FILTERING IN THE TIME-FREQUENCY DOMAIN FOR THE DETECTION OF COMPACT OBJECTS

D. Herranz[a], *J. L. Sanz*[a] *and E. E. Kuruoğlu*[b]

[a] Instituto de Física de Cantabria, CSIC–UC
Av. los Castros s/n, 39005, Santander, Spain
phone: + (34) 942201534, fax: + (34) 942200935, email: herranz@ifca.unican.es
web: http://max.ifca.unican.es/webcmb/people/herranz.html

[b] Istituto di Scienza e Tecnologie dell'Informazione "A. Faedo", CNR
via G. Moruzzi 1, 56124, Pisa, Italy
phone: + (39) 0503153128 , fax: +(39) 0503152810 , email: Ercan.Kuruoglu@isti.cnr.it

ABSTRACT

In this work we propose a detection method for compact sources in the Time-Frequency domain. By exploring the joint Time-Frequency distribution of data and by calculating the time-frequency profile of the compact sources it is possible to improve the detectability of faint point sources. We propose the detection of sources by identification of local maxima (peaks) in the time-frequency Wigner-Ville distribution of the data, previously filtered with a two-dimensional correlator filter. The filter serves the double purpose of enhancing the signal and of reducing cross-terms derived from the Wigner-Ville transform. We test our method and compare it with a the case of the detection in the time domain (filtered with the appropriate one-dimensional correlator), for a situation in which we have faint sources embedded in stationary white Gaussian noise. The detection in the time-frequency domain gives us better significance levels of faint sources (signal-to-noise ratio ~ 1) than the detection in just the time domain.

1. INTRODUCTION

The detection and estimation of the intensity of compact objects embedded in a background plus instrumental noise is relevant in different contexts, e.g. astrophysics, cosmology, medicine, teledetection, radar, etc. A typical situation in microwave astronomy, for example, is the detection of far and faint galaxies embedded in the diffuse emission of the Cosmic Microwave Background (CMB), our Galaxy and the instrumental noise of the telescope detectors. Typically those galaxies have angular scales that are smaller than the angular size of the beam of the antenna, hence they are referred to as '*point sources*' in the literature. More in general, we talk about *compact sources* (CS): objects/signals that are spread out over a small region.

Different techniques have proven useful in the literature. In the context of microwave astronomy, some of the proposed techniques are frequentist, such as the standard matched filter [1,2], wavelets like the standard Mexican Hat [3–5] and other members of its family [6] and, more generally, filters based on the Neyman-Pearson approach using the distribution of maxima [7–9]. Besides, Bayesian methods have also been recently developed [10, 11].

All the previous methods make use of the fact that CS have a characteristic profile (spatial or temporal) that is different from the diffuse noise they are embedded in. For example, the matched filter is just the correlation of the data with the source profile (weighted by the noise correlation function), wavelets are useful because they identify features with a given scale and so on.

Another possibility, not much explored yet, is to detect CS in the frequency domain; for example, it is possible to devise detectors that work in Fourier space. Again, the key point is that CS are compact as well in frequency domain and therefore it could be possible to separate them from other components if they are not band-limited or if their spectral distribution differs sufficiently from the profile of the CS.

Since CS are compact in both time and frequency domain, an interesting possibility is to detect them in the joint Time-Frequency domain. This way the full distinctiveness of CS with respect to the other components (diffuse signals and noise) can be taken into account. Time-frequency detectors have been studied in the literature (see for example [12–14]); besides, the problem of blind time-frequency source separation in astrophysics has been addressed in several works [15, 16]. In this work we propose a detection method specifically tailored for compact sources. The method is based on filtering of the joint Time-Frequency domain, in which we can calculate the profile of the sources. In Section 2 we will shortly review the fundamental of Time-Frequency distributions and we will propose the detection method. In Section 3 we compare with simulations the proposed detection method in the Time-Frequency domain to a standard detection method in the time domain only. Finally, in Section 4 we summarise our conclusions.

