ADAPTIVE ANALYTICAL SIMPLIFIED CONSTANT MODULUS ALGORITHM: A BSS ALGORITHM FOR MIMO SYSTEMS WITH TIME-VARYING ENVIRONMENTS

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ABSTRACT

An adaptive Analytical Simplified Constant Modulus Algorithm (adaptive-ASCMA) which accomplishes blind source separation and carrier phase recovery is proposed. This algorithm is applied on Multiple Input Multiple Output (MIMO) communication systems and uses some analytical methods to minimize the Simplified Constant Modulus (SCM) cost function. A study was done to find the subspace containing the solution of the minimization problem. With comparable computational complexities, the adaptive ASCMA accomplishes better performances in Bit Error Rate (BER) and Signal to Interference and Noise Ratio (SINR) terms than the adaptive-ACMA with time varying environment.

1. INTRODUCTION

MIMO systems, created by the Bell Labs [1], are wireless communications techniques which uses several antennas at emission and reception and has a high transmission capacity. At the receiver the channel characteristics are known via a training sequence, but in situations like interception such sequences can't be used then Blind Source Separation (BSS) is considered.

BSS algorithms allow to obtain transmitted symbols from only received signals. With BSS we obtein the sources directly sontrary to methods which estimate the channel like the Kalman filter used in blind context [2]. This type of method requires a decoder to obtain sources, that is increase the complexity. Among possible adaptive BSS, there are the classical Constant Modulus Algorithm (CMA) [3, 4] , its simplified version called SCMA [5] and the adaptive-Analytical Constant Modulus Algorithm (ACMA) described in [6, 7]. The SCMA and the CMA use, to minimize their cost function, a stochastic gradient descent, so they may be slow to converge and with time-varying environments they are less accurate. On the other hand, the adaptive-ACMA uses some analytical methods to minimize the cost function, thus it converges quickly and can be used on varying environments.

The SCMA allows to accomplish equalization and carrier phase recovery simultaneously contrary to the CMA and adaptive-ACMA. Therefore, with the CMA and adaptive-ACMA, a carrier loop must be used to recover the carrier phase after the BSS algorithm what increases the complexity of these algorithms.

So, we propose a new algorithm, called ASCMA, to minimize the cost function SCM by using some analytical method. Simulations show that adaptive-ASCMA allows to obtain a higher SINR and lower BER than adaptive-ACMA and SCMA. Furthermore, phase recovery is accomplished conjointly with the BSS by using our algorithm.

This paper is organized as follows. In section 2 we briefly review the system model and the principle of BSS. Next the proposed algorithm is described in section 3. Simulation results are discussed in section 4, and conclusions are given in section 5

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2. PROBLEM FORMULATION

2.1. Notations

We first introduce some basic notation and properties used. Lower-case boldface letters are used to denote vectors and upper-case boldface letters to denote matrices. In addition:

- $(.)^T$ denotes the transpose
- $(.)^H$ denotes the transpose conjugate
- $tr(.)$ denotes the trace operator
- $\mathbf{0}_{N_t}$ is an $N_t \times N_t$ matrix with zeros components
- $\mathbf{I}_{N_t}$ is the $N_t \times N_t$ identity matrix
- $E[.]$ is the expectation operator
- $\oplus$ denotes the direct sum
- $\Re$ denotes the real part of complex
- $\Im$ denotes the imaginary part of complex
- $vec(\mathbf{B})$ stacking of the columns of $\mathbf{B}$ into a vector
- $\otimes$ is the Kronecker product
- $\mathbf{A} \circ \mathbf{B} = (a_1 \otimes b_1 \; a_2 \otimes b_2 \; \ldots)$

2.2. System model

Let us consider a MIMO system where N_t and N_r represent respectively the number of transmit and receive antennas. We assume that N_t independent and identically distributed and mutually independent zero-mean discrete-time sequences $\mathbf{x}(k)$ are transmitted at time instant k through a $N_r \times N_t$ MIMO time-varying memoryless channel $\mathbf{H}(k)$.

