A NEW METHOD FOR SNR-ESTIMATION IN IMPULSE RESPONSE MEASUREMENTS

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ABSTRACT

Methods for measuring the impulse response of a linear transmission system and system identification algorithms in general must be robust against noise in the measured system response. To handle the noise it is of great advantage to know the instantaneous signal-to-noise ratio (SNR), especially in situations with changing noise conditions. In this paper we present a new approach for estimating the SNR during an impulse response measurement by means of the so-called sliding window correlation (SWiC) as introduced in this paper. The performance of the proposed method is evaluated by means of simulation results.

1. INTRODUCTION

In many applications such as room or loudspeaker equalization [1] the knowledge of the impulse response of a transmission system is of great importance. Most of these applications have in common, that one has to deal with disturbing noise. As a consequence the methods for evaluating the system impulse response have to be robust against such noise.

When the system under test is time-invariant, one way to deal with noise is to use a periodic signal to excite the transmission system and average suitably over a long time, e.g., by taking the mean of successive periods of the measured signal [2]. One question here is for the necessary averaging time to gain a system impulse response with a desired SNR. For this purpose, one has to know the SNR in the measured system response as precisely as possible.

In this paper, we propose a new method for estimating the instantaneous SNR during the measurement of a linear time-invariant system where the measured system response is disturbed by additive white noise with time-variant power. This new approach is based on the correlation features of the input and the error signal of the normalized least mean square algorithm (NLMS). For exploiting these features, a sliding window correlation (SWiC) is introduced in this paper.

First, in Section 2 we will characterize the class of signals we use to excite the system under test, the so-called perfect sequences. The utilized NLMS algorithm is summarized and the properties of the input and the error signal needed later on are presented in Section 3. In Sections 4 and 5 we introduce the SWiC and describe the new method for estimating the SNR by means of this measure. Finally we present simulation results and draw the conclusions in Sections 6 and 7, respectively.

2. PERFECT SEQUENCES

Perfect sequences ([3], [4]) have shown to be advantages for performing system identification, especially in combination with the NLMS algorithm [5], [6], [7]. The main property of a sequence that makes it perfect is that all out-of-phase values of its periodic autocorrelation function must be equal to zero. Let $p(n)$ be a perfect sequence of length N_p, with n being the time index, and $\tilde{p}(n)$ its periodic repetition with period N_p. The periodic autocorrelation function $r_{pp}(\lambda)$ is then given by

$$r_{pp}(\lambda) = \sum_{i=0}^{N_p-1} \tilde{p}(i) \cdot \tilde{p}(i+\lambda) = \begin{cases} E_p, & \lambda \bmod N_p = 0 \\ 0, & \lambda \bmod N_p \neq 0 \end{cases} \tag{1}$$

with E_p being the energy of the sequence $p(n)$

$$E_p = \sum_{i=0}^{N_p-1} p^2(i). \tag{2}$$

It can be shown that the magnitude of the discrete Fourier transform (DFT) of length N_p of a perfect sequence is a constant (with k being the frequency index)

$$|\mathrm{DFT}\{p(n)\}| = |P_k| = \sqrt{E_p}. \tag{3}$$

According to (3) a real-valued, perfect sequence can easily be constructed in the frequency domain by combining a constant magnitude with any odd-symmetrical phase [3]. In many applications it is advantageous to use a sequence with a low crest factor (ratio of peak to root-mean-square), i.e., with high energy efficiency. The so-called odd-perfect sequences fulfill this requirement and methods exist to construct them for many signal lengths [3], [4].

3. NLMS ALGORITHM

Due to its simplicity and good stability features the normalized least mean square (NLMS) algorithm [8] has gained widespread use in various kinds of signal processing applications. Figure 1 shows the basic setup of the NLMS for estimating the impulse response of a system under test. The possibly noisy system response $\tilde{y}(n)$ is compared to