2. FILTERING OF TIME-FREQUENCY DISTRIBUTIONS

Time-Frequency distributions can be described in a unified approach by means of Cohen's distribution [17, 18]

$$P(t,\omega) = \frac{1}{4\pi^2}\int\int\int e^{-i\theta t - i\tau\omega + i\theta u}\phi(\theta,\tau)\times s^*\left(u-\frac{1}{2}\tau\right)s\left(u+\frac{1}{2}\tau\right)du\,d\tau\,d\theta. \quad (1)$$

Here t denotes the time variable and ω the frequency variable. Unless otherwise stated, all integrals in this paper run from $-\infty$ to ∞. Eq. (1) is a bilinear distribution obtained from a certain signal $s(t)$ by applying a symmetric lag $\pm\tau/2$ and by weighting it by a *kernel function* $\phi(\theta,\tau)$ that must satisfy a set of conditions (see [17] for a list of the different conditions that the kernel must satisfy) in order to guarantee that $P(t,\omega)$ is a valid time-frequency distribution. There are many different kernels that satisfy the required options. Therefore, there are many different ways to define time-frequency distributions. The comparison among the different kernels and time-frequency distributions is out of the scope of this work.

One of the simplest and historically most used time-frequency distributions is the Wigner-Ville [19, 20] distribution:

$$W(t,\omega) = \frac{1}{2\pi}\int e^{-i\tau\omega} s^*\left(t-\frac{1}{2}\tau\right) s\left(t+\frac{1}{2}\tau\right) d\tau. \quad (2)$$

The Wigner-Ville distribution can be obtained from the general formula (1) by making $\phi(\theta,\tau)=1$. This simplifies calculations a lot. Besides, the Wigner-Ville has some good properties: it is real, time and frequency shifts in the signal produce time and frequency shifts in the distribution, its properties under time and frequency scaling are readily determined, and it has the following inversion property:

$$s(t) = \frac{1}{s^*(0)}\int W\left(\frac{t}{2},\omega\right) e^{i\omega t} d\omega. \quad (3)$$

For these reasons we will use the Wigner-Ville distribution in this paper. However, the method we present here could be easily generalised for other time-frequency distributions.

We are interested in the detection of compact sources with a given waveform. An interesting situation is the case of point sources observed with a instrument with a known point spread function, for example stellar objects or very far galaxies observed with a radiometer whose antenna beam is Gaussian-shaped. As the point source passes across the field of view of the telescope, the radiometer registers its intensity as a function of time:

$$g(t) = A \exp\left[-\frac{t^2}{2R^2}\right], \quad (4)$$

where A is the *amplitude* (or central intensity) of the source and R is the width of the beam, and we have assumed for simplicity that the source is centred at the position $t_0 = 0$. Let $G(\omega)$ be the Fourier transform of $g(t)$. Then we have

$$G(\omega) = A\, R \exp\left[-\frac{\omega^2 R^2}{2}\right]. \quad (5)$$

Using equations 2 and 4 it is straightforward to obtain the Wigner-Ville distribution corresponding to a Gaussian-shaped object:

$$W_g(t,\omega) = A^2\left(\sqrt{2}R\right)\exp\left[-\left(\frac{t}{R}\right)^2 - (\omega R)^2\right]. \quad (6)$$

Eq. (6) associates to the source, at any time t, its local power spectrum and it is normalised so that the marginals of $W_g(t,\omega)$ (the instantaneous energy and spectrum of $g(t)$) are satisfied.

Now let us consider that the source is embedded in stationary white noise. Stationary white noise is characterised by a flat power spectrum at any time instant t. Noise tends to mask the presence of the sources, making it difficult to detect them. The theory of detection of signals in stationary white noise is well established (see for example [21]) and it seems to be little else to be said on the topic. We could, for example, use a detector based on the correlator:

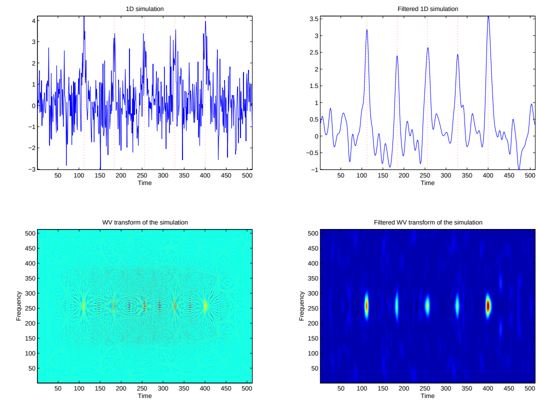

Figure 1: A example of the two approaches examined in this paper: in the two top panels, the one-dimensional approach of filtering the data directly. In the two bottom panels, the two-dimensional approach of filtering the Wigner-Ville transform of the data instead.

