N}\, x_{k,1}\, x_{k,2} \cdots x_{k,N}]^T$. One has $\tilde{\mathbf{x}}_k = \mathbf{P}_{\text{cp}} \mathbf{x}_k$, where $\mathbf{P}_{\text{cp}} \doteq [\mathbf{\Lambda}_{P\times M}^T\, \mathbf{I}_M]^T$ and $\mathbf{\Lambda}_{P\times M}$ is obtained by picking the last P rows of the $M \times M$ identity matrix $\mathbf{I}_M$.

After the P/S operation, the data are transmitted through S channels $\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_S$. Multiple channels may be available as a result of deploying several receive antennas and/or of oversampling the received signal. The channels are modeled as finite impulse response (FIR) filters of length $L_h + 1$. If the transmitter and receiver are synchronized, then the received data at the s-th branch ($s \in \{1, 2, \cdots, S\}$) is given by

$$\tilde{r}_{s,kM+m} = \sum_{l=0}^{L_h} h_{s,l} \tilde{x}_{kM+m-l} + \tilde{b}_{s,kM+m}, \tag{1}$$

where $\tilde{b}_{s,kM+m}$ are the noise samples (assumed temporally and spatially white), and $m \in \{1, \cdots, M\}$ is the sample index within a block. For the sake of a compact notation, the decision delay is omitted in the received signal.

When $P \geq L_h + 1$, demodulation can be implemented by means of an FFT operation followed by a bank of single-tap frequency domain equalizers. On the other hand, when the channel length exceeds the CP duration, a TEQ is needed before the FFT. For SIMO transceivers, channel shortening is performed by S TEQs $\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_S$ of length $L_w + 1$, whose outputs are given by

$$\tilde{y}_{s,kM+m} = \sum_{l=0}^{L_w} w_{s,l} \tilde{r}_{s,kM+m-l} = \mathbf{w}_s^T \tilde{\mathbf{r}}_{s,kM+m}, \tag{2}$$

where $\mathbf{w}_s \doteq [w_{s,0}, w_{s,1}, \cdots, w_{s,L_w}]^T$ and $\tilde{\mathbf{r}}_{s,kM+m} \doteq [\tilde{r}_{s,kM+m}, \tilde{r}_{s,kM+m-1}, \cdots, \tilde{r}_{s,kM+m-L_w}]^T$. After adding the S TEQ outputs, one obtains

$$\tilde{y}_{kM+m} = \sum_{s=1}^{S} \tilde{y}_{s,kM+m} = \sum_{l=0}^{L_c} c_l \tilde{x}_{kM+m-l} + \sum_{s=1}^{S} \sum_{l=0}^{L_w} w_{s,l} \tilde{b}_{s,kM+m-l}, \tag{3}$$

where the effective equivalent channel of length $L_c + 1$ is

$$c_i = \sum_{s=1}^{S} c_{s,i} = \sum_{s=1}^{S} \sum_{l=0}^{L_h} h_{s,l} w_{s,i-l}. \tag{4}$$

At the receiver, the effective equivalent channel output is S/P converted into M parallel substreams to form the block $\tilde{\mathbf{y}}_k \doteq [\tilde{y}_{k,1}, \tilde{y}_{k,2}, \cdots, \tilde{y}_{k,M}]^T$, the cyclic prefix is removed from the received block $\tilde{\mathbf{y}}_k$ to obtain the k-th received block vector $\mathbf{y}_k \doteq [y_{k,1}, y_{k,2}, \cdots, y_{k,N}]^T = [\tilde{y}_{kM+P+1}, \tilde{y}_{kM+P+2}, \cdots, \tilde{y}_{(k+1)M}]^T = \mathbf{R}_{\text{cp}} \tilde{\mathbf{y}}_k$ where $\mathbf{R}_{\text{cp}} \doteq [\mathbf{0}_{N\times P}, \mathbf{I}_N]$ discards the first P symbols of $\tilde{\mathbf{y}}_k$. Thus, the discrete-equivalent MC SIMO system can be modeled in matrix-vector form as

$$\begin{aligned} \mathbf{y}_k &= \mathbf{C}_{\text{isi}} \mathbf{x}_{k-1} + \mathbf{C}_{\text{ici}} \mathbf{x}_k + \mathbf{C}_{\text{circ}} \mathbf{x}_k + \mathbf{W} \mathbf{b}_k \\ &= \mathbf{C} \hat{\mathbf{x}}_k + \mathbf{W} \mathbf{b}_k, \end{aligned} \tag{5}$$

