INVESTIGATION OF PARAMETER EFFECTS AND TRUNCATED MULTISTATIC DATA MATRIX IN DECOMPOSITION OF TIME REVERSAL OPERATOR METHOD

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ABSTRACT

Acoustic waves are commonly used in the detection, localization and focusing fields. DORT, a French acronym for decomposition of time reversal operator, is a novel method in which active detection and focusing of acoustic waves using arrays of transceivers is performed. The DORT method extracts relevant information from collected data and forms a multistatic data matrix (MDM) that enables the detection and focusing of signals on scattering objects. In this paper, the parameters of the DORT method are studied and their effects on the focusing performance are investigated. In addition, a truncated MDM method, which achieves both time and storage saving over the full MDM method, is proposed.

Index Terms — *DORT method, multistatic data matrix, parameter effects, time reversal, truncation.*

1. INTRODUCTION

Time reversal signal processing (TRSP) is a novel technique for focusing waves to a designated target. This technique has been developed in many applications such as lithotripsy, nondestructive testing, imaging [9], underwater communication [8] and electromagnetic [11]. As the time reversal process is very flexible, it can be easily used in an iterative mode [1]. The wave transmitted after one time reversal process leads to a second reflected wave that can also be measured and time reversed. In the case of multiple point-like scatterers, the iterative time reversal (ITR) process will focus on the most reflective target (or scatterer) in the medium. The theory of ITR has shown that the iteration converges and leads to an invariant of the time reversal operator (TRO) [1]. Note that the TRO is obtained from the multistatic data matrix (MDM) that can be measured from array of transducers.

However, as the ITR process cannot focus on weaker point-like scatterers in a multiple scattering medium [1], [2], the DORT method, which is based on the ITR process, was developed and proven able to perform selective focusing on weaker point-like scatterers efficiently [2], [3]. The DORT method explores the scatterers' information by decomposing the TRO and relating each eigenvector of the TRO with a corresponding point-like scatterer in a multiple scattering medium. The back-propagation of the eigenvector in the time reversal array (TRA) can selectively focus signal on the corresponding scatterer.

The DORT method has been developed for cases containing well-resolved scatterers [2]-[4], non-well-resolved scatterers [5], and both [6]. The DORT method is also presented in two approaches, including experimental techniques [1]-[4] and numerical techniques [6], [7]. Other variations of the DORT include the beam-space DORT method which was employed in underwater environment [8] and the focused DORT method for medical treatment [9].

The DORT method includes many parameters, thus this paper investigates the impact of these parameters on its focusing performance. The paper has two main objectives. The first is to investigate the parameters of the DORT method so as to optimize focusing performance versus cost. The second is to propose a truncated MDM (TMDM) to achieve recording time saving, storage space saving and smaller computational load compared to the full MDM used in conventional DORT method.

2. DESCRIPTION OF THE DORT METHOD

2.1 Time Reversal Operator

In a multiple-target medium, a single time reversal process (TRP) does not lead to a one-point focused wave, but the process can be iterated to focus on the most reflective target [1]. The main concept behind time reversal is to express the received signals as a general function of the transmitted ones. The inter-element impulse response $k_{lm}(t)$ is signal received at the l^{th} transducer after a signal $\delta(t)$ has been emitted from the m^{th} transducer, as shown in Fig. 1.

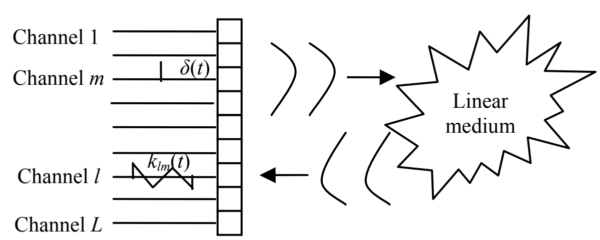

Figure 1 - Inter-element impulse responses measurement.