Assuming a carrier frequency offset δ_f, the $N_r \times 1$ vector of received signals $\mathbf{y}(k)$ can be expressed as:

$$\mathbf{y}(k) = \mathbf{H}(k).\mathbf{x}(k)e^{j2\pi k\delta_f T_s} + \mathbf{b}(k)$$

where $\mathbf{y}(k) = \begin{pmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_{N_r}(k) \end{pmatrix}$, $\mathbf{x}(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_{N_t}(k) \end{pmatrix}$,

$\mathbf{b}(k) = \begin{pmatrix} b_1(k) \\ b_2(k) \\ \vdots \\ b_{N_r}(k) \end{pmatrix}$, $\mathbf{H}(k) = \begin{pmatrix} h_{11}(k) & \cdots & h_{1N_t}(k) \\ \vdots & \ddots & \vdots \\ h_{N_r1}(k) & \cdots & h_{N_rN_t}(k) \end{pmatrix}$

where $\mathbf{b}(k)$ is the $N_r \times 1$ vector of additive noise samples and $1/T_s$ is the baud rate of the transmitter.

2.3. BSS principle

To recover blindly the transmitted sources $\mathbf{x}(k)$ an algorithm of BSS is used. In order to realize the BSS, some hypotheses must be verified:

- $N_r \geq N_t$
- **H** have iid complex components, is unitary else the received signals $\mathbf{y}(k)$ must be prewhitened before applying BSS.
- The noise is additive, white and gaussian with zero mean and a covariance matrix $\mathbf{C}_b = E[\mathbf{b}\mathbf{b}^H] = \sigma_b^2 \mathbf{I}_{N_r}$.
- The sources are zero mean discrete-time sequences, with a covariance matrix $\mathbf{C}_x = E[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{N_t}$, and they must be mutually independent at a given time instant and identically distributed.

In order to recover the source signals, the received signals $\mathbf{y}(k)$ are processed by a $N_r \times N_t$ matrix equalizer $\mathbf{W}(k)$ that produces the $N_t \times 1$ vector output $\mathbf{z}(k)$. The receiver output can be represented as (figure 1):

$$\begin{aligned} \mathbf{z}(k) &= \mathbf{W}^H(k)\mathbf{y}(k) \\ &= \mathbf{W}^H(k)\left(\mathbf{H}(k).\mathbf{x}(k)e^{j2\pi k\delta_f T_s} + \mathbf{b}(k)\right) \\ &= \mathbf{G}^H(k)\mathbf{x}(k) + \mathbf{b}'(k) \end{aligned}$$

where $\mathbf{z}(k)$ should ideally match the N_t transmitted signals $\mathbf{x}(k)$, $\mathbf{b}'(k) = \mathbf{W}^H(k)\mathbf{b}(k)$ is the colored noise at the equalizer output and $\mathbf{G}(k) = \mathbf{H}^H(k)e^{-j2\pi k\delta_f T_s}\mathbf{W}(k)$ is the $N_t \times N_t$ global response matrix. The matrix $\mathbf{W}$ is feasible to separate the sources, except for a possible permutation and an arbitrary rotation.

Generally the channel matrix $\mathbf{H}$ is not unitary, so we must prewhiten the received signals $\mathbf{y}(k)$ to verify the second hypothesis. So, the $N_r \times N_t$ prewhitening filter $\mathbf{F}(k)$ is applied on the received signals $\mathbf{y}(k)$:

$$\underline{\mathbf{y}}(k) = \mathbf{F}^H\mathbf{y}(k)$$

where $\underline{\mathbf{y}}(k)$ is the $N_t \times 1$ vector of prewhitened received signals. Once the received signals pre-whitened, an $N_t \times N_t$ equalization matrix $\mathbf{T}(k)$ is searched in order to separate the mixing of sources $\underline{\mathbf{y}}(k)$ (figure 1):

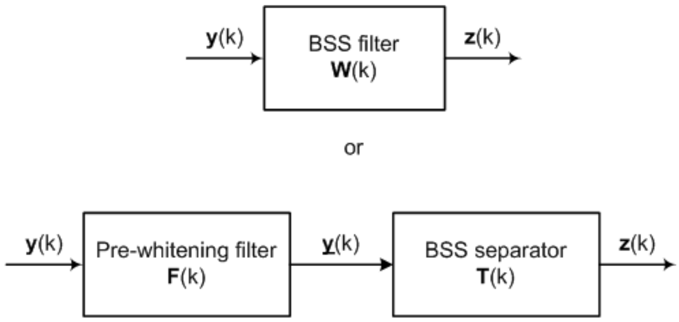

Fig. 1. Scheme for the blind source separation

$$\begin{aligned} \mathbf{z}(k) &= \mathbf{T}^H(k)\underline{\mathbf{y}}(k) \\ &= \mathbf{T}^H(k)\mathbf{F}^H(k)\mathbf{y}(k) \\ &= \mathbf{W}^H(k)\mathbf{y}(k) \end{aligned}$$

where **W=FT**.

