

# CODING ASSISTED BLIND MIMO EQUALIZATION AND DECODING

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## ABSTRACT

Despite the widespread use of forward-error coding (FEC), most MIMO blind channel estimation techniques ignore its presence, and instead make the simplifying assumption that the transmitted symbols are uncoded. However, FEC induces code structure in the transmitted sequence that can be exploited to improve blind multiple input multiple output (MIMO) channel estimates. In this work we exploit the iterative channel estimation and decoding performance for blind MIMO equalization. Experiments show the attractive improvements by exploiting the existence of coding structures.

## 1. INTRODUCTION

In wireless MIMO systems, all practical receivers are designed based on requirements of acquiring channel state information (CSI) and the channel needs to be estimated in advance before decoding operations. However, obtaining an accurate estimation can be difficult mission in some environments. For example, if the channel response varies rapidly with time, if the channel is very singular and the signal to noise (SNR) is low. Moreover, with the ever-growing demands of increasing data rate and requirements of saving the limited bandwidth, several blind or semi-blind systems have been studied in the last decade. The typical subspace method described in [1] utilizes the orthogonality between the channel matrix and the noise subspace in order to compensate for extra degrees of freedom in the noise subspace. The main drawback of subspace-based MIMO channel estimation is that it needs the number of received antennas to be larger than the the number of transmit antennas, otherwise, it requires pre-coding preprocessing. Other schemes [2] using singular value decomposition (SVD) employ a simple block pre-coding structure. The advantage is that the CSI can be recovered without ambiguity when applying a proper modulation. Nevertheless, this advantage is implemented at the cost of the decreasing the number of transmitting antennas. waste of bandwidth. The most popular equalizers using Godard's method [3] or the constant modulus algorithm (CMA) [4] essentially employ a linear equalization and these methods encounter difficulty if the channel matrix is not well conditioned, in which case the maximum likelihood (ML) receiver is much more robust.

On the other hand, FEC coding, which restricts the transmitted sequence to a limited coding space so as to increase the minimum distance, can correct the potentially wrong decoding due to noise contamination. Nevertheless, there is little work relating FEC to MIMO channel estimations. Although the independent, identically distributed (i.i.d.) assumption is not kept any more, it has been shown that FEC

does not impair the performance of some blind equalization techniques [5]. Using FEC incurs extra calculations, however, with powerful digital processors, contributions of FEC with reasonable and affordable complexity were explored in [6]. Looking from a broader angle, we can take blind equalization as part of a decoding process to FEC codes. Thus, there exist a blind equalization and channel correcting scheme that together approximate the Shannon bound. Such methods combine the blind iterative channel estimation and turbo equalization. As illustrated in figure(1), the equalizer uses the channel estimates to compute soft information of the transmitted symbols. The channel estimator then applies these soft symbols to improve the channel estimates, which in turn yields better symbol estimates, and so forth. In contrast, the FEC aware channel estimator based on the soft symbols, which take them as *a priori* information, feeds this information into the decoder in order to get more reliable soft bit information. Next, this posterior information is back to the channel estimator, and so on. Such a scheme explores FEC benefits in blind equalization. Some previous research has explored the FEC property on blind channel estimation [7]. However, this was only employed in single input single output channels and small constellations, where the convergence space is rather smooth and are less likely to become trapped in undesirable stationary points of the iterative scheme. These techniques do not simply extend to the high dimension (MIMO) systems since the high dimensionality of the MIMO channel make the convergence difficult. In the recent work [8], an efficient hybrid system for blind equalization of large constellation MIMO systems was proposed, in which the scheme used Expectation Maximization (EM) algorithm for the large multi-dimensional channel estimate and enjoyed a fast convergence with low computational complexity. However, it still suffered a loss of performance when the channel was close to singular or the noise was large.

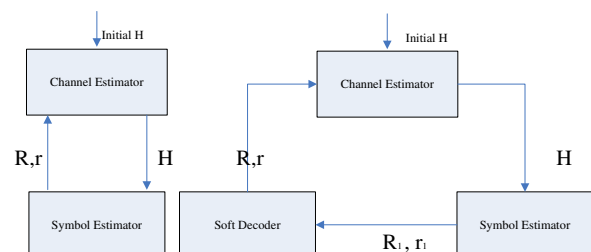


Figure 1: Joint channel estimation and symbol detection.

