SEQUENTIAL ESTIMATION OF THE RANGE AND THE BEARING USING THE ZERO-FORCING MUSIC APPROACH

Mohammed Nabil El korso, Guillaume Bouleux‡, Rémy Boyer* and Sylvie Marcos**

*Laboratoire des Signaux et Systémes (L2S),
Université Paris-Sud XI (UPS), CNRS, SUPELEC,
3 rue Joliot Curie, 91190 Gif-Sur-Yvette, France
{elkorso, remy.boyer, marcos}@lss.supelec.fr
‡Laboratoire d'Analyse des Signaux et des Processus Industriels (LASPI),
Université Jean Monnet, IUT de Roanne,
20 avenue de Paris, 42334 Roanne Cedex, France
guillaume.bouleux@univ-st-etienne.fr

ABSTRACT

In this paper, we consider the range and bearing estimation of near-field narrow-band sources from noisy data observed across a passive sensor array. For some difficult scenarios as for correlated and largely spaced sources at low SNRs, or correlated and closely spaced sources, the Near FieLd (NFL) version of the MUltiple SIgnal Classification (MUSIC) algorithm is no longer reliable. In this paper, we adapt and extend the sequential Zero-Forcing MUSIC (ZF-MUSIC) algorithm, which avoids the delicate search of multiple maxima, to the NFL context. In order to compare the NFL ZF-MUSIC with the Second-Order Statistics Weighted Prediction (SOS-WP) algorithm, we assumed an uniform sampling in the spatial domain. However, the proposed algorithm can be exploited for general array geometries. Finally numerical simulations show that the variance of the proposed algorithm achieves the Cramér-Rao Bound (CRB) in difficult scenarios and for sufficient Signal to Noise Ratio (SNR).

1. INTRODUCTION

The localization of radiating sources by an array of passive sensors arises in several applications, such as radar, sonar, seismology, geophysics, radio astronomy and oceanography. A variety of algorithms [1] has been proposed to solve the estimation problem of the bearing of far-field narrowband sources. In this scenario, the propagating waves can be considered to have a planar wavefront when they reach the sensor array. However, when the sources are located in the near-field region, the waves impinging on the sensors can no longer be assumed to have a planar wavefront. Thus the techniques based on the far-field assumption show unsatisfactory performance. A good approximation of the nonlinear propagation delay can be done by a second order Taylor expansion, called the Fresnel approximation.

Many localization methods for far-field sources can be extended to the NFL context. Among all the existing methods [2], we focus on sequential estimation schemes. There exists essentially two families of sequential estimation schemes: (1) based on the Fourier transform, such as the RELAX approach [3] -this algorithm uses a relaxation based method to minimise the maximum likelihood function- and (2) based on a high-resolution scheme such as the Sequential MUSIC algorithms [4]. Those algorithms adopt a step by step deflation procedure to reduce the signal subspace dimension. At each step, the observed signal is then projected onto the null space of the previously estimated bearings.The zero-forcing approach, called ZF-MUSIC and introduced recently in the far-field case [5], is based on a different principle. It is a sequential high-resolution MUSIC approach which leaves unchanged the signal subspace but scales appropriately the MUSIC pseudo-spetrum. In this paper, we present the extended method of the sequential ZF-MUSIC algorithm in the NFL context. Results and simulations show that the proposed algorithm has a lower variance than the standard NFL MUSIC [6] and the Second-Order Statistics Weighted Prediction (SOS-WP) [7] algorithms. Furthermore, the variance of the NFL ZF-MUSIC achieves the CRB in difficult scenarios and for sufficient SNR.

This project is funded by both region Île de France and Digeteo Research Park.

2. PROBLEM FORMULATION

Consider M radiating near-field and narrowband sources observed by an Uniform Linear Array[1] (ULA) of N sensors with interelement spacing d. The signal at the output of the $(n+1)^{th}$ sensor can be described by

$$x_n(t) = \sum_{l=1}^{M} s_l(t) e^{j\tau_{nl}} + v_n(t), \quad t = 1,\dots,T, \quad n = 0,\dots,N-1,$$

where T $(T > N)$ is the number of snapshots, $s_l(t)$ is the l^{th} radiating signal, $v_n(t)$ is the additive complex white Gaussian noise, which is assumed to be temporally and spatially uncorrelated, of density probability function $\mathcal{N}(0,\sigma^2)$ where σ is a positive real number and τ_{nl} denotes the time delay associated with the signal propagation time from the l^{th} source to the $(n+1)^{th}$ sensor. Consequently τ_{nl} can be expressed as [6]

$$\tau_{nl} = \frac{2\pi r_l}{\lambda}\left(\sqrt{1 + \frac{n^2 d^2}{r_l^2} - \frac{2nd\sin\theta_l}{r_l}} - 1\right), \qquad (1)$$

where r_l and θ_l are the range and the bearing of the l^{th} source, and λ is the signal wavelength. A good approximation of the time delay τ_{nl}, which corresponds to the so-called Fresnel