3. THE ADAPTIVE ANALYTICAL SCMA

In this section the following properties of the Kronecker product are used:

$$vec(\mathbf{a}\mathbf{b}^H) = \mathbf{b}^* \otimes \mathbf{a}$$
$$vec(\mathbf{ABC}) = (\mathbf{C}^H \otimes \mathbf{A})vec(\mathbf{B})$$
$$vec(\mathbf{A}diag(\mathbf{b})\mathbf{C}) = (\mathbf{C}^H \circ \mathbf{A})\mathbf{b}$$

3.1. Cost function

The adaptive Analytical SCMA uses the SCM cost function proposed by [5]:

$$J(\mathbf{T}(k)) = \sum_{l=1}^{N_t}\left(\Re^2(z_l(k)) - R_r\right)^2$$

where $R_r = \frac{E\left[|\Re(\mathbf{x}(k))|^4\right]}{E\left[|\Re(\mathbf{x}(k))|^2\right]}$, to bring out the real part of $z_l(k)$, it is written as:

$$\begin{aligned} z_l(k) &= \mathbf{t}_l(k)^H\underline{\mathbf{y}}(k), l \in \{1, \ldots, N_t\} \\ &= \tilde{\mathbf{t}}_l(k)^T\tilde{\underline{\mathbf{y}}}(k) + j\tilde{\mathbf{t}}_l^T\overline{\underline{\mathbf{y}}}(k) \end{aligned}$$

where $\tilde{\mathbf{t}}_l(k) = \begin{pmatrix} \Re(\mathbf{t}_l(k)) \\ \Im(\mathbf{t}(k)) \end{pmatrix}$, $\tilde{\underline{\mathbf{y}}}(k) = \begin{pmatrix} \Re(\underline{\mathbf{y}}(k)) \\ \Im(\underline{\mathbf{y}}(k)) \end{pmatrix}$,
$\overline{\underline{\mathbf{y}}}(k) = \begin{pmatrix} -\Im(\underline{\mathbf{y}}(k)) \\ \Re(\underline{\mathbf{y}}(k)) \end{pmatrix}$

As inspired by [7], by using the Kronecker product properties, $\Re^2(z_l(k))$ and $\Im^2(z_l(k))$ can be written as:

$$\begin{aligned} \Re^2(z_l(k)) &= \tilde{\mathbf{t}}_l(k)^T(\tilde{\underline{\mathbf{y}}}(k)\tilde{\underline{\mathbf{y}}}^T(k))\tilde{\mathbf{t}}_l(k) \\ &= (\tilde{\underline{\mathbf{y}}}(k) \otimes \tilde{\underline{\mathbf{y}}}(k))^T(\tilde{\mathbf{t}}_l(k) \otimes \tilde{\mathbf{t}}_l(k)) \end{aligned}$$

From [7], by using a QR factorization, the cost function becomes:

$$\tilde{\mathbf{t}}(k) = \arg \min_{\substack{\mathbf{d}_l(k)=\tilde{\mathbf{t}}_l(k)\otimes\tilde{\mathbf{d}}_l(k) \\ \|\tilde{\mathbf{d}}_l(k)\|=R}} \sum_{l=1}^{N_t} \mathbf{d}_l^T(k)\mathbf{C}(k)\mathbf{d}_l(k)$$

where the constraint $\|\tilde{\mathbf{d}}_l(k)\| = R$ avoids the trivial solution $\tilde{\mathbf{d}}_l(k) = \mathbf{0}$.