where $\mathbf{C} \doteq [\mathbf{C}_{\text{isi}} \quad \mathbf{C}_{\text{ici}} + \mathbf{C}_{\text{circ}}]$ is the $N \times 2N$ Toeplitz matrix of the effective equivalent channel; the vector $\hat{\mathbf{x}}_k \doteq [\mathbf{x}_{k-1}^T\, \mathbf{x}_k^T]^T$ is the concatenation of two consecutive MC blocks; and $\mathbf{b}_k \doteq [\mathbf{b}_{1,kM+P+1}^T, \mathbf{b}_{2,kM+P+1}^T, \cdots, \mathbf{b}_{S,kM+P+1}^T]^T$ is the noise vector, where $\mathbf{b}_{s,kM+P+1} \doteq [\tilde{b}_{s,kM+P+1}, \tilde{b}_{s,kM+P+2}, \cdots, \tilde{b}_{s,(k+1)M}]^T$. The equivalent equalizer matrix is $\mathbf{W} \doteq [\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_S]$ where $\mathbf{W}_s$ is the $N \times N$ convolution matrix of the s-th branch TEQ. $\mathbf{C}_{\text{circ}}$, $\mathbf{C}_{\text{ici}}$ and $\mathbf{C}_{\text{isi}}$ are the $N \times N$ Toeplitz matrices that produce, respectively, the desired component of the received signal, the inter-carrier component and the inter-symbol component. $\mathbf{C}_{\text{circ}}$ is the circulant part of the channel matrix, whose first column and row are given by $[c_0, \ldots, c_P, 0, \ldots, 0]^T$ and $[c_0, 0, \ldots, 0, c_P, \ldots, c_1]$, respectively, $\mathbf{C}_{\text{isi}}$ is an upper triangular matrix, whose first row is given by $[0, \ldots, 0, c_{L_c}, \ldots, c_{P+1}]$ and $\mathbf{C}_{\text{ici}} \doteq [\mathbf{C}_1^T, \mathbf{C}_2^T]^T$ where $\mathbf{C}_1 \doteq [\mathbf{0}_{(L_c-P+1),N-(L_c-P)}, \mathbf{C}_3]$, $\mathbf{C}_3$ is $(L_c - P + 1) \times (L_c - P)$ lower triangular matrix whose first column is given by $[0, c_{P+1}, c_{P+2}, \ldots, c_{L_c}]^T$ and $\mathbf{C}_2$ is $(N - L_c + P) \times N$ Toeplitz matrix, whose first column and row are given by $[c_{P+1}, \ldots, c_{L_c}, 0, \ldots, 0]^T$ and $[c_{P+1}, 0, \ldots, 0, c_{L_c}, \ldots, c_{P+2}]$, respectively. Note that if the overall channel is within the CP extension (i.e. $c_i = 0$ for $i > P$), then $\mathbf{C}_{\text{isi}} = \mathbf{0}$ and $\mathbf{C}_{\text{ici}} = \mathbf{0}$.

An FFT operation demodulates the received MC symbol $\mathbf{y}_k$. The resulting block $\mathbf{s}_k \doteq [s_{k,1}, s_{k,2}, \cdots, s_{k,N}]^T$ is given by

$$\mathbf{s}_k = \mathbf{F}\mathbf{C}\hat{\mathbf{x}}_k + \mathbf{F}\mathbf{W}\mathbf{b}_k = \mathbf{F}\mathbf{C}\hat{\mathbf{F}}\hat{\mathbf{z}}_k + \mathbf{F}\mathbf{W}\mathbf{b}_k, \tag{6}$$

where $\hat{\mathbf{F}} \doteq \mathbf{I}_2 \otimes \mathbf{F}^H$ and $\hat{\mathbf{z}}_k \doteq [\mathbf{z}_{k-1}^T\, \mathbf{z}_k^T]^T$. Taking account of all previous relations, we derive next a blind shortening scheme based on the maximization of the useful-to-null subcarrier energy ratio.