[1] In order to compare the NFL ZF-MUSIC with the SOS-WP algorithm [7], we assumed an uniform sampling in the spatial domain. However, this assumption can be easily relaxed to more general geometries.

region, i.e., $0.62(d^3(N-1)^3/\lambda)^{1/2} < r_l < 2d^2(N-1)^2/\lambda$, consists of the second-order Taylor expansion, which is given by [7]

$$\tau_{nl} \approx \omega_l n + \phi_l n^2 + O\left(\frac{d^2}{r_l^2}\right), \quad (2)$$

where $O\left(\frac{d^2}{r_l^2}\right)$ represents terms of order larger or equal to $\frac{d^2}{r_l^2}$, and ω_l and ϕ_l are the so-called electric angles which are given by $\omega_l = -2\pi\frac{d}{\lambda}\sin(\theta_l)$, and $\phi_l = \pi\frac{d^2}{\lambda r_l}\cos^2(\theta_l)$. Then the signal can be expressed as

$$x_n(t) = \sum_{l=1}^{M} s_l(t) e^{j\left(\omega_l n + \phi_l n^2\right)} + v_n(t). \quad (3)$$

Rather than estimating the range and bearing directly, we will estimate the electric angles. Once the estimation of the electric angles is done, we can deduce easily the range and the bearing according to [7]:

$$\theta_l = -\arcsin\left(\frac{\omega_l \lambda}{2\pi d}\right), \quad (4)$$

$$r_l = \frac{\pi d^2}{\lambda \phi_l}\cos^2(\theta_l). \quad (5)$$

Consequently the received signal vector $\mathbf{x}(t)$ can be expressed as $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$, where $\mathbf{x}(t) = [x_1(t)\dots x_N(t)]^T$, $\mathbf{s}(t) = [s_1(t)\dots s_M(t)]^T$ and $\mathbf{v}(t) = [v_1(t)\dots v_N(t)]^T$ denote the received signal vector, the source signal vector and the noise vector, respectively. The array manifold matrix is given by

$$\mathbf{A} = [\mathbf{a}(\omega_1, \phi_1)\dots\mathbf{a}(\omega_M, \phi_M)], \quad (6)$$

where the elements of the steering vector are given by

$$[\mathbf{a}(\omega_l, \phi_l)]_{n+1} = e^{j(\omega_l n + \phi_l n^2)},\ n = 0, \dots, N-1. \quad (7)$$

The problem here addressed is then the estimation of the couple $\{\omega_l, \phi_l\}$, $l = 1, \dots, M$ given the noisy data $\mathbf{X} = [\mathbf{x}(1) \quad \mathbf{x}(2)\dots\mathbf{x}(T)]$.

3. NFL ZERO-FORCING SEQUENTIAL MUSIC ALGORITHM

The sequential ZF-MUSIC algorithm [5] can be modified to estimate the electric angles. Let $\hat{\mathbf{R}}_\mathbf{X}$ be the sample spatial covariance of the noisy observation. According to the data model (3), we have

$$\hat{\mathbf{R}}_\mathbf{X} = \frac{1}{T}\mathbf{X}\mathbf{X}^H, \quad (8)$$

and we assume that $\lim_{T\to\infty}\hat{\mathbf{R}}_\mathbf{X} = E[\mathbf{X}\mathbf{X}^H] = \mathbf{A}\mathbf{R}_\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I}$, where $\mathbf{R}_\mathbf{S}$ is the source covariance matrix. We sort the eigenvalues λ_i of the matrix $\hat{\mathbf{R}}_\mathbf{X}$ in descending order $\lambda_1 \geq \cdots \geq \lambda_M \geq \lambda_{M+1} \geq \cdots \geq \lambda_N$, then the expression (8) can be written as

$$\hat{\mathbf{R}}_\mathbf{X} = \sum_{n=1}^{N} \lambda_n \mathbf{u}_n \mathbf{u}_n^H, \quad (9)$$