In this paper, we propose a blind MIMO receiver that combines a soft channel estimation and a soft MIMO equalizer and decoder. A similar idea appears in [9], where the au-

thor used pilot sequence and a Wiener filter to initial and update the channel respectively. This Minimum Mean Square Error (MMSE) based iterative channel estimators uses soft information from the output of the decoder to improve the mean square error of the channel estimates. However, taking the mean values of the data symbols calculated by the posteriori probabilities, is not an efficient way to improve channel estimates. In our hybrid design, we improve the receiver's performance through efficiently incorporating the soft bit information from the decoder into the EM channel estimator since it is a suitable choice to exploit the a priori knowledge. Our system includes an efficient independent component analysis (ICA) method which is extremely suitable for QAM modulation, a selective sphere decoder (SD) process that computes the likelihood values (soft information) and a simple error correcting operation. This scheme has low complexity and improved convergence, being more likely to converge to desirable stationary point. Moreover, through sending the bit interleaved coded modulation (BICM) bits on differently fast fading channels, we further make use of the temporal diversity. Combining this with the spatial diversity caused by the statistical independencies of transmitted sequences, blind estimate and adjustment of the channel matrix can be operated simultaneously. Then this provide us outstanding performance of MIMO blind equalization and decoding, as we will see.

We consider a  $M \times M$  system,

$$Y = HS + N. \quad (1)$$

Where  $Y \in \mathbb{C}^{M \times T}$  is the matrix containing observed signals from the receiver antennas, and  $S \in \mathbb{C}^{M \times T}$  is the complex discrete source signals.  $N \in \mathbb{C}^{M \times T}$  is the matrix of the noise with covariance,  $\Sigma$ , which is assumed uncorrelated with source signals and  $T$  is the sampled points of observations.  $H \in \mathbb{C}^{M \times M}$  is the unknown linear square channel matrix whose elements are drawn independently from a Rayleigh distribution and we assume that it is invertible. Note that,  $H$  is instantaneous but we do not guarantee it is orthogonal. For the transmission of a frame of  $K_b$  bits, the transmitter encodes the  $K_u$  information bits using a convolutional code of rate  $r$ , where  $K_u = K_b \times r$ . The coded bits are interleaved and mapped into QAM symbols, forming a sequence of  $K_s = K_b / \log_2 P$  symbols. Then the QAM sequence of symbols is split into  $M$  substreams corresponding to one Rayleigh fading channel, and is transmitted in parallel from each one of the  $M$  antennas. The problem arises above not only in MIMO systems, but also in multiuser DS/CDMA systems [10]. It further reduces to SIMO blind equalization when there is only one source signal or when fractionally spaced equalization is employed in single antenna communication systems.

## 2. SOFT MIMO EQUALIZER AND DECODER

The optimal ML receiver has the exponential complexity with the signal modulation size and number of transmit antennas, thus limiting real time applications. The SD, on the other hand, is capable of achieving near ML performance and can be designed to provide the soft (likelihood) output information, and a simple error correcting operator. Such a low-complexity structure could enable iterative equalization for fast wireless Rayleigh channels. An important requirement of this blind MIMO equalization is to calculate

the soft information both for the channel estimation and the soft MIMO detector and decoder. Given that the study of the channel coding has been extensively researched in the literature, we introduce the techniques used in this paper briefly. The Bahl–Cocke–Jelinek–Raviv (BCJR) algorithm [11] is used to compute a posterior probability (APP) of inputs to a finite state machine with noisy received signals. It has been applied to many channel correcting codes such as, turbo and LDPC codes. For a BPSK modulation scheme, it takes the form of the log-likelihood ratio (LLR):

$$L(b_k) = \ln \frac{p(b_k = 1|Y, \tilde{H})}{p(b_k = 0|Y, \tilde{H})}. \quad (2)$$

Where  $\tilde{H}$  is the channel estimate. This LLR value shows the reliability of the information bit. Given the convolutional code at transmitters, the well known BCJR algorithm calculates the APP exactly if we know the channel state information. While a full complexity BCJR soft decoder is used in this work, an efficient sliding window type scheme [12] can be applied in practical applications, which leads to suboptimal performance with a much lower complexity.

## 3. EM CHANNEL ESTIMATION

Most prior work in blind iterative channel identification can be tied to the EM algorithm of [13]. It is a general methodology for maximum likelihood or maximum a posteriori estimation. The first use of EM with soft symbol estimates was proposed in [14]. An adaptive version of EM was applied to the identification problem in [15]. Some modified EM algorithms were proposed in [16] [17]. The EM algorithm updates are analytically simple and numerically stable for distributions that belong to the exponential family.