3. USEFUL-TO-NULL SUBCARRIER ENERGY RATIO MAXIMIZATION

In MC systems it is common to have unused subcarriers embedded in the spectrum, either as pilot subcarriers or as completely unmodulated (i.e., null) subcarriers. The FFT bins corresponding to the null subcarriers contain contributions from the noise and the ICI due to an excessively long channel. When the channel is perfectly shortened, the latter contribution is zero, and therefore the minimization of the energy measured at these bins makes sense as a shortening criterion. In order to maximize the signal to interference and noise ratio (SINR) of each subcarrier, however, attention should be paid as well to the effect of the TEQ on the data-carrying tones. We propose a novel criterion based on the maximization of the ratio of the energy measured at the useful tones to that at the null tones. This maximization is performed after the FFT operation, so that the cost function can be written as

$$J \doteq \frac{\sum_{n\in V} E[|s_{k,n}|^2]}{\sum_{l\in U} E[|s_{k,l}|^2]}. \quad (7)$$

where U and V are respectively the sets of null and useful subcarriers, and $E[\cdot]$ is the expectation operation. In order to maximize J with respect to the TEQ taps, let us write the TEQ output vector $\mathbf{y}_k$ from (5) as

$$\mathbf{y}_k = \sum_{s=1}^{S} \mathbf{R}_{s,k}\mathbf{w}_s = \mathbf{R}_k\mathbf{w}, \quad (8)$$

where $\mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \cdots, \mathbf{w}_S^T]^T$, and

$$\mathbf{R}_{s,k} \doteq [\ \tilde{\mathbf{r}}_{s,kM+P+1} \quad \tilde{\mathbf{r}}_{s,kM+P+2} \quad \cdots \quad \tilde{\mathbf{r}}_{s,kM+M}\ ]^T \quad (9)$$

is the $N\times(L_w+1)$ TEQ input data matrix for the s-th branch, whereas $\mathbf{R}_k \doteq [\ \mathbf{R}_{1,k} \quad \mathbf{R}_{2,k} \quad \cdots \quad \mathbf{R}_{S,k}\ ]$. The output of the n-th FFT frequency bin is then given by

$$s_{k,n} = \mathbf{q}_n^H \mathbf{R}_k \mathbf{w}. \quad (10)$$

where $\mathbf{q}_n \doteq \frac{1}{\sqrt{N}}[1\, e^{j\frac{2\pi}{N}n}\, e^{j\frac{2\pi}{N}2n} \cdots e^{j\frac{2\pi}{N}(N-1)n}]^T$. To evaluate the FFT output energy, the proposed approach needs the vectors $\mathbf{q}_n^H\mathbf{R}_k$, which seems to require multiple costly FFT operation per received symbol. However, due to the Toeplitz structure of $\mathbf{R}_{s,k}$, one (sliding) FFT per symbol per branch turns out to be sufficient. Let $\mathbf{g}_{s,k,n}$ be the required vector at branch s, symbol index k, and frequency bin n:

$$\mathbf{g}_{s,k,n} = \mathbf{q}_n^H \mathbf{R}_{s,k} \doteq [\ g_{s,k,n}^0 \quad g_{s,k,n}^1 \quad \cdots \quad g_{s,k,n}^{L_w}\ ].$$

The vector $[\ g_{s,k,1}^0 \quad g_{s,k,2}^0 \quad \cdots \quad g_{s,k,N}^0\ ]^T$ is obtained as the FFT of the first column of $\mathbf{R}_{s,k}$, whereas the remaining $g_{s,k,n}^{i+1}$ terms are recursively obtained by the relation

$$g_{s,k,n}^{i+1} = g_{s,k,n}^{i} e^{-j\frac{2\pi}{N}n} + \frac{1}{\sqrt{N}}\left(\tilde{r}_{s,kM+P-i} - \tilde{r}_{s,kM+M-i}\right),$$

for $i\in\{0,\cdots,L_w-1\}$. Collecting the received data in the vector $\mathbf{g}_{k,n} \doteq [\ \mathbf{g}_{1,k,n} \quad \mathbf{g}_{2,k,n} \quad \cdots \quad \mathbf{g}_{S,k,n}\ ]^H$, we can express the output of the n-th FFT bin as

$$s_{k,n} = \mathbf{q}_n^H \mathbf{R}_k \mathbf{w} = \mathbf{g}_{k,n}^H \mathbf{w}. \quad (11)$$

Therefore, the cost J from (7) turns out to be a generalized Rayleigh quotient in terms of the TEQ tap vector $\mathbf{w}$:

$$J(\mathbf{w}) = \frac{\mathbf{w}^H\left(\sum_{n\in V} E[\mathbf{g}_{k,n}\mathbf{g}_{k,n}^H]\right)\mathbf{w}}{\mathbf{w}^H\left(\sum_{l\in U} E[\mathbf{g}_{k,l}\mathbf{g}_{k,l}^H]\right)\mathbf{w}} = \frac{\mathbf{w}^H\mathbf{A}\mathbf{w}}{\mathbf{w}^H\mathbf{B}\mathbf{w}}. \quad (12)$$

Therefore $J(\mathbf{w})$ is maximized at the dominant generalized eigenvector of the matrix pair ($\mathbf{A}$, $\mathbf{B}$). In the next section we present an update algorithm based on this cost, in order to have the TEQ adaptively track possible channel variations.