Let $e_m(t)$,$1\le m\le L$, be the L input signals, then the output signals $r_l(t)$, $1\le l\le L$, where L is number of the transducers are given by:

$$r_l(t) = \sum_{m=1}^{L} k_{lm}(t) * e_m(t) \cdot \qquad (1)$$

A temporal Fourier transforms leads to the relation:

$$R_l(\omega) = \sum_{m=1}^{L} K_{lm}(\omega) E_m(\omega) \cdot \quad (2)$$

Equation (2) can be expressed in a matrix form:

$$\mathbf{R} = \mathbf{KE}\,, \quad (3)$$

where $\mathbf{K} = (K_{lm}(\omega))_{1 \le m, l \le L}$ is a multistatic data matrix (MDM).

In the ITR process, the convergence of the process is dependent on the behavior of the TRO $\mathbf{K}^H\mathbf{K}$, where $\mathbf{K}^H$ is a transpose conjugate of matrix **K** [1]. The TRP follows the reciprocity theorem, which indicates that the point source and the observer can be reversed without altering the acoustic field [2]. Thus, **K** is a symmetrical matrix and the TRO can be simplified to $\mathbf{K}^*\mathbf{K}$. From a mathematical point of view, the diagonalization of the TRO $\mathbf{K}^*\mathbf{K}$ is equivalent to singular value decomposition (SVD) of the MDM **K** as

$$\mathbf{K} = \mathbf{U}\Lambda\mathbf{V}^H\,, \quad (4)$$

where Λ is a real diagonal matrix of the singular values; **U** and **V** are unitary matrices. The eigenvalues of TRO $\mathbf{K}^*\mathbf{K}$ are the squares of the singular values of MDM **K** and its eigenvectors are the columns of **V**.

In the DORT method, let the first transmitted signal be $\boldsymbol{V}$, an eigenvector of the TRO $\mathbf{K}^*\mathbf{K}$ be associated to the eigenvalue λ, then after a TRP, the received signal is $\lambda\boldsymbol{V}^*$, which is proportional to the conjugate of $\boldsymbol{V}$ [3]. Hence, the eigenvectors of the TRO correspond to waveforms that are invariants of the TRP.

2.2 The DORT Method in Well-Resolved Scatterers Case

In this paper, we consider the DORT method in a well-resolved scatterers case. The scatterers are defined as well-resolved when it is possible to focus on only one scatterer, without sending energy to the others. The scatterers are also considered as point-like and isotropic, as shown in Fig. 2. Thus, number of significant eigenvalue of the TRO corresponds to number of the scatterer and each eigenvector of the TRO associated to only one scatterer in the medium.

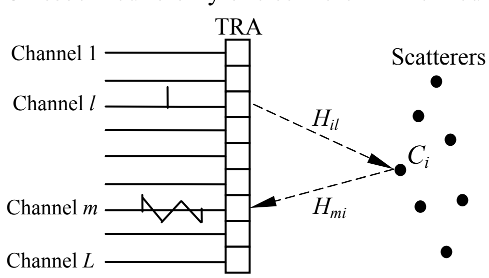

Figure 2 - Propagating paths in a well-resolved scatterers case.

The DORT method is an experimental technique that includes a mathematical processing of the measured data [3]. This method comprises of three steps to selectively focus signal on each scatterer. The first and second steps deal with the detection of scatterers while the third step focuses the signal on the targeted scatterer.

In the first step, the inter-element impulse response (IEIR) is measured. This measurement requires L transmit-receive operations. The first transducer of the array is excited with a signal $e(t)$. The signals received on all L transducers are stored. This operation is repeated for all the transducers in TRA. The components of **K** are obtained by a Fourier transform of each signal.