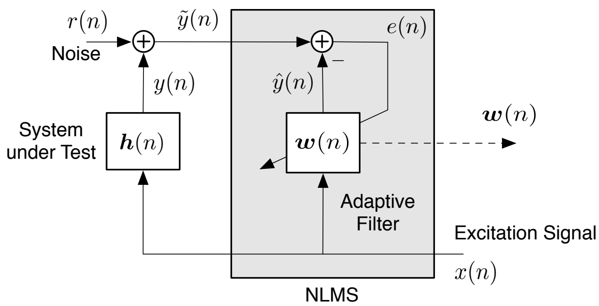

Figure 1: NLMS algorithm with the excitation signal $x(n)$, the system impulse response $h(n)$, the system response $y(n)$, the noise signal $r(n)$, the measured system response $\tilde{y}(n) = y(n) + r(n)$, the error signal $e(n) = \tilde{y}(n) - \hat{y}(n)$, and the adaptive filter vector $\boldsymbol{w}(n)$.

the arbitrary excitation signal $x(n)$ filtered with the current state of the adaptive filter with the coefficients $\boldsymbol{w}(n)$. The error signal $e(n)$ is then used to adapt the filter coefficients for the next time step in a gradient descent approach.

The time-recursive adaptation rule of the NLMS is defined as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \alpha \cdot e(n) \frac{\boldsymbol{x}(n)}{E_x(n)}. \tag{4}$$

with the step size factor α, $0 < \alpha < 2$, and

$$\boldsymbol{x}(n) = (x(n), x(n-1), \ldots, x(n-N_w+1))^T \tag{5}$$

$$\boldsymbol{w}(n) = (w_0(n), w_1(n), \ldots, w_{N_w-1}(n))^T \tag{6}$$

$$e(n) = \tilde{y}(n) - \hat{y}(n) = \tilde{y}(n) - \boldsymbol{w}^T(n) \cdot \boldsymbol{x}(n) \tag{7}$$

where N_w is the length of the adaptive filter. The fastest convergence is gained, when the step size is $\alpha = 1$. With a periodic perfect sequence as excitation it can be shown that after an initialization phase of one period $\boldsymbol{w}(n)$ is fully adapted within exactly one period after a system change ([5], [7]) if N_w is greater than or equal to the length of the impulse response of the system under test. For any periodic excitation signal the energy in (4) becomes a constant $E_x(n) = E_x$.

As will be shown later, the relation between the noise signal $r(n)$, the NLMS input signal $\tilde{y}(n)$ and the NLMS error signal $e(n)$ in the case of a periodic perfect sequence used as excitation signal $x(n) = \tilde{p}(n)$ are of interest for the SNR estimation proposed in this paper. Thus, the properties of these signals are described in more details.

When the system under test, which is assumed to be linear and time-invariant, is excited with a periodic perfect sequence with a period length of N_p the system response $y(n)$ must also be periodic, i.e.,

$$y(n) = y(n - N_p). \tag{8}$$

Thus, for the noisy NLMS input signal $\tilde{y}(n)$ the following relations are valid:

$$\begin{aligned} \tilde{y}(n) &= y(n) + r(n) \\ \tilde{y}(n-N_p) &= y(n) + r(n-N_p). \end{aligned} \tag{9}$$

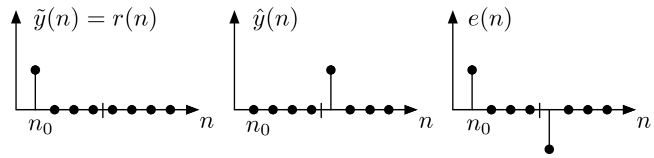

Figure 2: Illustration of the relation between $r(n)$, $\tilde{y}(n)$, $\hat{y}(n)$ and $e(n)$ for the special case of $\boldsymbol{h}(n) = 0$ for all n, $\boldsymbol{w}(n_0) = 0$ at time instance n_0 and $r(n) = \delta(n_0)$. A period length $N_p = 4$ of the excitation signal is assumed.

The step size is now chosen to be $\alpha = 1$. As will be shown in the appendix the recursive formula for the adaptive filter can then be reformulated as

$$\boldsymbol{w}(n) = \sum_{i=1}^{N_p} \tilde{y}(n-i) \frac{\boldsymbol{x}(n-i)}{E_x}. \tag{10}$$

Substituting (10) into (7) and taking the orthogonality features of the perfect sequence excitation $x(n)$ into account yields

$$\begin{aligned} e(n) &= \tilde{y}(n) - \left(\sum_{i=1}^{N_p} \tilde{y}(n-i) \frac{\boldsymbol{x}(n-i)}{E_x} \right)^T \boldsymbol{x}(n) \\ &= \tilde{y}(n) - \tilde{y}(n-N_p). \end{aligned} \tag{11}$$

With (9) we get

$$e(n) = r(n) - r(n-N_p). \tag{12}$$

As a consequence, the error signal $e(n)$ can be calculated from the noisy NLMS input signal only. Thus, it is not necessary to actually calculate the NLMS recursion equations if only the error signal is of interest.