Consider the system model, equation (1). The EM algorithm estimates the channel  $H$  based on received signals  $Y = \{y_t\}_1^T$ . It minimizes the log likelihood,  $-\log P(Y|H)$ , by iteratively minimizing,

$$H^{k+1} = \arg \min_H E \{-\log P(Y|H^k, S)P(S|Y, H^k)\}, \quad (3)$$

where  $H^k$  is the  $k$ th estimates of channels. This EM iteration in (3) guarantees to converge to a local minimum of  $P(Y|H)$  [18].

The update of the equation (3) can be written in a closed-form solution [19] as follows,

$$r = \sum_{t=1}^T y_t E \{s_t | Y, H^k\} \quad (4)$$

$$R = \sum_{t=1}^T E \{s_t s_t^* | Y, H^k\} \quad (5)$$

$$H^{k+1} = R^{-1} r \quad (6)$$

Equations (4) and (5) depend on first-order statistics and the second-order statistics of the symbols respectively. Note that the computation of (4) and (5) also requires the posterior probabilities  $P(s_t | Y, H)$  and  $P(s_t s_t^* | Y, H)$ , which can be produced by the BCJR algorithm instead of from the symbol estimators directly when using an error correcting code. We emphasize that posterior probabilities exploiting the coding structure can improve the channel estimation significantly.

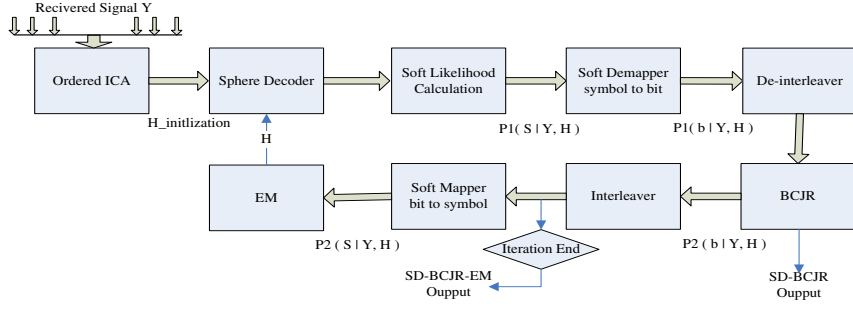


Figure 2: Receiver architecture of the proposed coding assisted system.

An important problem in the performance of the EM algorithm is the appropriate selection of the initial estimate. If the starting point is far away the desired point and the likelihood surface is complicated, the EM is liable to convergence to the local minimum. Its convergence has been studied by many researchers [20] and references therein. The recent work [21] demonstrated in digital modulation systems, that the EM algorithm shows Newton-like convergence. Thus the EM algorithm guarantees fast convergence and is suitable for real time applications in wireless communications. However, there is still the important but unsolved problem of whether the EM algorithm can converge to the correct solution, i.e., the consistent solution of the true channel parameters.

#### 4. ITERATIVE PROCEDURE

In this work, a blind MIMO channel equalization algorithm is proposed in which the BCJR and EM algorithms are iterated. Given initial estimates  $H_{ini}$  from an efficient ICA method [8], the SD-BCJR algorithm computes the signal posterior probabilities  $P(s_n|Y, H^k, \Sigma)$  by utilizing the code structure and then feeds these to the EM algorithm. The EM algorithm uses these posterior probabilities to evaluate the conditional expectations in (4) and (5). Thus we update new channel state information by (6) and pass this back to the SD-BCJR algorithm again. As the iterations proceed, estimates become more accurate and the posterior symbol probabilities become more precise.

Figure (2) shows the receiver structure using iterative equalization, whereby a soft equalizer interacts with a soft input soft output error control decoder.

In multiple-input multiple-output (MIMO) channels, this soft decoding strategy is given below:

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#### Algorithm 1 Coding assisted blind MIMO equalization

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1. Blindly estimate the MIMO channel state information with the statistic properties of received signals.
  2. Estimate the soft bits, i.e., the LLR of each transmitted bit, using the list version of sphere decoding or its variants.
  3. Make the soft bits information more reliable through a simple BCJR soft decoder.
  4. Update the channel state information by the EM algorithm with the soft bit information input and feed it back to the channel estimator for further improvement.
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Usually, a good MIMO decoder is required to produce re-

liable soft-bit estimates at the first stage before soft decoding is performed. In our simulation, an efficient nonlinear ICA approach [8] is used to get an initial channel estimate. Note that this nonlinearity can partly solve the phase ambiguity of ICA since the exploration of the independence between real and imaginary parts in QAM modulations. There is still a  $k\pi/2$  phase ambiguity. Such problem can be solved by further differential encoding techniques.