The matrix $\mathbf{C}(k)$ is defined as:

$$\mathbf{C}(k) = \lambda\mathbf{C}(k-1) + \beta(k)\mathbf{c}(k)\mathbf{c}^T(k) \qquad (1)$$

with $0 < \lambda < 1$, $\beta(k) = \frac{\alpha(k-1)}{\alpha(k)}\lambda(1-\lambda)$, $\beta > 0$
$\alpha(k) = \lambda\alpha(k-1) + 1 - \lambda$ and

$$\mathbf{c}(k) \quad = \quad \underline{\tilde{\mathbf{y}}}(k)\otimes\underline{\tilde{\mathbf{y}}}(k) - \mathbf{p}(k-1)/\alpha(k-1)$$

where

$$\mathbf{p}(k) \quad = \quad \lambda\mathbf{p}(k-1) + (1-\lambda)(\underline{\tilde{\mathbf{y}}}(k)\otimes\underline{\tilde{\mathbf{y}}}(k))$$

3.2. Search of the subspace containing the solution

To find the subspace which contains the solution we propose the following analysis. In this section, we will use two antennas at emission ($N_t = 2$) to make the theoretical study. Now, it will be shown that vectors $\mathbf{d}_l$, $\forall l \in \{1\ldots 2\}$, which minimize the function $\sum_{l=1}^{2}\mathbf{d}_l^T\mathbf{C}\mathbf{d}_l$ under the hypotheses $\mathbf{d}_l = \tilde{\mathbf{t}}_l\otimes\tilde{\mathbf{t}}_l$ and $\|\tilde{\mathbf{d}}_l\| = R$, are in the minor subspace of $\mathbf{C}$ spanned by a linear combination of eigenvectors of $\mathbf{C}$ associated to smallest eigenvalues different of 0.

First, we use properties of $\mathbf{C}$ to reformulate the optimization problem:

Theorem 1 *The matrix* 16×16 $\boldsymbol{C}$ *is symmetric and semi-define positive.*

Lemma 1 $\boldsymbol{d}_l^T\boldsymbol{C}\boldsymbol{d}_l \geq 0$, $\forall l \in \{1\ldots 2\}$ *and so the optimization problem becomes:*

$$\tilde{\boldsymbol{t}}(k) = \arg \min_{\substack{\boldsymbol{d}_l(k)=\tilde{\boldsymbol{t}}_l(k)\otimes\tilde{\boldsymbol{t}}_l(k) \\ \|\tilde{\boldsymbol{d}}_l(k)\|=R}} \sum_{l=1}^{2} \boldsymbol{d}_l^T(k)\boldsymbol{C}(k)\boldsymbol{d}_l(k)$$

$$\Leftrightarrow$$

$$\tilde{\boldsymbol{t}}(k) = \arg \min_{\substack{\boldsymbol{d}_l(k)=\tilde{\boldsymbol{t}}_l(k)\otimes\tilde{\boldsymbol{t}}_l(k) \\ \|\tilde{\boldsymbol{d}}_l(k)\|=R}} \boldsymbol{d}_l^T(k)\boldsymbol{C}(k)\boldsymbol{d}_l(k), \quad \forall l \in \{1\ldots 2\}$$

Proof 1 *By construction* $\boldsymbol{c}\boldsymbol{c}^T$ *is symmetric and semi-define positive and by using the equation 1, we deduce that* $\boldsymbol{C}(k)$ *is symmetric and semi-define positive too.*

Afterward, the nullspace and image of $\mathbf{C}(k)$ will be used, they are defined as:

$$\ker(\mathbf{C}) \quad = \quad \left\{\mathbf{y}\in\mathbb{R}^{16}/\mathbf{C}\mathbf{y} = 0_{R^{16}}\right\}$$
$$image(\mathbf{C}) \quad = \quad \left\{\mathbf{z}\in\mathbb{R}^{16}/\mathbf{z} = \mathbf{C}\mathbf{y},\ \mathbf{y}\in\mathbb{R}^{16}\right\}$$

$\ker(\mathbf{C})$ and $image(\mathbf{C})$ are subspaces of $\mathbb{R}^{16}$.