In the second step, the TRO **K*K** is diagonalized at a chosen frequency. In the case of well-resolved, isotropic and point-like scatterers, each eigenvector of the TRO is associated to a scatterer in the scattering medium [3]. For example, the eigenvector associated to the i^{th} scatterer is $\mathbf{H}_i^*$, where $\mathbf{H}_i = \left[H_{i1} \ldots H_{il} \ldots H_{iL}\right]$, consists of the propagating paths between the i^{th} scatterer and all the transducers in the TRA. Otherwise, the eigenvalue corresponding to the i^{th} scatterer depends on the reflectivity of this scatterer and the propagating paths as

$$\lambda_i = \left|C_i\right|^2 \left(\sum_{l=1}^{L} \left|H_{il}\right|^2 \right)^2, \quad (5)$$

where C_i is the reflectivity coefficient of the i^{th} scatterer.

In the third step, the signal at the TRA is back-propagated using the phase and amplitude provided by the eigenvector found in the second step. The signals focus only on the corresponding scatterer in the medium. This step can be done both experimentally [1] - [4] and numerically [6], [7].

2.3 The DORT Simulation

The DORT is simulated in the finite-difference time-domain (FDTD) [11], as shown in Fig. 3. The dimension of the field is fixed at 200×200 cells. Each cell is equal to λ/10. Ten point-like scatterers are placed randomly with a minimum spacing of 3λ that satisfies as well-resolved scatterers case. The scatterer (in circle) is chosen as the target. The reflectivity coefficient of the target is 10 times larger than other scatterers in the medium. The multiple scattering among scatterers is also considered in the DORT simulations in FDTD. The input signal is a Gaussian modulated sine wave, typical of the type of signals send to a piezoelectric emitter. The DORT simulations are conducted in 12 different experiment set-ups to investigate parameters' effects, as shown in Table I.

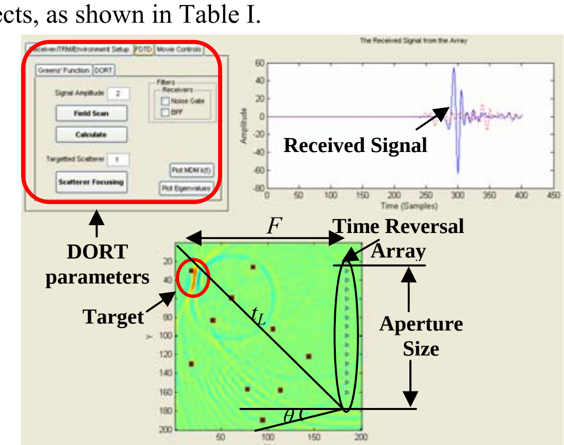

Figure 3 - Simulation set-up of the DORT method.

To optimize focusing performance versus cost, the parameters of the DORT method are investigated in Section 3. To achieve better focusing performance, the forming of the MDM in the DORT method requires longer recording time and larger storage spaces [2], [3], [7]. Thus, in Section 4, we examine a truncated version of the MDM that is able to reduce recording time, computational load and storage space requirement without affecting on its focusing performance.

TABLE I
FDTD SIMULATION SET-UPS

Set-up No	Time Reversal Array (TRA)			
	X	Y	Size (Number of transducers)	Spacing
1	185	3	40	5
2	185	6	20	10
3	185	3	14	15
4	185	10	10	20
5	185	15	10	15
6	185	35	10	10
7	185	57	10	5
8	185	67	10	3
9	185	12	8	28
10	185	70	10	5
11	185	90	10	5
12	185	110	10	5

3. INVESTIGATION OF PARAMETERS IN DORT METHOD

Figure 3 shows some of the DORT parameters that are discussed in this paper. Parameter t_L is defined as the time length that signal travels from the first (or last) element of the TRA to the furthest corner of the field of interest. Parameter t_R is defined as the recording time of the received signal in the TRA. In IEIR measurements, the signal is propagated from TRA. This signal is multiply reflected among scatterers and boundaries before back-propagating to the TRA. Thus, t_R is usually a multiple of t_L. The elevation angle, θ, is the angle between the first (or last element) of the TRA and the highest (or lowest) scatterer.