The relation between $r(n)$, $\hat{y}(n)$ and $e(n)$ is illustrated in Figure 2 for the special case of $\boldsymbol{h}(n) = \boldsymbol{0}$ for all n, $\boldsymbol{w}(n_0) = \boldsymbol{0}$ at time instance n_0 and $r(n) = \delta(n_0)$, i.e., a dirac pulse at $n = n_0$. As $\boldsymbol{h}(n) = \boldsymbol{0}$ the measured system response is equal to the noise signal for all n. The error signal in the first period then is equal to the noise signal. In the second period it is equal to the sign inverted noise signal of the first period. This relation can be proved for an arbitrary but fixed $\boldsymbol{h}(n)$.

4. SLIDING WINDOW CORRELATION

The new measure used later for estimating the SNR is called sliding window correlation (SWiC). For an arbitrarily long signal $x(n)$ the SWiC $c_x^{(W,D)}(n)$ at time index n is given by the correlation between two segments of length W and a displacement D, i.e., with the first segment starting at time index n and the second starting at time index $n + D$:

$$c_x^{(W,D)}(n) = \frac{1}{W} \sum_{i=0}^{W-1} x(n+i) \cdot x(n+i+D). \tag{13}$$

Thus, the SWiC characterizes the correlation between two segments of the given signal with a fixed distance D as a function of time. For a perfect sequence $\tilde{p}(n)$ with a period length N_p the SWiC with $W = N_p$ results in

$$c_{\tilde{p}}^{(N_p,D)}(n) = \begin{cases} E_p, & D \bmod N_p = 0 \\ 0, & D \bmod N_p \neq 0 \end{cases} \quad (14)$$

5. SNR ESTIMATION

For estimating the SNR we make the following assumptions:

- The excitation signal is a periodic perfect sequence with period N_p.
- The length of the adaptive filter is equal to N_p with N_p being equal to or greater than the length of the impulse response of the system under test.
- The system under test is linear and time-invariant.
- The noise signal is white. Its power may change over time.

We define the signal power S to be the power of the system response $y(n)$

$$S = \mathrm{E}\{y^2(n)\} \quad (15)$$

and the noise power N to be the power of the noise signal $r(n)$

$$N = \mathrm{E}\{r^2(n)\}. \quad (16)$$

For a non-stationary signal the power of this signal can be approximated by its short-term power averaged over N_p samples

$$\hat{\mathrm{E}}\{z^2(n)\} = \frac{1}{N_p} \sum_{i=0}^{N_p-1} z^2(n+i). \quad (17)$$

Thus, we define the time-variant noise power $N(n)$ as the short-term power of the noise signal $r(n)$

$$N(n) = \hat{\mathrm{E}}\{r^2(n)\} = \frac{1}{N_p} \sum_{i=0}^{N_p-1} r^2(n+i) \quad (18)$$

and the signal power to be the short-term power of the system response. This yields the approximated SNR at time index n:

$$\mathrm{SNR}(n) \approx \frac{\hat{\mathrm{E}}\{y^2(n)\}}{\hat{\mathrm{E}}\{r^2(n)\}} = \frac{\sum_{i=0}^{N_p-1} y^2(n+i)}{\sum_{i=0}^{N_p-1} r^2(n+i)}, \quad (19)$$

which is used as the reference SNR for the rest of this paper.

As the signals $r(n)$ and $y(n)$ normally cannot be measured directly in a real system, they have to be estimated. A common assumption is, that the misadjustment of the adaptive filter is sufficiently small, so that after an initialization phase the signals $r(n)$ and $y(n)$ can be approximated by $e(n)$ and $\hat{y}(n)$, respectively. This yields the following estimated SNR

$$\mathrm{SNR}'_{Ramadan}(n) = \frac{\sum_{i=0}^{N-1} \hat{y}^2(n+i)}{\sum_{i=0}^{N-1} e^2(n+i)} \quad (20)$$

As this approach is used by Ramadan and Poularikas [9] to estimate the SNR, in this paper this method is called the Ramadan-method.