Property 1 $\ker(\boldsymbol{C})\oplus image(\boldsymbol{C}) = \mathbb{R}^{16}$ *where* $\oplus$ *denotes the direct sum, i.e.,* $\ker(\boldsymbol{C})\cap image(\boldsymbol{C}) = \{0\}$ *and* $\ker(\boldsymbol{C})$ *and* $image(\boldsymbol{C})$ *are subspace of* $\mathbb{R}^{16}$.

Definition 1 *By construction, the only dependence obtained on columns of* $\boldsymbol{C}$ *are:* $C_2 = C_5$, $C_3 = C_9$, $C_4 = C_{13}$, $C_7 = C_{10}$, $C_8 = C_{14}$, $C_{12} = C_{15}$, *where* C_n *represents the* n^{th} *column of* $\boldsymbol{C}$. *So,* $dim[\ker(\boldsymbol{C})] = 6$. *A basis* B_{ker} *of the nullspace is defined as:*

$$\begin{aligned} B_{ker} &= \{e'_1, e'_2, e'_3, e'_4, e'_5, e'_6\} \\ &= \{e_2 - e_5, e_3 - e_9, e_4 - e_{13}, e_7 - e_{10}, e_8 - e_{14}, e_{12} - e_{15}\} \end{aligned}$$

where e_i, $i\in\{1,\ldots,16\}$ *are vectors in the standard basis of* $\mathbb{R}^{16}$.
The e'_i *are linearly independent and verify* $\boldsymbol{C}e'_i = 0$.
Thanks to the rank-nullity theorem:

$$rank(\boldsymbol{C}) + dim(\ker(\boldsymbol{C})) = dim(\mathbb{R}^{16}),$$

and since the dimension of $\ker(\boldsymbol{C})$ *is equal to 6, the dimension of* $Image(\boldsymbol{C})$ *is equal to 10. And* B_{im} *is a basis of image:*

$$\begin{aligned} B_{im} &= \{e''_1, e''_2, e''_3, e''_4, e''_5, e''_6, e''_7, e''_8, e''_9, e''_{10}\} \\ &= \{e_1, e_6, e_{11}, e_{16}, e_2 + e_5, e_3 + e_9, e_4 + e_{13}, \\ &\quad e_7 + e_{10}, e_8 + e_{14}, e_{12} + e_{15}\} \end{aligned}$$

The e''_i *are linearly independent, verify* $\boldsymbol{C}e''_i \neq 0$ *and we can verify the property 1: the* e'_i *and the* e''_i *are linearly independent, then* $\{e'_1,\ldots,e'_6,e''_1,\ldots,e''_{10}\}$ *is a basis for* $\mathbb{R}^{16}$.

By using the nullspace basis of $\mathbf{C}$, we deduce the following theorem (the proof of this theorem is shown in appendix 6):

Theorem 2 *In the nullspace of* $\boldsymbol{C}$, *only vector* $0_{\mathbb{R}^{16}}$ *has a Kronecker structure.*

By using the properties 1, we deduce the following lemma:

Lemma 2 *Let* E, *the set of vectors in* $\mathbb{R}^{16}$ *having a Kronecker structure:* $E = \{\boldsymbol{p}\in\mathbb{R}^{16}/\boldsymbol{p} = \boldsymbol{t}\otimes\boldsymbol{t}, \boldsymbol{t}\in\mathbb{R}^4\}$, *then* $E\subset Image(\boldsymbol{C})$.

Finally, we search to minimize the cost function:

$$\begin{cases} J(\tilde{\mathbf{t}}_l) = \mathbf{d}_l^T\mathbf{C}\mathbf{d}_l, \forall l\in\{1\ldots 2\} \\ \text{under } \mathbf{d}_l = \tilde{\mathbf{t}}_l\otimes\tilde{\mathbf{t}}_l,\ \|\tilde{\mathbf{d}}_l\| = R \end{cases}$$

Vectors which minimize $J(\tilde{\mathbf{t}}_l)$ are eigenvectors $\mathbf{v}_n$ corresponding to the smallest eigenvalues different of zero of $\mathbf{C}$, and vectors which minimize $J(\tilde{\mathbf{t}}_l)$ under the hypotheses $\mathbf{d}_l = \tilde{\mathbf{t}}_l\otimes\tilde{\mathbf{t}}_l$ and $\|\tilde{\mathbf{t}}_l\| = R$ are spanned by the eigenvectors $\mathbf{v}_n$: $\mathbf{d}_l = span(\mathbf{v}_1,\ldots,\mathbf{v}_n)$.