#### 5. SIMULATIONS

We consider a MIMO system with four transmitters, four receivers and QAM16 modulation are employed. The channel  $H$  is a  $4 \times 4$  complex instantaneous matrix, which is constant for each block interval (256 symbols), and it follows a Rayleigh Fading distribution.  $N$  follows the complex additive white Gaussian distribution. The results have been obtained for transmitting blocks of  $K_b = 4032$  bits. Each antenna adds four pilot symbols to solve the permutation and phase ambiguity problem of ICA, as we call it ordered ICA in figure (2). A rate of  $r = 1/2$  parallel concatenated convolutional code of memory 3 with two nonsystematic convolutional (NSC) code has been used. The generator polynomials are  $G_1(D) = 1 + D + D3 + D4$ ,  $G_2(D) = 1 + D3 + D4$  and the interleaver is set to pseudo random. The soft symbol information was calculated at each antenna. The SD [22] is employed first to get soft information. Only two iterations of the channel estimation are employed in EM updates. In order to explore the full contribution provided by FEC, this work computes the symbol likelihood based on 16 points on each dimension. This calculation could be simplified by a list sphere decoder (LSD) [23] or list-fixed-complexity sphere decoder (LFSD) [24] but with a performance degradation.

In comparison with other effective methods of blind MIMO equalization, such as the *Split Threshold nonlinear function* and the SD-EM approach [8], our scheme shows promising performance for this type of problem. The former used an efficient score function which is specified for QAM signals to obtain a good separability. The later proposed an efficient hybrid blind MIMO equalization and decoding scheme. It directly applied soft information into EM channel update, despite the spacial diversity caused by FEC. Figure (3) illustrates this significant separability improvements with the aid of channel code. This performance is measured by the distance of the estimated channel from the true value. We define  $P = H_{est}^{-1}H_{real}$

$$ICI(P) = \frac{1}{n} \sum_i \sum_j \left[ \left( \frac{|P_{ij}|}{\max |P_{ij}|} \right)^2 - 1 \right], \quad (7)$$

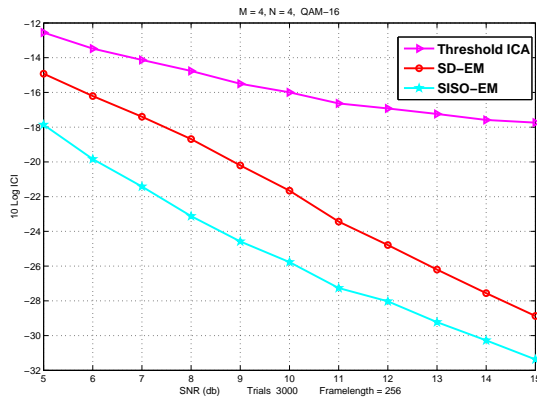


Figure 3: Channel separability of the split threshold nonlinear ICA, the SD-EM and the coding assisted SD-EM algorithm with a rate  $r = 1/2$  convolutional code over different SNR.

where ICI stands for inter component interference.

Figure (4) shows the BER performance. Comparisons were set up with a pilot assisted transmission scheme and two blind methods methoded above. Training based V-blast [25] scheme used 24 symbols per block as pilot sequence and applied a Least squares algorithm into channel estimate and followed by MMSE detection, nulling, cancellation and ordering. The BER was measured before the error correcting operations. Obviously, the coding assisted iterative structure improves the system performance significantly since it can help avoid the EM algorithm from becoming trapped in a local minimum.

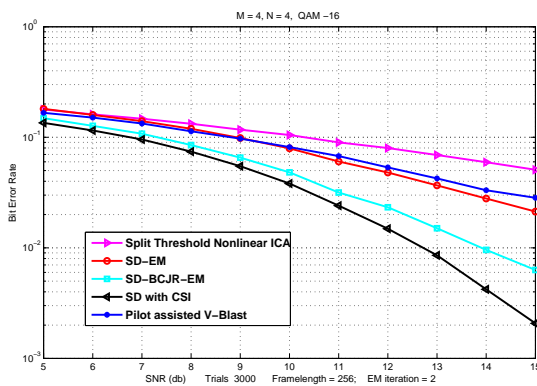


Figure 4: BER performance of the pilot assisted V-blast scheme, the split threshold nonlinear ICA, the SD-EM, the coding assisted SD-EM algorithm and known CSI SD with a rate  $r = 1/2$  convolutional code.

