DESIGN OF A RECTANGULAR FREQUENCY INVARIANT BEAMFORMER WITH A FULL AZIMUTH ANGLE COVERAGE

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Abstract. A novel design of the frequency invariant beamformer based on a rectangular array is proposed. There are two unique features about this design: there is no taped delay-line (TDL) or any other temporal processing involved and the resultant beamformer has a full 360° azimuth angle coverage. Two design examples are provided with a satisfactory frequency invariant property.

Keywords. broadband arrays, frequency invariant beamformer, rectangular arrays, tapped delay-lines.

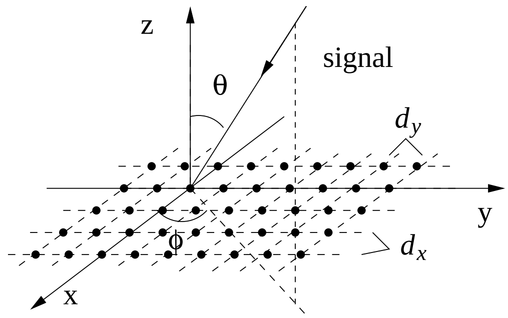

Figure 1: A uniformly spaced rectangular array, where the signal impinges from the direction (θ, ϕ).

1. INTRODUCTION

Broadband beamforming has found many applications in various areas ranging from sonar and radar to wireless communications and it is usually achieved by the use of tapped delay-lines (TDLs) or FIR/IIR filters in its discrete form [1, 2, 3], which can form a frequency dependent response for each of the received broadband sensor signals to compensate the phase difference for different frequency components. Frequency invariant beamformer (FIB), which can achieve a beam pattern independent of frequency, and hence with a constant beamwidth, is a special class of broadband beamformers and traditionally its operation also requires the employment of TDLs or FIR/IIR filters [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

However, it is possible to perform broadband beamforming without employing any temporal processing by replacing the TDLs with the so-called sensor delay-lines (SDLs) and in this structure only one single weight is attached to each sensor [16, 17]. Some efforts have been made to design frequency invariant beamformers based on such kind of array structures [18, 19]. In this paper, we will address the frequency invariant beamforming design problem based on a rectangular array with full 360° azimuth angle coverage. This is a modification to the proposed design method in [18], where, due to the symmetry of the introduced substitution, it can only simulate the response of a broadband linear array and form a response along the azimuth angle ϕ for the range $\phi \in [0° \ 180°]$; while for the range $\phi \in [-180° \ 0°]$, it simply repeats itself. This leads to two main beams pointing to two opposite directions.

However, in many applications it is important to have a beamforming capability over the full azimuth angle range and here a modified approach will be proposed to design such a frequency invariant beamformer pointing to a specific direction, while maintaining the desired attenuation over the remaining directions in the full azimuth range.

This paper is organised as follows. In Section 2, the original FIB design based on the rectangular array without TDLs is first reviewed, followed by the proposed modification. Design examples are provided in Section 3 and conclusions drawn in Section 4.

2. FREQUENCY INVARIANT BEAMFORMING FOR RECTANGULAR ARRAYS

Fig. 1 shows an equally spaced rectangular array with a signal coming from a direction (θ, ϕ). The spacing of the adjacent array elements in the x and y directions is d_x and d_y, respectively.

The response of the array with respect to temporal frequency $\omega\ rad/s$ and angle of arrival (θ, ϕ) of the impinging signal is given by

$$P(\omega, \theta, \phi) = \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y) e^{-j\frac{k\omega \sin\theta \cos\phi d_x}{c}} e^{-j\frac{l\omega \sin\theta \sin\phi d_y}{c}} , \quad (1)$$

where $D(kd_x, ld_y)$ is the response of the sensor at the position (kd_x, ld_y), $k, l = \ldots, -1, 0, 1, \ldots$, and c is the wave propagation speed. Note that $D(kd_x, ld_y)$ is a constant and independent of frequency, since there are no TDLs or any other frequency dependent processing for each received sensor signal.

With the following substitutions

$$\begin{aligned} \omega_1 &= \frac{\omega \sin\theta \cos\phi d_x}{c} \\ \omega_2 &= \frac{\omega \sin\theta \sin\phi d_y}{c} , \end{aligned} \quad (2)$$

we have

$$P(\omega_1, \omega_2) = \sum_{k,l=-\infty}^{\infty} D(kd_x, ld_y)\, e^{-jk\omega_1} e^{-jl\omega_2} . \quad (3)$$

We can see that the beam pattern of such a rectangular array can be obtained by first applying a 2-D (two-dimensional) Fourier transform to the array's coefficients $D(kd_x, ld_y)$ according to (3) and then using the above substitutions in (2).