Now, we will show how to track the solution $\mathbf{d}_l$.

3.3. Tracking of the solution $\mathbf{d}_l$

To track vector $\mathbf{d}_l$, we use, like in [7] the NOOJA (Normalized Orthogonal Oja) algorithm [8]. This algorithm extracts adaptively the minor subspace $\mathbf{D} = (\mathbf{d}_1,\ldots,\mathbf{d}_{N_t})$, spanned by the eigenvectors corresponding to the smallest eigenvalues different of zero, of the autocorrelation matrix $\mathbf{C}$ of the signal $\mathbf{c}(k)$ ($\mathbf{C} = E\left[\mathbf{c}\mathbf{c}^H\right]$) by maximizing the following cost function:

$$\begin{aligned} J(\mathbf{D}) &= E\left\|\mathbf{c}(k) - \mathbf{D}\mathbf{D}^T\mathbf{c}(k)\right\|^2 \\ &= tr(\mathbf{C}) - 2tr(\mathbf{D}^T\mathbf{C}\mathbf{D}) + tr(\mathbf{D}^T\mathbf{C}\mathbf{D}\mathbf{D}^T\mathbf{D}) \end{aligned}$$

The vectors $\mathbf{d}_l(k)$ obtained by the NOOJA must satisfy the constraint $\mathbf{d}_l(k) = \tilde{\mathbf{t}}_l(k) \otimes \tilde{\mathbf{t}}_l(k)$. To do that we use, like Van Der Veen [7], the following method:
First, $\tilde{\mathbf{T}}(k-1) \circ \tilde{\mathbf{T}}(k-1)$ is computed and regarded as the current estimate of the subspace basis ($\mathbf{D}(k-1) = \tilde{\mathbf{T}}(k-1) \circ \tilde{\mathbf{T}}(k-1)$). Then, by using this basis, the subspace update is performed thanks to the NOOJA, giving $\mathbf{D}(k)$. The last step of this algorithm is the mapping of the columns $\mathbf{d}_l(k)$ of $\mathbf{D}(k)$ to a Kronecker-product:

$$\begin{aligned} \mathbf{d}_l(k) &= \tilde{\mathbf{t}}_l(k) \otimes \tilde{\mathbf{t}}_l(k) = vec(\tilde{\mathbf{t}}_l(k)\tilde{\mathbf{t}}_l^T(k)) \\ vec^{-1}(\mathbf{d}_l(k)) &= \mathbf{D}_l(k) = \tilde{\mathbf{t}}_l(k)\tilde{\mathbf{t}}_l^T(k) \end{aligned}$$

Then, a power iteration [9] is applied, which takes the form:

$$\tilde{\mathbf{t}}_l(k) = \mathbf{D}_l(k)\tilde{\mathbf{t}}_l(k-1)$$

3.4. Estimated symbols

To obtain the transmitted symbols, the complex matrix $\mathbf{T}$ is rebuilt from $\tilde{\mathbf{T}}$:

$$\mathbf{t}_{lm} = \tilde{\mathbf{t}}_{lm} + j\tilde{\mathbf{t}}_{l(m+N_t)},\ l,m \in 1,\ldots,N_t$$

To guarantee that outputs BSS converge to independent solutions, $\mathbf{T}$ must be orthogonal [10]. This can be formulated as:

$$\mathbf{T'} = \arg\min_{\mathbf{T'}^H\mathbf{T'}=\mathbf{I}} \|\mathbf{T} - \mathbf{T'}\|_F^2$$

The solution of this optimization problem is done in [9]. First a singular value decomposition of $\mathbf{T}$ as $\mathbf{T} = \sum \sigma_j \mathbf{u}_j \mathbf{v}_j^H$ is computed. Then, the singular values of $\mathbf{T}$ are replaced by 1:

$$\mathbf{T'} = reorth(\mathbf{T}) = \sum \mathbf{u}_j \mathbf{v}_j^H$$

Finally, the matrix $\mathbf{T'}$ is applied on the pre-whitened received signals:

$$\hat{\mathbf{s}}(k) = \mathbf{T'}^H(k)\underline{\mathbf{y}}(k)$$

4. SIMULATION RESULTS

In this section we present simulation results illustrating the performances of the adaptive-ASCMA. Each transmitted signal is drawn from 4-QAM constellation and undergoes a time-varying MIMO channel. The considered channel Doppler shift (f_dT_s) is equal to $3,3.10^{-3}$. A two-transmit four-receive type of scenario is considered. We run the adaptive-ASCMA and the adaptive-ACMA with a forget factor $\lambda = 0.99$ and the SCMA with a stepsize $\mu = 5.10^{-2}$.