To derive an alternative more accurate method for estimating the SNR we recall the property (12) of the NLMS error signal $e(n)$ found in Section 3 for the excitation signal $x(n) = \tilde{p}(n)$ being a periodic perfect sequence:

$$e(n) = r(n) - r(n - N_p).$$

This relation shows that in the case of a linear time-invariant system excited with a perfect sequence and the step size of the NLMS being set to $\alpha = 1$, the error signal of the NLMS is equal to the sum of the original and the shifted and sign inverted noise signal. The shift is exactly one period. Thus, the expectation of the product of the error signal and a shifted version of it with the displacement being $D = N_p$ yields

$$\begin{aligned} &\mathrm{E}\{e(n) \cdot e(n+N_p)\} = \\ &\quad = \mathrm{E}\{(r(n) - r(n-N_p)) \cdot (r(n+N_p) - r(n))\} \\ &\quad = -\mathrm{E}\{r^2(n)\} + \mathrm{E}\{r(n) \cdot r(n+N_p)\} \\ &\qquad + \mathrm{E}\{r(n) \cdot r(n-N_p)\} \\ &\qquad - \mathrm{E}\{r(n+N_p) \cdot r(n-N_p)\}. \end{aligned} \quad (21)$$

For a stationary white noise signal all terms but the first become zero and we get

$$\mathrm{E}\{e(n) \cdot e(n+N_p)\} = -\mathrm{E}\{r^2(n)\} \quad (22)$$

This relation is also valid – at least approximately – if the noise signal is sufficiently stationary for the duration of three periods. In this case (22) is (approximately) equal to (18) with a minus sign. Thus, (22) can be calculated by means of the SWiC of the NLMS error signal with a window length $W = N_p$ and a displacement $D = N_p$

$$c_e^{(N_p,N_p)}(n) = \frac{1}{N_p} \sum_{i=0}^{N_p-1} e(n+i) \cdot e(n+i+N_p). \quad (23)$$

For a time-invariant system we know that in the case of a periodic excitation signal the output of the system must also be periodic:

$$y(n) = y(n - N_p). \quad (24)$$

Thus, the expectation of the NLMS input signal $\tilde{y}(n)$ multiplied by a shifted version of it with a displacement of $D = N_p$ results in

$$\begin{aligned} &\mathrm{E}\{\tilde{y}(n) \cdot \tilde{y}(n+N_p)\} = \\ &\quad = \mathrm{E}\{(y(n) + r(n)) \cdot (y(n+N_p) + r(n+N_p))\} \\ &\quad = \mathrm{E}\{y^2(n)\} + \mathrm{E}\{y(n) \cdot r(n)\} \\ &\qquad + \mathrm{E}\{y(n) \cdot r(n+N_p)\} \\ &\qquad - \mathrm{E}\{r(n) \cdot r(n+N_p)\}. \end{aligned} \quad (25)$$

If the noise signal is white and not correlated to the system output $y(n)$ all terms but the first become zero and (25) yields the power of the system output. Analog to the

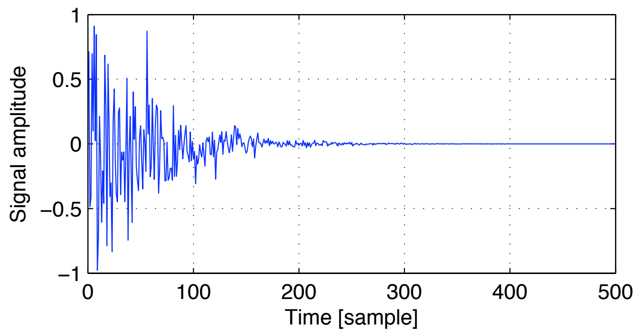

Figure 3: System impulse response $\boldsymbol{h}$ of length $N_h = N_p = 500$. It is a random signal with an exponential envelope.