From (2), we have

$$\frac{\omega_2 d_x}{\omega_1 d_y} = \tan\phi . \quad (4)$$

Thus, the substitution ϕ is given by

$$\phi = \arctan \frac{\omega_2 d_x}{\omega_1 d_y} . \quad (5)$$

To achieve a frequency invariant beam pattern $P(\phi)$, which is independent of the elevation angle, we can express $P(\phi)$ as a function of ω_1 and ω_2 by the substitution in Equation (5) and then apply the 2-D inverse Fourier transform to obtain $D(kd_x, ld_y)$.

Since

$$\phi = \arctan \frac{\omega_2 d_x}{\omega_1 d_y} = \arctan \frac{-\omega_2 d_x}{-\omega_1 d_y} , \quad (6)$$

the resultant response $P(\omega_1, \omega_2)$ has a symmetric response on the (ω_1, ω_2) plane, which leads to real-valued coefficients $D(kd_x, ld_y)$ after applying the inverse transform. However, the problem is, the function $\tan\phi$ is periodic with a period of π. Therefore, the resultant beam pattern of the proposed design is also a periodic function of the azimuth angle ϕ with a period of $180°$.

A simple remedy to this problem is to modify the substitution in Equation (5). Assume $\omega \geq 0$, then according to Equation (2), we have

$$\begin{aligned} &\omega_1 \geq 0 \quad \text{for} \quad \phi \in [-\frac{\pi}{2}\ \frac{\pi}{2}] \\ &\omega_1 \leq 0 \quad \text{for} \quad \phi \in [-\pi\ -\frac{\pi}{2}] \cup [\frac{\pi}{2}\ \pi] \\ &\omega_2 \geq 0 \quad \text{for} \quad \phi \in [0\ \pi] \\ &\omega_2 \leq 0 \quad \text{for} \quad \phi \in [-\pi\ 0] . \end{aligned} \quad (7)$$

Then new set of substitutions is given by

$$\phi = \begin{cases} \arctan \frac{\omega_2 d_x}{\omega_1 d_y} & \text{for } \omega_1 > 0 \\ \frac{\pi}{2} & \text{for } \omega_1 = 0\ \&\ \omega_2 > 0 \\ -\frac{\pi}{2} & \text{for } \omega_1 = 0\ \&\ \omega_2 < 0 \\ \arctan \frac{\omega_2 d_x}{\omega_1 d_y} - \pi & \text{for } \omega_1 < 0\ \&\ \omega_2 < 0 \\ \arctan \frac{\omega_2 d_x}{\omega_1 d_y} + \pi & \text{for } \omega_1 < 0\ \&\ \omega_2 > 0 \\ a & \text{for } \omega_1 = 0\ \&\ \omega_2 = 0 \end{cases} , \quad (8)$$

where a is a scalar with an arbitrary value since the case "$\omega_1 = 0$ & $\omega_2 = 0$" corresponds to the case $\omega = 0$ and the array is not supposed to form any beam to the DC signals.

Now, given the desired frequency invariant response $P(\phi)$, the design of a uniformly spaced rectangular array with a full azimuth coverage can be briefly described as follows:

Step 1. Using the substitutions in Equation (8) in $P(\phi)$, we obtain $P(\omega_1, \omega_2)$, defined over one period $\{\omega_1, \omega_2\} \in [-\pi; \pi)$.

Step 2. Applying a 2-D inverse Fourier transform to $P(\omega_1, \omega_2)$ returns the desired coefficients $D(kd_x, ld_y)$ for the corresponding sensors. As an approximation, we can employ the 2-D inverse discrete Fourier transform (IDFT) by sampling $P(\omega_1, \omega_2)$ on the (ω_1, ω_2) plane over the range $[-\pi; \pi)$.

One key issue is in this design is to find a realisable desired frequency invariant beam pattern $P(\phi)$. We can not obtain it through a narrowband linear array design method as in the original design since the resultant beam pattern will be a periodic function of ϕ with a period π. One solution is to use an FIR filter design method to obtain a lowpass filter with a response $H_0(\Omega)$, which has a maximum response at the normalised frequency $\Omega = 0$. Then the desired response $P(\phi)$ is obtained by directly replacing Ω in $H_0(\Omega)$ by $\phi - \phi_0$, i.e.

$$P(\phi) = H_0(\phi - \phi_0) , \quad (9)$$

where ϕ_0 is the main beam direction. We can also design a bandpass FIR filter $H_1(\Omega)$ with a maximum response at

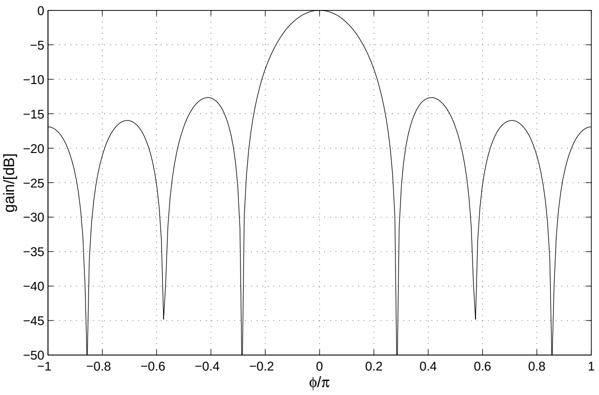

Figure 2: The desired response with a main beam in the direction $\phi = 0°$.