In this paper, we investigate four main parameters including the TRA aperture (D), the TRA size (L), the waveguide effect and the recording time (t_R). The MDM is decomposed to calculate the eigenvalue and eigenvector. The target has a larger reflectivity coefficient than the other scatterers and is the furthest distance from the TRM. Thus, the eigenvector corresponding to the largest eigenvalue is chosen to focus signal on the target. The focusing performance criterion is evaluated by the signal-to-ripple ratio (SRR), which is defined as the ratio between peak main-lobe signal and peak side-lobe signal, is calculated in decibel (dB). The higher SRR indicates better focusing performance on the target.

3.1 TRA Aperture and Elevation Angle

In this subsection, two parameters (t_R and L) are fixed as $t_R=3t_L$ and $L=10$. The TRA aperture is calculated as

$$D = (L-1)\Delta\,, \tag{6}$$

where Δ is the spacing between two neighboring transducers.

Thus, when the spacing (Δ) decreases, the TRA aperture (D) also decreased, which in turn, increases the elevation angles θ. From Table II, we can see that as the TRA aperture (D) decrease, the SRRs decrease from 12 dB to 10 dB. In set-up #8, target focusing is not achieved. Hence, smaller TRA aperture causes poorer focusing performance on target. The reason is smaller aperture received less reflected signals in IEIR measurement. As a result, decomposition of $\mathbf{K}^*\mathbf{K}$ does not provide accurate eigenvectors associated to the scatterers.

To evaluate the aperture effect in focusing performance, we consider the ratio between TRA aperture and the distance between target and TRA as

$$r = D / F\,, \tag{7}$$

where F is the distance between the target and TRA.

TABLE II
FOCUSING PERFORMANCE IN DIFFERENT TRA APERTURE

Set-up No	t_R	Aperture Size	SRR	Elev^
4	3.0	18.0λ	12.0 dB	-6.91°
5	3.0	13.5λ	12.0 dB	-5.19°
6	3.0	9.0λ	11.5 dB	1.74°
7	3.0	4.5λ	10.0 dB	9.29°
8	3.0	2.7λ	NA**	12.64°

Elev^ is the elevation angle from the TRA to the lowest or highest scatterer.
NA** is unable to focus.

In this experiment, when the distance is fixed as F=16.5λ, we observe that the minimum TRA aperture to achieve focusing on target is D=4.5λ. Thus, the ratio r is required to be larger than 3/11.

3.2 TRA Size

In this subsection, the recording time parameter is fixed as $t_R=3t_L$. The number of transducer (L) varies from 8 to 40, corresponding to set-ups #1 to #4 and #9.

TABLE III
FOCUSING PERFORMANCE IN DIFFERENT TRA SIZES

Set-up No	t_R	TRA Size	Aperture Size	SRR
1	3.0	40	19.5λ	12.0 dB
2	3.0	20	19.0λ	12.0 dB
3	3.0	14	19.5λ	12.0 dB
4	3.0	10	18.0λ	12.0 dB
9	3.0	8	19.6λ	NA**

From Table III, it is observed that in order to focus on target in a scattering environment of 10 random scatterers, a minimum of 10 transducers are required. This implies that the DORT method requires more number of transducers than number of scatterers to achieve selectively focusing. The simulation results tally with theoretical results that are mentioned in previous papers [2] - [4].

3.3 Waveguide Effect

This experiment is carried out in the free space (without waveguide) and in a waveguide medium. The simulation set-ups in both cases are similar with fixed aperture size but the TRA is moved vertically from middle to FDTD boundary, corresponding to set-ups #10 - #12.