First, to examine the performance of joint source separation and carrier phase recovery, we compare the output constellation of the adaptive-ASCMA, the adaptive-ACMA and the SCMA (figure 2). We considered the case of a carrier frequency offset $\delta_f T_s = 10^{-4}$ with SNR=30dB. Here only the adaptive-ASCMA can remove the offset without the use of a carrier tracking loop. The equalizer output of adaptive-ACMA and SCMA is spinning due to the carrier frequency offset and the time-varying channel, so a carrier recovery is necessary after them. Normally, the SCMA allows to track the carrier phase, but in this case the carrier phase is too high to obtain a good tracking. Then, we use the signal to interference and noise ratio (SINR) criterion defined as:

$$SINR_m = \frac{|g_{mm}|^2}{\sum_{l,l\neq m}|g_{lm}|^2 + \mathbf{T}_m\mathbf{C}_b\mathbf{T}_m^H}$$

$$SINR = \frac{1}{N_t}\sum_{m=1}^{N_t} SINR_m$$

where $SINR_m$ is the SNR at the m^{th} output, $g_{lm} = \mathbf{H}_m^H\mathbf{W}_l$, where $\mathbf{W}_l$ and $\mathbf{H}_m$ are the l^{th} and the m^{th} column vector of the matrices $\mathbf{W}$ and $\mathbf{H}$ respectively. The figure 3 represents the SINR with a SNR of 15 dB. The adaptive-ASCMA achieves higher SINR than adaptive-ACMA and SCMA.

The figure 4 shows the average BER versus SNR where the average is a time average. The lower BER is achieved by the adaptive-ASCMA, followed by the adaptive-ACMA and by the SCMA. The proposed adaptive-ASCMA allows to obtain a gain of 2 dB in SNR terms compared with the adaptive-ACMA for a BER equal to 10^{-3}.

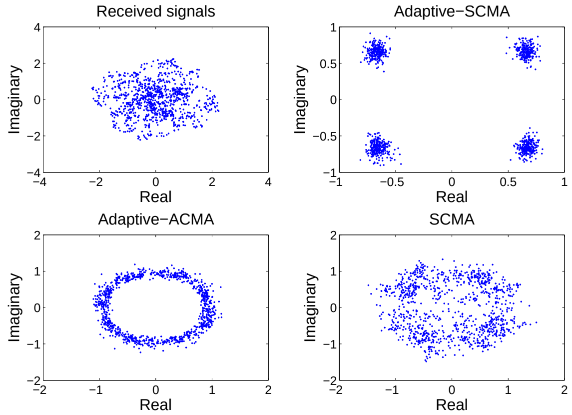

Fig. 2. Constellations of a 4-QAM with a carrier frequency offset. SNR=30dB, $N_t = 2$, $N_r = 4$, $\delta f T_s = 10^{-4}$, $f_d T_s = 0.0033$

5. CONCLUSION

In this paper we have presented a new BSS algorithm for time-varying environments. The proposed algorithm is called adaptive-ASCMA and implements the SCM cost function by using some analytical methods. Moreover, we have shown which subspace contains the solution of our optimization problem. This algorithm can accomplish blind separation and carrier phase recovery simultaneously contrary