$\Omega = \Omega_0$, $\Omega_0 \in [-\pi\ \pi]$; then use the substitution $P(\phi) = H_1(\phi)$ to obtain the desired response with a main beam pointing to $\phi = \Omega_0$.

3. DESIGN EXAMPLES

Now we give two design examples based on a 19×19 uniform spaced rectangular array. The frequency range of interest is between 400 Hz and 1600 Hz with a signal propagation speed $c = 340m/s$ and an array spacing $d_x = d_y = \lambda_{min}/2 = 34000/(2 \times 1600)cm \approx 10cm$. Note this setting can be translated into other frequencies (both acoustic and electromagnetic) by changing the array spacing according to the wavelength of the signal. Suppose the desired response $P(\phi)$ is given by the following equation

$$P(\phi) = \frac{1}{7}\sum_{n=-3}^{3} e^{jn\phi}\ , \tag{10}$$

which is the response of a lowpass 7-tap FIR filter with uniform weighting. The desired response in this case has a main beam pointing to $\phi = 0°$, as shown in Fig. 2.

The resultant beam pattern for the elevation angle $\theta = 90°$ over the frequency range of interest is shown in Fig. 3, which exhibits a satisfactory frequency invariant property.

Next, we show an example with a main beam direction $\phi_0 = \frac{\pi}{2}$ $(90°)$. The desired response is given by

$$P(\phi) = \frac{1}{7}\sum_{n=-3}^{3} e^{jn(\phi-\phi_0)}\ . \tag{11}$$

The resultant beam pattern is shown in Fig. 4, with again a clear frequency invariant property.

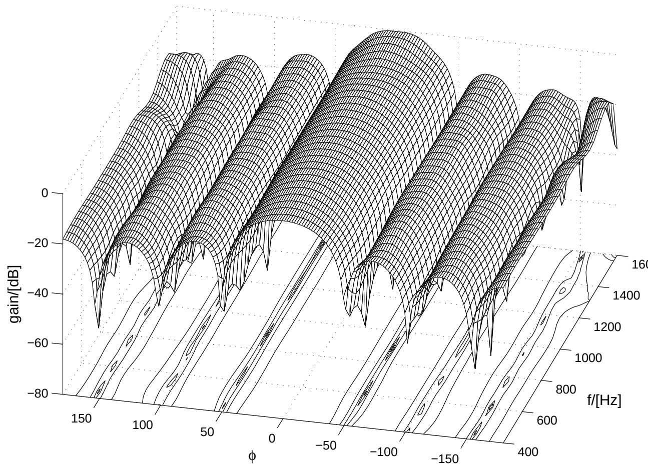

Figure 3: A design example with a broadside main beam ($\theta = 90$).

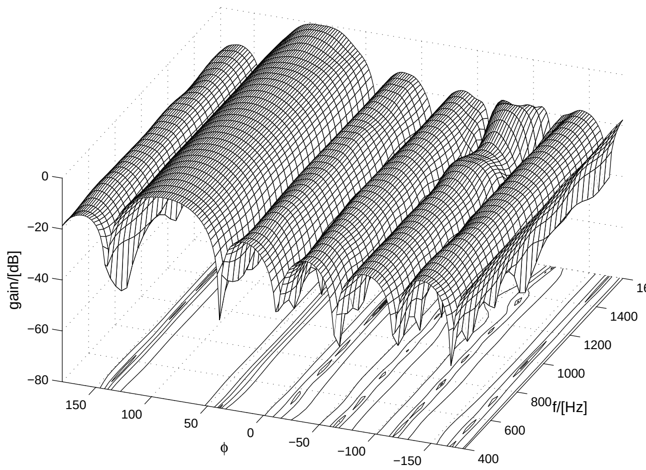

Figure 4: A design example with an off-broadside main beam ($\theta = 90$).

4. CONCLUSIONS

A novel design for a frequency invariant beamformer based on a rectangular array has been proposed without employing TDLs or any other temporal filtering processes. Different from a previously proposed design, it has a beamforming capability over the full 360° azimuth angle range. Design examples show that it can achieve a satisfactory frequency invariant response over the frequency range of interest.

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