For a setup similar to the system above but which provides an increased time diversity, we apply a codeword to four and eight channel realizations in which the channel realizations are independent. The BER performance is measured after the BCJR algorithm. Figure (5a) and figure (5b), in the next page, show us that the performance is also improved by the time diversity. Through the interleaving operation, poor channel estimates can be set right with high quality information from the well conditioned channel estimate, then the

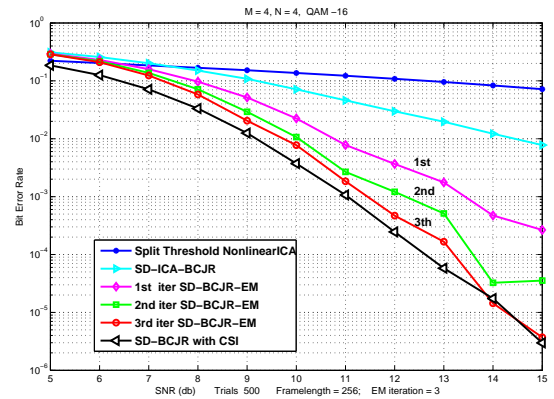


Figure 6: Iterative improvements of the coding assisted SD-EM algorithm in a  $4 \times 4$  system with 16-QAM modulation.

channel estimate can be changed to be more reliable with further iterations. Obviously, we can see the joint contribution of time diversity and channel coding to the blind MIMO equalization and decoding.

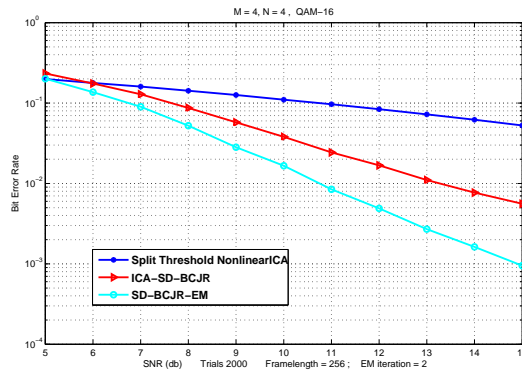
Our final simulation indicates the iterative gain of the coded soft channel estimation measured over 500 Monte Carlo runs. Similar to the setup above with 4 fixed channel realizations. One channel matrix is very singular with a condition number over 26 and it is difficult to estimate with a linear estimator blindly. Other channels are good in terms of the channel condition number. Clearly, from figure (6), we can see that the performance progresses towards the optimal curve with CSI known at the receiver. This shows us that the BER of this blind algorithm is almost identical to that for a BCJR equalizer with perfect channel information.

## 6. CONCLUSION

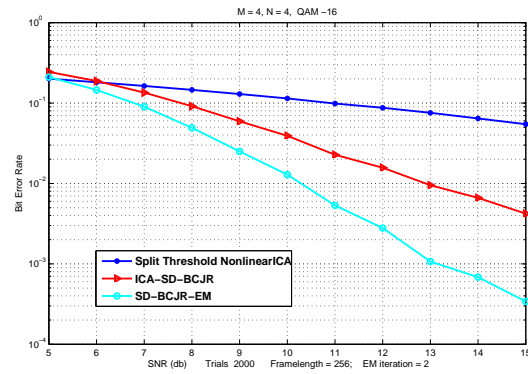
We have proposed a coding assisted MIMO blind equalization and decoding scheme. By utilizing posteriori information, it provides substantial gain over the uncoded system. The existence of coding structures partly solves the problems of the EM getting trapped in a local minimum when the channel is close to singular or SNR is low. This happens frequently when the number of receiver antennas, the size and the dimension of the data are large. The new scheme appears to avoid local minimum and converge to the global minimum or at least a good approximation of it. Moreover, this system uses an efficient ICA approach and a simple BCJR decoding without any extra computation load. This low complexity should make it suitable for tracking fast channel variations in real time applications.

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(a) 4 blocks form a codeword



(b) 8 blocks form a codeword

Figure 5: BER improvements by utilizing time diversity and channel coding. Triangle line is the performance of the SD following the BCJR algorithm and the circle line is the performance of the coding assisted SD-EM algorithm.

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