$$\Psi_g(t) \propto g(t). \quad (7)$$

But we intend to go a little bit further. Time-frequency analysis is generally used in the context of non-stationary signals and/or noise, where the study of the local properties of the data can give useful insights about the statistical nature of the signals. Here, however, we hope to show that it may be useful to apply time-frequency analysis even to *stationary* noise. The key idea is as follows:

Valid time-frequency distributions (including the Wigner-Ville distribution) must satisfy the condition of the marginals: if we integrate $P(t,\omega)$ over the frequency for a given t we must obtain the instantaneous energy at t, and if we integrate over time for a given frequency ω we must obtain the power spectrum at ω. If we apply a time-frequency transform (for example Wigner-Ville) to stationary white noise its energy is distributed uniformly across the entire (t,ω) plane, whereas if we apply it to a noiseless Gaussian signal we obtain we obtain eq. (6): the energy of the signal is still concentrated in a compact elliptic region of area $\propto \pi\times R\times(1/R)=\pi$. Therefore, by going to the time-frequency plane we expect the source to stand out more clearly among the noise.

The problem is not as easy. The bilinear form of the Wigner-Ville transform make it appear interference cross-terms, both between different sources and between sources and noise. For example, let us consider the case of two Gaussian signals, one located at $t=0$ and the other at $t=t_0$, with no noise:

$$s(t) = A_1 \exp\left[-\frac{t^2}{2R^2}\right] + A_2 \exp\left[-\frac{(t-t_0)^2}{2R^2}\right]. \quad (8)$$

Then the corresponding Wigner-Ville distribution would be

$$P_s(t,\omega) = \left(\sqrt{2}R\right) e^{-(\omega R)^2} \times [A_1^2 e^{-t^2/R^2} + A_2^2 e^{-(t-t_0)^2/R^2} + 2A_1A_2\cos(t-t_0) e^{-\left(\frac{t-t_0/2}{R}\right)^2}]. \quad (9)$$

The last term is a quickly oscillatory interference cross-term, located at the position $t_0/2$ and whose absolute amplitude is comparable to the 'true' point sources. Similarly, interference terms do appear between the signal and noise peaks and between different noise peaks. Interference terms are a serious nuisance and, if not taken into account properly, can lead to spurious detections. One possible solution is to choose wisely the kernel so that the cross-terms are cancelled [22]. Other possibility is to realize the oscillating nature of the cross-term and to do some post-processing that effectively removes them. A way to do this is convolving the time-frequency image with a suitable filter. We can define, in analogy to the one-dimensional correlator in eq. (7), a two-dimensional correlator:

$$\Psi_W(t,\omega) \propto W_g(t,\omega). \quad (10)$$

This filter will serve the double purpose of removing cross-terms and improving the detectability of point sources in the time-frequency plane [14].

In the following, we are going to test this idea using one-dimensional simulations of white noise plus Gaussian sources. On the one hand, we will use the correlator (7) to the simulations and detect the sources on the filtered data. On the other hand, we are going to apply a discrete Wigner-Ville transform to the simulations, then to use the two dimensional correlator (10) and detect in the resulting two-dimensional image. Figure 1 shows in its first panel (top-left) the time series containing the sources mixed with the noise. The position of the sources is marked with vertical dot lines. The second panel (top-right) shows the time series after filtering; it is possible to see how much easier would be to detect the sources in the filtered data. The third panel (bottom-left) shows the discrete Wigner-Ville transform of the time series. Note that the cross-terms make it very difficult to see anything meaningful. However, in the last panel (bottom-right) we can see the filtered time-frequency image, where sources are clearly detectable.