4. A POWER ITERATION ALGORITHM FOR ADAPTIVE IMPLEMENTATION

We develop an adaptive scheme based on the power iteration algorithm to maximize the generalized Rayleigh quotient (12). To this end, with $\lambda\in(0,1]$ a forgetting factor, the correlation matrices $\mathbf{A}$, $\mathbf{B}$ can be recursively estimated as

$$\begin{aligned}\hat{\mathbf{A}}(k) &\doteq \sum_{j=0}^{k}\lambda^{k-j}\sum_{n\in V}\mathbf{g}_{j,n}\mathbf{g}_{j,n}^H \\ &= \lambda\hat{\mathbf{A}}(k-1) + \sum_{n\in V}\mathbf{g}_{k,n}\mathbf{g}_{k,n}^H \quad (13)\end{aligned}$$

$$\begin{aligned}\hat{\mathbf{B}}(k) &\doteq \sum_{j=0}^{k}\lambda^{k-j}\sum_{l\in U}\mathbf{g}_{j,l}\mathbf{g}_{j,l}^H \\ &= \lambda\hat{\mathbf{B}}(k-1) + \sum_{l\in U}\mathbf{g}_{k,l}\mathbf{g}_{k,l}^H \quad (14)\end{aligned}$$

The power iteration algorithm [16] seeks the dominant generalized eigenvector of a matrix pair $(\mathbf{X},\ \mathbf{Y})$ by iteratively computing $\bar{\mathbf{v}}_{k+1} = \mathbf{Y}^{-1}\mathbf{X}\mathbf{v}_k$ and $\mathbf{v}_{k+1} = \bar{\mathbf{v}}_{k+1}/||\bar{\mathbf{v}}_{k+1}||$. This technique can be adopted for the TEQ adaptation: with each new received block, the correlation matrices (13)-(14) are updated first, and then the TEQ is updated by a power-like iteration

$$\bar{\mathbf{w}} = \hat{\mathbf{B}}^{-1}(k)\hat{\mathbf{A}}(k)\mathbf{w}(k), \qquad \mathbf{w}(k+1) = \frac{\bar{\mathbf{w}}}{||\bar{\mathbf{w}}||}. \quad (15)$$

The inverse of the correlation matrix $\hat{\mathbf{B}}(k)$ can be carried out directly using the Matrix Inversion Lemma (see e.g. [15]). First, let us rewrite the recursive relation of the correlation matrix as

$$\begin{aligned}\hat{\mathbf{B}}_1(k) &= \lambda\hat{\mathbf{B}}(k-1) + \mathbf{g}_{k,U(1)}\mathbf{g}_{k,U(1)}^H, && (16)\\ \hat{\mathbf{B}}_i(k) &= \hat{\mathbf{B}}_{i-1}(k) + \mathbf{g}_{k,U(i)}\mathbf{g}_{k,U(i)}^H, \quad i=2,\ldots,u, && (17)\\ \hat{\mathbf{B}}(k) &= \hat{\mathbf{B}}_u(k), && (18)\end{aligned}$$

where $U(i)$ denotes the i-th element of the set U, and $u=|U|$ is the number of null subcarriers. Let now $\hat{\mathbf{P}}(k) \doteq \hat{\mathbf{B}}^{-1}(k)$, and also $\hat{\mathbf{P}}_i(k) \doteq \hat{\mathbf{B}}_i^{-1}(k)$, for $i\in\{1,\cdots,u\}$. Then

$$\begin{aligned}\hat{\mathbf{P}}_1(k) &= \frac{1}{\lambda}\left[\hat{\mathbf{P}}(k-1) - \frac{\hat{\mathbf{P}}(k-1)\mathbf{g}_{k,U(1)}\mathbf{g}_{k,U(1)}^H\hat{\mathbf{P}}(k-1)}{\lambda + \mathbf{g}_{k,U(1)}^H\hat{\mathbf{P}}(k-1)\mathbf{g}_{k,U(1)}}\right],\\ \hat{\mathbf{P}}_i(k) &= \hat{\mathbf{P}}_{i-1}(k) - \frac{\hat{\mathbf{P}}_{i-1}(k)\mathbf{g}_{k,U(i)}\mathbf{g}_{k,U(i)}^H\hat{\mathbf{P}}_{i-1}(k)}{1+\mathbf{g}_{k,U(i)}^H\hat{\mathbf{P}}_{i-1}(k)\mathbf{g}_{k,U(i)}}\\ &\qquad \text{for } i=2,\ldots,u, \quad (19)\\ \hat{\mathbf{P}}(k) &= \hat{\mathbf{P}}_u(k). \quad (20)\end{aligned}$$