TABLE IV
FOCUSING PERFORMANCE IN MEDIUM WITHOUT AND WITH WAVEGUIDE

Set-up No	t_R	Free Space		Waveguide	
		SRR	Elev^	SRR	Elev^
10	3.0	11.0 dB	-6.91°	12.0 dB	-6.91°
11	3.0	NA**	-5.19°	12.0 dB	-5.19°
12	3.0	NA**	1.74°	11.5 dB	1.74°

From Table IV, we can see that the DORT method in free space can not focus signal when the TRA is near the FDTD boundary such as set-ups #11 and #12. On the other hand, the DORT method still focuses signal on target in waveguide medium although the TRA is near the waveguide boundary that is placed at FDTD boundary. The reason is that the aperture size is larger than the actual aperture size due to virtual images of TRA in waveguide medium [12]. This improves focusing performance on the target in waveguide medium versus free space (without waveguide).

3.4 Recording Time

In this experiment, the recording time varies in the range t_R=2-4t_L and the TRA size varies from 10 to 40 corresponding to set-ups #1 to #4 to give more comprehensive results.

TABLE V
FOCUSING PERFORMANCE IN DIFFERENT RECORDING TIME

t_R	Set-up #1	Set-up #2	Set-up #3	Set-up #4
4.0	12.0 dB	12.0 dB	12.0 dB	12.0 dB
3.0	12.0 dB	12.0 dB	12.0 dB	12.0 dB
2.9	12.0 dB	11.5 dB	11.5 dB	11.5 dB
2.8	11.5 dB	11.0 dB	10.5 dB	10.5 dB
2.7	11.5 dB	10.5 dB	10.0 dB	9.5 dB
2.5	10.5 dB	10.0 dB	9.5 dB	9.0 dB
2.4	8.5 dB	8.5 dB	8.0 dB	7.5 dB
2.0	6.0 dB	6.0 dB	6.0 dB	5.5 dB

From Table V, when t_R=3t_L and t_R=4t_L, the same SRR results are obtained because for $t_R \geq 3t_L$, there is no reflected signal from the IEIR measurement, thus the MDMs are similar with any $t_R \geq 3t_L$. If $t_R \leq 2.5t_L$, focusing performance is greatly degraded. Thus, the conventional range of recording time t_R is between 2.5-3t_L.

4. A PROPOSED TRUNCATED MUTISTATIC DATA MATRIX (TMDM)

In the DORT method, the measurement of the IEIR is the first and most important step in achieving good focusing. In fact, larger data in IEIR measurement in the time domain produces a more accurate MDM and eigenvectors in the frequency domain, and thus results in better focusing on the target. From Table V, the full MDM is defined as the MDM with t_{RO}=3t_L resulting in the best SRR. However, this MDM also requires long recording time, large storage space and high computational load in DORT implementation [13].

In a scattering environment, at later recording times, the recorded signals in TRA are weaker. The tail end of a full IEIR has negligible amplitudes that cause insignificant data to construct MDM in frequency domain, as shown in Fig. 4. Thus, the truncated multistatic data matrix (TMDM) is proposed to achieve time and storage space saving over the full MDM in conventional DORT method. The proposed method comprises of three steps: (i) truncate the rear end of full IEIR; (ii) zero-pad the truncated responses; (iii) apply Fast Fourier transformation (FFT) to achieve TMDM $\mathbf{K_T}$ in frequency domain, as shown in Fig. 5. The computation load of FFT in TMDM is reduced much compared to full MDM.

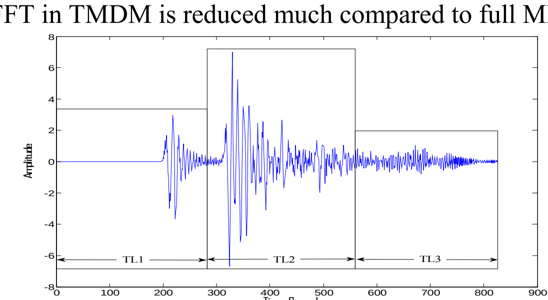

Figure 4 – A full inter-element impulse response measurement.