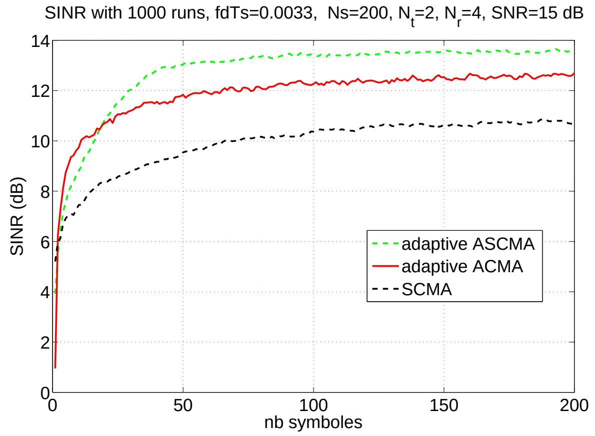

Fig. 3. SINR of a 4-QAM without a carrier frequency offset versus $E_b/N_0.N_t = 2$, $N_r = 4$, $\delta fT_s = 0$, $f_dT_s = 0.0033$ and $SNR = 15dB$

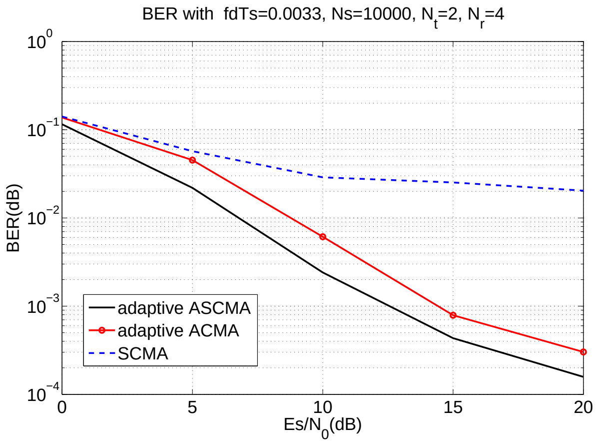

Fig. 4. BER of a 4-QAM without a carrier frequency offset versus E_b/N_0. $N_t = 2$, $N_r = 4$, $\delta_f T_s = 0$, $f_d T_s = 0.0033$

to the adaptive-ACMA. Simulations results have shown that the adaptive-ASCMA has better performances in SINR and BER terms compared to the adaptive-ACMA with comparable computational complexities.

6. APPENDIX

6.1. Proof of Theorem 2

Let $\mathbf{x} \in \mathbb{R}^{16}$, $\mathbf{x} = (x_1, \ldots, x_{16})^T$. By supposing $\mathbf{x} \in \ker(\mathbf{C})$,

$$\mathbf{x} = \sum_{i=1}^{6} \alpha_i e'_i,\ \alpha_i \in \mathbb{R}$$

We have

- $x_1 = x_6 = x_{11} = x_{16} = 0$
- $x_2 = -x_5$, $x_3 = -x_9$, $x_4 = -x_{13}$, $x_7 = -x_{10}$, $x_8 = -x_{14}$, $x_{12} = -x_{15}$

So, $\mathbf{X} = vec^{-1}(\mathbf{x})$ is a (4×4) skew-symmetric matrix. $\mathbf{X}$ and $vec^{-1}(\mathbf{t} \otimes \mathbf{t})$, with $\mathbf{t} = (t_1\ t_2\ t_3\ t_4)^T \in \mathbb{R}^4$ are equal to:

$$\mathbf{X} = \begin{pmatrix} 0 & -x_2 & -x_3 & -x_4 \\ x_2 & 0 & -x_7 & -x_8 \\ x_3 & x_7 & 0 & -x_{12} \\ x_4 & x_8 & x_{12} & 0 \end{pmatrix}$$

$$vec^{-1}(\mathbf{t} \otimes \mathbf{t}) = \begin{pmatrix} t_1^2 & t_1t_2 & t_1t_3 & t_1t_4 \\ t_1t_2 & t_2^2 & t_2t_3 & t_2t_4 \\ t_1t_3 & t_2t_3 & t_3^2 & t_3t_4 \\ t_1t_4 & t_2t_4 & t_3t_4 & t_4^2 \end{pmatrix}$$

$vec^{-1}(\mathbf{t} \otimes \mathbf{t})$ is a (4×4) symmetric matrix and its values on diagonal are positive while $\mathbf{X}$ is a skew-symmetric matrix. So, vectors $\mathbf{x}$ don't have a Kronecker structure, except $0_{\mathbb{R}^{16}}$.

7. REFERENCES

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