3. SIMULATIONS

In order to quantify the previous ideas we have performed a set of toy simulations. Each simulation consists of 512 samples ('pixels'). For each simulation we have generated a random realisation of stationary white Gaussian noise, with standard deviation $\sigma = 1$ (in arbitrary units), to which we add five identical point sources with a given amplitude A distributed in regularly spaced grid so that there is no overlap among sources. We filter the simulation with the correlator (7), with a normalisation chosen so that the amplitude of the sources is unchanged after filtering. We filter separately the noise alone and the total simulation (noise+sources) with the same filter; the noise-only filtered simulation will be used later to establish the significance of the detections.

In parallel, we compute the discrete Wigner-Ville transform of the noise, on the one hand, and the total simulation, on the other hand. The output of the Wigner-Ville transform are two 512×512 pixel images (due to the symmetry of the time-frequency transform, these images are redundant: the part corresponding to negative frequencies is symmetric to the part corresponding to positive ones). We filter both images with the two-dimensional asymmetric correlator (10), normalised to preserve the amplitude of the sources. Again, we keep the part corresponding to noise for later use.

We identify detections by looking for peaks (local maxima) in the filtered simulations. We are interested in testing whether the significance of the detections, as a function of the input source amplitude A, is better in the filtered Wigner-Ville images or in the filtered one-dimensional real space. We have simulated 40 different values of A, from $A = 0.7$ to $A = 3.5$ (in the same arbitrary units of the noise, note that we are simulating rather low signal-to-noise sources, from -1.5 to 5.4 dB). We made 1000 simulations for each value of the amplitude (5000 sources). In total, we have simulated 200000 sources in 40000 different noise realisations.

For each source we find (we consider a detection any peak in the filtered image whose position corresponds to the position of a input source) we have calculated its significance. In order to calculate it, we use the statistics of the noise peaks we have saved during the process (tens of millions of peaks in both cases). The significance of a detection with an observed amplitude $\hat{A}$ is the fraction of peaks of the noise that have intensities $\geq \hat{A}$.

Results can be seen in Figure 2. The solid line shows the average significance of the detected sources as a function of the input value of A for the one-dimensional case (filtering in real space). The dashed line shows the same for the two-dimensional case (filtering in the Wigner-Ville space). The dashed curve is below the solid one: that means that is more difficult to mistake a peak generated by noise for a true source in the two-dimensional case than in the one-dimensional case.

3.1 Non-stationary noise

The main goal of this paper has been to show that time-frequency analysis, that has been traditionally applied for non-stationary noise and/or signals, can also improve the detection of compact sources embedded in stationary noise. In this section we will briefly show that even in the case on non-stationary noise similar improvements can be observed. For this purpose, we have performed a set of simulations similar to the ones described in the previous section, but considering non-stationary noise.

In order to make the study more realistic, we have taken the instrumental noise pattern of a typical row of pixels belonging to a CMB image of the ESA *Planck* mission [23] at 70 GHz. The non-stationary noise pattern can be seen in Figure 3. The non-stationarity of the noise comes from a combination of the non-isotropic scanning strategy of the satellite and the map-making algorithm used for obtaining the images. Since the physical units of the images are not relevant for this exercise, we have normalised the noise pattern to an average value of 1 (in arbitrary units). In the same arbitrary units, the variance of the rms values is 0.21. Note that, since at the moment of the preparation of this paper *Planck* has not been launched yet, the rms noise pattern comes from a simulation.

We have repeated the same procedure as explained in the previous section. The results can be seen in Figure 4. The qualitative agreement of figures 2 and 4 indicates that the proposed technique can work well even under non-stationary conditions.

4. CONCLUSIONS AND DISCUSSION

In this paper, we have proposed a new method to detect point sources embedded in stationary white noise based on the filtering of time-frequency distributions. The time-frequency transform spreads the energy of the noise over a large area on the time-frequency space whereas point sources remain compact objects in the same space. This makes them easier to detect among noise fluctuations. In order to quantify this, we have compared a standard detection procedure based on the standard correlator to its equivalent in the Wigner-Ville transformed data. We have performed 40000 simulations of stationary white Gaussian noise plus low signal-to-noise sources, and computed the significance of the detections in the two approaches. The approach based on the filtering of the time-frequency images leads to better significance levels.