In the following parts, the TMDM is compared to full MDM. The comparison criterions include recording time saving, storage space saving, signal-to-ripple ratio (SRR) and spatial focusing.

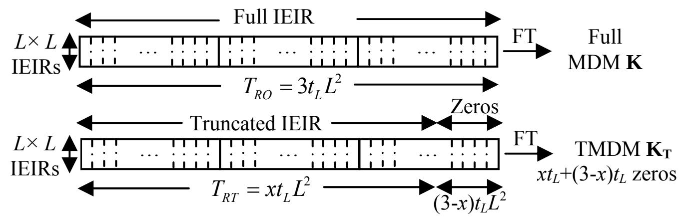

Figure 5 - Illustration of full MDM and TMDM.

4.1 Recording Time Saving and Storage Space Saving

In full MDM, each data measurement requires $t_{RO} = 3t_L$ where t_L is time length. The total recording time for L×L data is

$$T_{RO} = 3t_L L^2 \,. \tag{8}$$

Otherwise, in TMDM, each data measurement requires $t_{RT} = xt_L$ where x is a factor in the range [2.5, 3), as shown in Fig. 4. Thus, the total recording time requirement in L×L data of TMDM is

$$T_{RT} = xt_L L^2 \,. \tag{9}$$

Therefore, the recording time saving when using TMDM is:

$$T_S = T_{RO} - T_{RT} = (3-x)t_L L^2 \,. \tag{10}$$

Moreover, the recoding time saving also reduces storage space requirement in the IEIR measurement. If S is the storage space needed to save a frame of data, per element, then the total storage space, S_T, needed to store the truncated IEIR when using TMDM is:

$$S_T = xt_L L^2 S \,. \tag{11}$$

Otherwise, the total storage space needed when using the full MDM is:

$$S_O = 3t_L L^2 S \,. \tag{12}$$

Thus, the storage space saving when using TMDM is:

$$S_S = S_O - S_T = (3-x)t_L L^2 S \,. \tag{13}$$

From (8), (10), (12) and (13), we can see that the TMDM can achieve recording time and storage space saving ratio is

$$R = (3-x)/x \,. \tag{14}$$

This saving ratio is up to 20% with x=2.5, as seen in Fig. 6. Thus, TMDM reduces significant recording time and storage space requirement.

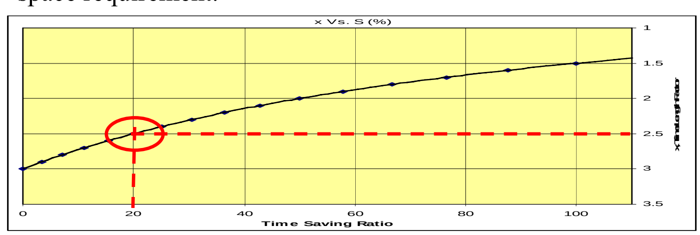

Figure 6 - TMDM saving ratio R.

Besides, the recording time saving T_S and storage space saving S_S are proportional to the square of number of transducer (L^2). In DORT experiments [2], [3], the number of transducer is very large, thus the TMDM achieves very significant saving in time and storage space.

4.2 Signal-to-Ripple Ratio (SRR)

Figure 7 shows the received signals in the target of full MDM, TMDM with 2.5t_L+0.5t_L zeros and TMDM with 2.8t_L+0.2t_L zeros. The main and ripple of received signals using TMDM are stronger than that in full MDM. Thus, the

SRRs using TMDM are slightly smaller about 0-0.8 dB (depending on truncation length) compared to full MDM.

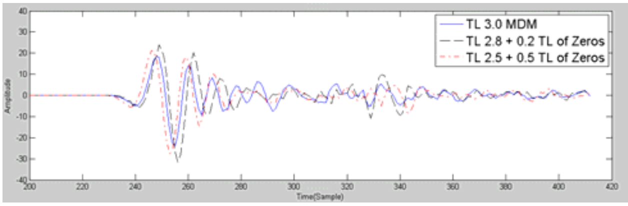

Figure 7 - Received signals at target in full MDM, TMDM with $2.5t_L$+$0.5t_L$ zeros and TMDM with $2.8t_L$+$0.2t_L$ zeros.