Using these, the adaptive algorithm can be implemented without explicit matrix inversion. The matrix $\hat{\mathbf{P}}(0)$ is typically initialized as a scaled identity matrix $\rho\mathbf{I}_{S(L_w+1)}$, where ρ is a large positive constant. In terms of complexity, the proposed adaptive implementation requires approximately $u(S(L_w+1))^2$ complex multiply-accumulate operations per update plus a division for normalization.

5. NUMERICAL EXAMPLES

This section simulates the blind channel shortener approach derived in this paper and compares its performance to that of previously proposed channel shortening algorithms. The simulation parameters are typical of the wireless LAN standard IEEE 802.11a : The cyclic prefix is 16 samples, the FFT size is 64 and 16 QAM signaling is used in the data-carrying tones. We assume that the subcarriers with indices in the set $U = \{1, 27, 28, \cdots, 37\}$ are not modulated. Hence, in this setting there is a total of $u = 12$ null subcarriers.
The channel is assumed to have 27 taps per branch. The real and imaginary parts of the taps are drawn from independent zero-mean Gaussian distributions with equal variance (i.e. Rayleigh model), with exponential power delay profiles as in [17]. It is assumed that frame synchronization has already been established. The system has $S = 2$ antennas and the TEQ has order $L_w = 24$. Fig. 2 shows the magnitude of the impulse response of a channel realization, as well as that of the effective shortened channel using the TEQ design produced by the maximization of (12), under SNR = 30 dB.

The performance of the proposed blind TEQ design is compared to other approaches: the MSSNR (which is a trained scheme) and the CNA designs derived respectively in [4] and [12]. The results are obtained by carrying out 100 independent realizations with each run using a different set of symbols, noise and channels. The SSNR (Shortened SNR as defined in [4]), averaged over these 100 independent realizations, is the same for all of approaches, as shown in Table 1. However, the SSNR is a heuristic measure, and thus a large SSNR value does not necessarily translate into optimal performance. Ultimately, the SINR (Signal to Noise and Interference Ratio) in each subchannel of the MC system is what matters. Fig. 3 shows the attained SINR as a function of the subcarrier index, for the same channel realization as in Fig. 2. Compared to the MSSNR and CNA solutions, the proposed TEQ consistently presents a higher SINR level for most of the subcarriers. This is further illustrated in Table 2, which lists the minimum, maximum and average values of the subcarrier SINR for different TEQ lengths. The improved behavior of the proposed design stems from the fact that, in contrast with previous approaches, it penalizes TEQ values that tend to introduce undesirable attenuation in the useful tones. This improved SINR behavior also translates into better performance in terms of uncoded BER (Bit Error Rate). For equal power loading, Fig. 4 shows the average BER versus SNR for the three TEQ designs. (It is assumed that perfect channel estimation is achieved at the last stage, in order to obtain the bank of single-tap frequency domain equalizers). The advantage of the proposed TEQ design is clear.

Finally, Fig. 5 shows the evolution of the cost function versus block number for the adaptive implementation of the proposed TEQ design. For a typical single realization, it is observed that the adaptive algorithm converges quite rapidly.

6. CONCLUSION

In Multicarrier systems, null subcarriers can be thought of as zero pilots, which are useful in order to shape the transmitted spectrum to avoid interference to or from other users/systems. They also enable carrier frequency offset estimation without channel knowledge, since they nullify the channel effect. To exploit the presence of these null subcarriers for the purpose of channel shortening a new criterion has been proposed, namely the maximization of the energy ratio of the useful subcarrier set to that of the set of null tones. It has been observed that this criterion results in performance improvement in terms of SINR with respect to previous approaches that only consider the minimization of the energy of the null subcarriers, or the maximization of the Shortening SSNR. This is due to the fact that these approaches do not take into account the effect of the TEQ on the data-carrying tones.