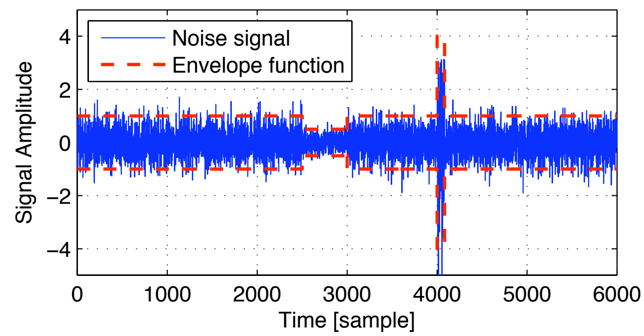

Figure 4: Noise signal and the multiplied envelope function.

noise power, this can be calculated by means of the SWiC of the NLMS input signal $\tilde{y}(n)$

$$c_{\tilde{y}}^{(N_p,N_p)}(n) = \frac{1}{N_p} \sum_{i=0}^{N_p-1} \tilde{y}(n+i) \cdot \tilde{y}(n+i+N_p) \quad (26)$$

With (23) and (26) the SNR can be estimated as

$$\text{SNR}'_{SWiC}(n) = \frac{c_{\tilde{y}}^{(N_p,N_p)}(n)}{-c_e^{(N_p,N_p)}(n)}. \quad (27)$$

6. SIMULATION AND RESULTS

To compare the proposed method with the Ramadan-method described above a linear time-invariant system with additive white Gaussian noise was simulated. The system was modeled by an FIR filter of length $N_h = 500$. The impulse response of this filter was chosen to be a random but fixed sequence with an exponentially falling envelope (see Figure 3).

As excitation signal an odd-perfect sequence with a period length of $N_p = N_h = 500$ was chosen. The additive white Gaussian noise signal was multiplied by an envelope function to simulate changing noise (see Figure 4). The noise was added to the system response $y(n)$. The simulation is run for 6000 samples, i.e., 12 periods.

The results of the simulation are shown in Figure 5. This comparison shows clearly, that the SNR estimate of the proposed method is much closer to the real SNR than the estimate of the method used by Ramadan and Poularikas.

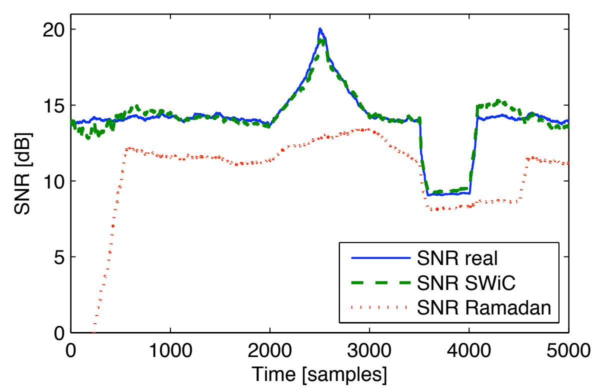

Figure 5: Results of the simulation. The figure shows the real SNR and the SNR estimates for the two compared methods. The y-axis goes only up to 5000 because a block of two periods, i.e., 1000 samples, is needed to estimate the SNR.

7. CONCLUSION

In this paper a new method for estimating the SNR during impulse response measurements with periodic perfect sequences was introduced. It is based on the correlation features of the measured system response. It was shown, that the true energy of the wanted as well as the noise signal can be approximated from the measured signal quite accurately by means of the SWiC.

As two periods of the input signal are needed to calculate the SNR, the use in applications with the demand for very low delay is limited. If, however, a delay of two periods is tolerable the proposed method delivers an accurate estimate of the current SNR.

It is of special interest that the proposed method is not only accurate but is also completely independent of the actual system identification algorithm, i.e., any suitable algorithm can be chosen while the results of the SNR estimation are not influenced by that choice or the accuracy of the estimated system impulse responses.

A typical szenario for the application of the proposed method is the measurement of a room impulse response. After having measured the response of the room, the SNR during the measurement can be estimated and the parts of the system response with low SNR, e.g., when a door was slammed, can be weighted differently to minimize the impact on the resulting impulse response.

The proposed approach for estimating the SNR opens up new perspectives for improving system identification methods in many applications that need an accurate estimate of the current SNR.