4.3 Spatial Focusing

Figures 8, 9 and 10 show the spatial focusing of full MDM and TMDM with $2.8t_L$+$0.2t_L$ zeros and $2.5t_L$+$0.5t_L$ zeros in FDTD [11]. The focusing spots are the darker spots marked in circles. The dynamic range of the focusing spots is also approximated as 20×1 cells (vertical × horizontal cells). Besides, the focusing spots are very close to the target in both TMDM and MDM. In summary, the comparison between TMDM and full MDM is illustrated in Table V.

TABLE V
COMPARISON OF PERFORMANCE BETWEEN TMDM AND FULL MDM

	Recording Time	Storage Space	SRR	Spatial Focusing
TMDM 2.5*TLs*+0.5*TLs*	$T_{RT}=2.5t_L L^2$	$S_T=2.5t_L L^2 S$	11.2 dB	20×1
Full MDM	$T_{RO}=3t_L L^2$	$S_O=3t_L L^2 S$	12 dB	20×1
Improvement	Saving 20%	Saving 20%	-0.8 dB	Same

5. CONCLUSIONS

This paper investigated the DORT parameters and proposed a truncated multistatic data matrix (TMDM) of the DORT method in focusing field. These investigations have disclosed the standard requirement of DORT parameters to achieve good focusing with minimum cost in different configurations. These results are also reliable in real experiment conditions. Besides, the proposed TMDM can achieve saving ratio up to 20% without much degradation of focusing performances in signal-to-ripple ratio (SRR) and spatial focusing compared to full MDM. Thus, the TMDM method is an appropriate approach in DORT experiments and implementations that require limitation of time, storage space and computation.

These findings provided a practical mean in implementing TMDM in many applications such as nondestructive testing, medical imaging [9], underwater acoustics [4], electromagnetism [11] etc.

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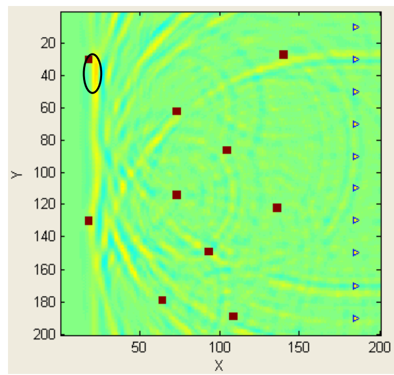

Figure 8 - Spatial focusing in full MDM.

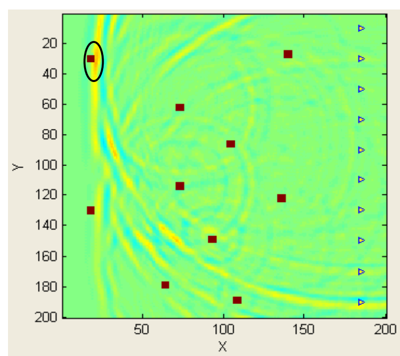

Figure 9 - Spatial focusing in TMDM with $2.5t_L$+$0.5t_L$ zeros.

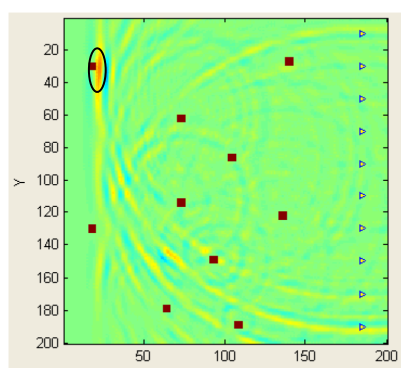

Figure 10 - Spatial focusing in TMDM with $2.8t_L$+$0.2t_L$ zeros.