	Proposed TEQ	CNA	MSSNR
SSNR	29.92 dB	29.75 dB	29.93 dB

Table 1: Shortened SNR for different equalizers

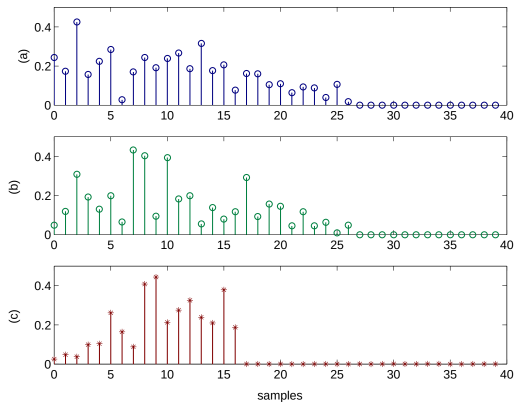

Figure 2: Shortened channel impulse response (c) compared to the original channels (a) and (b).

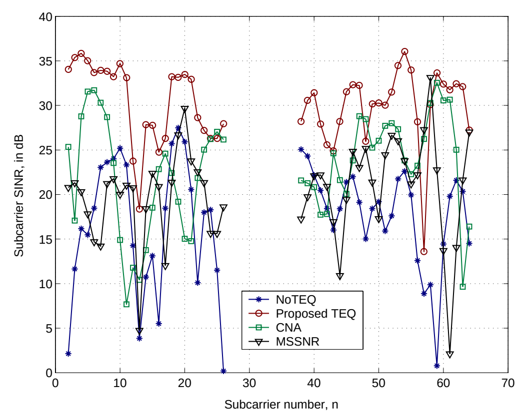

Figure 3: Measured subcarrier SINR for SNR = 30 dB.

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L_w	Proposed TEQ SINR (dB)			CNA SINR (dB)			MSSNR SINR (dB)		
	Min	Max	**Av.**	Min	Max	**Av.**	Min	Max	**Av.**
16	24.41	30.15	**27.82**	23.82	26.74	**26.12**	23.07	30.73	**26.51**
24	25.65	30.55	**28.35**	23.36	28.04	**26.34**	23.82	28.94	**26.67**
32	25.35	30.77	**28.39**	21.40	27.72	**25.45**	21.49	25.11	**23.44**
40	26.61	31.43	**28.98**	21.29	27.71	**25.06**	20.15	26.38	**23.27**
48	24.58	32.39	**29.25**	17.35	27.59	**23.82**	17.64	24.52	**21.43**

Table 2: Minimum, Maximum and Average of subcarrier SINR for different equalizers length (SNR = 30 dB)

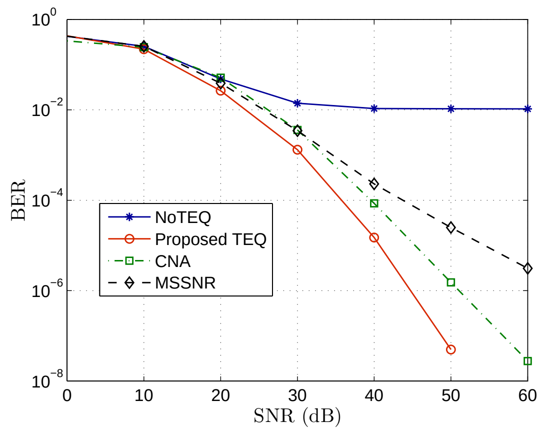

Figure 4: BER comparison for different TEQ designs.

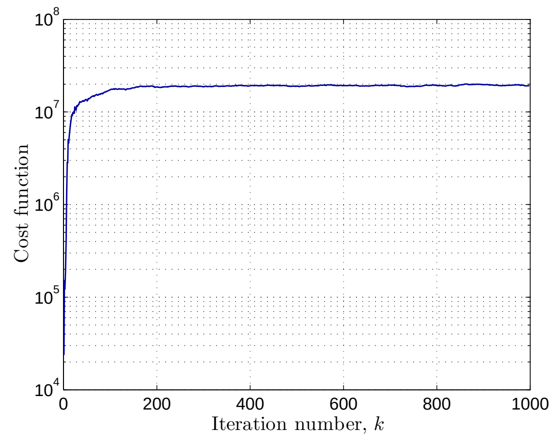

Figure 5: Adaptive implementation of the proposed TEQ design: evolution of the cost function versus block index. SNR = 30 dB, $\lambda = 0.99$.

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