We have chosen a very simple filtering scheme based on the well-known correlator. However, we admit that this is a too naive approach, as the time-frequency transform of a white Gaussian noise is not in general a Gaussian noise. In spite of this, the results are good enough to improve the results of the standard one-dimensional approach. But a more proper treatment of the problem would require filters that are specifically suited for the kind of noise that appears in time-frequency space, maybe a modification of the Neyman-Pearson filter proposed in [9]. This will be the subject of a future work.

Finally, we have considered as well the case of non-stationary noise, applying the same filtering scheme as in the stationary case. We observe similar results in terms of improvement of the time-frequency filtering over the standard one-dimensional approach. We wish to let it clear, however, that this cannot be considered as a full non-stationary approach to the problem, since the filters we have used are stationary. We have simply shown that the techniques proposed here can work even in non-stationary environments. Another direction of future work will be the development of a full non-stationary filtering scheme extension of these results to the regime of non-stationary noise able to generalise the results of this paper.

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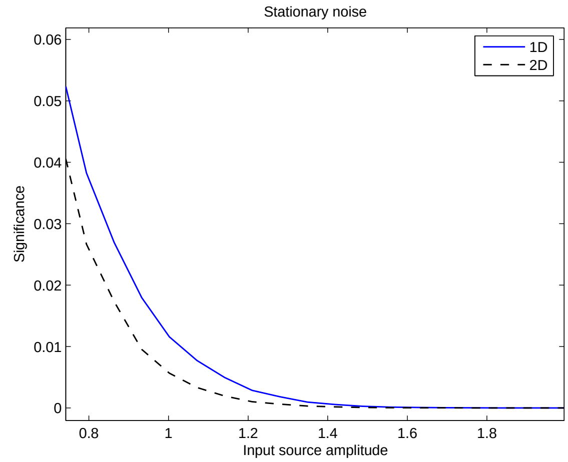

Figure 2: Average significance of the detections in the one-dimensional case (solid line) and the two-dimensional case (dotted line), as a function of the input source amplitude. Each point in the curves is the average over 1000 simulations.

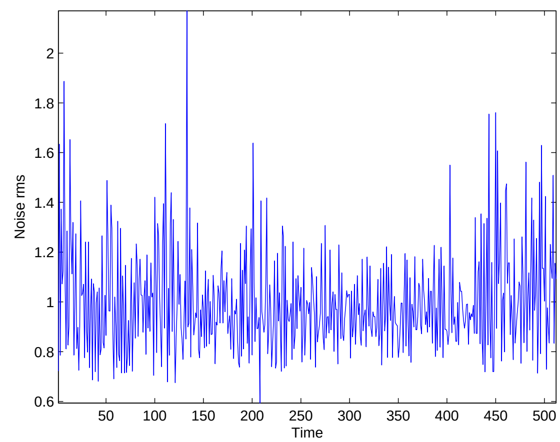

Figure 3: Non-stationary noise rms pattern for a row of pixels in a 70 GHz image of the *Planck* satellite.

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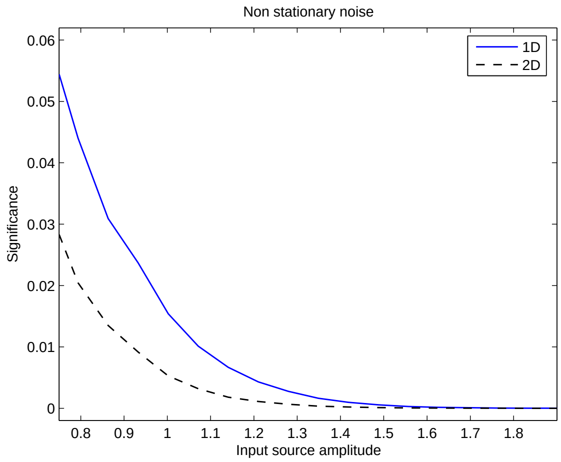

Figure 4: Average significance of the detections in the one-dimensional case (solid line) and the two-dimensional case (dotted line), as a function of the input source amplitude, for the non-stationary noise case. Each point in the curves is the average over 1000 simulations.

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