8. APPENDIX

To show that

$$\boldsymbol{w}(n) = \sum_{i=1}^{N_p} \tilde{y}(n-i) \frac{\boldsymbol{x}(n-i)}{E_x}$$

we write down the recursion equation (4) explicitly for N_p recursion. The filter state at time instance n_0 shall be

$$\boldsymbol{w}(n_0) = \boldsymbol{w}_0$$

and the step size factor is set to $\alpha = 1$.

$$\begin{aligned}
\boldsymbol{w}(n_0+1) &= \boldsymbol{w}_0 + e(n_0)\frac{\boldsymbol{x}(n_0)}{E_x} \\
&= \boldsymbol{w}_0 + (\tilde{y}(n_0) - \boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0))\frac{\boldsymbol{x}(n_0)}{E_x} \\
\boldsymbol{w}(n_0+2) &= \\
&= \boldsymbol{w}(n_0+1) + e(n_0+1)\frac{\boldsymbol{x}(n_0+1)}{E_x} \\
&= \boldsymbol{w}_0 + (\tilde{y}(n_0) - \boldsymbol{w}_0^T \cdot \boldsymbol{x_0}(n))\frac{\boldsymbol{x}(n_0)}{E_x} + \Big(\tilde{y}(n_0+1) \\
&\quad - \left(\boldsymbol{w}_0 + e(n_0)\frac{\boldsymbol{x}(n_0)}{E_x}\right)^T \boldsymbol{x}(n_0+1)\Big)\frac{\boldsymbol{x}(n_0+1)}{E_x}
\end{aligned}$$

With the orthogonality properties of the perfect sequence this can be reduced to

$$\begin{aligned}
\boldsymbol{w}(n_0+2) = \boldsymbol{w}_0 &+ (\tilde{y}(n_0) - \boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0))\frac{\boldsymbol{x}(n_0)}{E_x} \\
&+ (\tilde{y}(n_0+1) - \boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0+1))\frac{\boldsymbol{x}(n_0+1)}{E_x}
\end{aligned}$$

Continuing the recursion up to $n_0 + N_p$ we get

$$\begin{aligned}
\boldsymbol{w}(n_0+N_p) = \boldsymbol{w}_0 &+ \sum_{i=0}^{N_p-1} \tilde{y}(n_0+i)\frac{\boldsymbol{x}(n_0+i)}{E_x} \\
&- \sum_{i=0}^{N_p-1} \boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0+i)\frac{\boldsymbol{x}(n_0+i)}{E_x}
\end{aligned}$$

Taking the orthogonality feature of the perfect sequence into account and using that $\boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0+i)$ is a scalar, we can rewrite the second sum in the following way

$$\begin{aligned}
&\sum_{i=0}^{N_p-1} \frac{1}{E_x} \cdot \Big(\boldsymbol{w}_0^T \cdot \boldsymbol{x}(n_0+i)\Big) \cdot \boldsymbol{x}(n_0+i) \\
&\quad = \frac{1}{E_x} \cdot \sum_{i=0}^{N_p-1} \boldsymbol{x}(n_0+i) \cdot \Big(\boldsymbol{w}^T(n_0) \cdot \boldsymbol{x}(n_0+i)\Big) \\
&\quad = \frac{1}{E_x} \cdot \sum_{i=0}^{N_p-1} \Big(\boldsymbol{x}(n_0+i) \cdot \boldsymbol{x}^T(n_0+i)\Big) \cdot \boldsymbol{w}(n_0) \\
&\quad = \frac{1}{E_x} \cdot \boldsymbol{w}^T(n_0) \cdot \sum_{i=0}^{N_p-1} \boldsymbol{x}(n_0+i) \cdot \boldsymbol{x}^T(n_0+i) \\
&\quad = \frac{1}{E_x} \cdot \boldsymbol{w}^T(n_0) \cdot E_x \cdot \boldsymbol{I}^{(N_p)} = \boldsymbol{w}(n_0)
\end{aligned}$$

with $\boldsymbol{I}^{(N_p)}$ being the identity matrix of dimension N_p. Substituting this into the equation for $\boldsymbol{w}(n_0 - N_p)$ and substituting n_0 by n results in

$$\boldsymbol{w}(n_o + N_p) = \frac{1}{E_x} \sum_{i=0}^{N_p-1} \tilde{y}(n_0+i) \cdot \boldsymbol{x}(n_0+i).$$

q.e.d.

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