where $\mathbf{u}_n$ is the eigenvector associated with the eigenvalue λ_n. The noise-subspace is then spanned by $\mathbf{U} = [\mathbf{u}_{M+1}\dots\mathbf{u}_N]$ and the noise projector is defined by $\Pi^\perp = \mathbf{U}\mathbf{U}^H = \sum_{n=M+1}^{N}\mathbf{u}_n\mathbf{u}_n^H$. Consequently, the estimate of the electric angles of the l^{th} source are defined as the maximizers of the spectral form of the sequential NFL ZF-MUSIC given by

$$(\hat{\omega}_l, \hat{\phi}_l) = \arg\max_{(\omega,\phi)} \frac{f_l^{(N)}(\omega, \phi)}{\mathbf{a}(\omega,\phi)^H \Pi^\perp \mathbf{a}(\omega,\phi)}, \quad (10)$$

where $l \in [1\dots M]$ and the zero-forcing function is a quadratic function defined by

$$f_l^{(N)}(\omega, \phi) = \mathbf{a}_N(\omega,\phi)^H \mathbf{P}_l^\perp \mathbf{a}_N(\omega,\phi), \quad (11)$$

where $\mathbf{a}_N(\omega,\phi) = \frac{1}{\sqrt{N}}\mathbf{a}(\omega,\phi)$ and $\mathbf{P}_l^\perp = \mathbf{I} - \mathbf{P}_l$ is the orthogonal projector onto the space spanned by the $l-1$ previously estimated electric angles $(\hat{\omega}_1, \hat{\phi}_1), \dots, (\hat{\omega}_{l-1}, \hat{\phi}_{l-1})$, where

$$\mathbf{P}_l = \begin{cases} \mathbf{I} & \\ \mathbf{A}_l(\mathbf{A}_l^H\mathbf{A}_l)^{-1}\mathbf{A}_l^H & \text{for } l = 2, \dots, M \end{cases}$$

and

$$\mathbf{A}_l = [\mathbf{a}_N(\hat{\omega}_1, \hat{\phi}_1)\dots\mathbf{a}_N(\hat{\omega}_{l-1}, \hat{\phi}_{l-1})] \quad \text{for } l = 2, \dots, M$$

The search of the maximizers of (10) can be done by defining a set of discrete points spanning the electric angles intervals of interest and by performing a two dimensional search. However, according to (4) and (5), the Fresnel region, must be taken into account. Then the sequential NFL ZF-MUSIC algorithm can be described as follow:

> ***Init.*** Apply the spectral form of the sequential NFL ZF-MUSIC, with $\mathbf{P}_1^\perp = \mathbf{I}$, i.e., $f_1^{(N)}(\omega,\phi) = 1$. Then, compute the projector $\mathbf{P}_2^\perp = \mathbf{I} - \mathbf{a}_N(\hat{\omega}_1, \hat{\phi}_1)\mathbf{a}_N(\hat{\omega}_1, \hat{\phi}_1)^H$.
> ***Loop.*** For $l \in [2, \dots, M]$, compute the zero-forcing function according to (11) and solve the criterion (10).

For a fast computation, instead of performing a costly 2-D search, we can discretize our 2-D search with a large gap according to the Fresnel region. The previously estimated electric angles can be used as initial points. We then refine our estimation by using an unconstrained nonlinear optimization, like the *fminsearch* matlab function which is a simplex direct search method [8].

Fig. 1.(a) and Fig. 1.(b) represent the plot of the zero-forcing function for one source, $(\omega,\phi) = (1.2, 0.05)$ rad, and then for two sources, $(\omega_1,\phi_1) = (0.5, 0.015)$ rad and $(\omega_2,\phi_2) = (1.4, 0.06)$ rad, respectively. From Fig. 1.(a), we note that the zero-forcing function is equal to zero for the considered source without affecting too much the other area, and the zero-forcing function is almost equal to one everywhere else. Fig. 1.(b) shows that the latter function has two reversed peaks equal to zero for (ω_1,ϕ_1) and (ω_2,ϕ_2), this represents the cancellation of the zero-forcing function for the two considered electric angles. As the previous scheme, this latter function is almost equal to one everywhere else. Consequently, Fig. 1 suggests that the zero-forcing function $f_l^{(N)}(\omega,\phi)$ is equal to zero for all the previously estimated electric angles without affecting too much the other area.

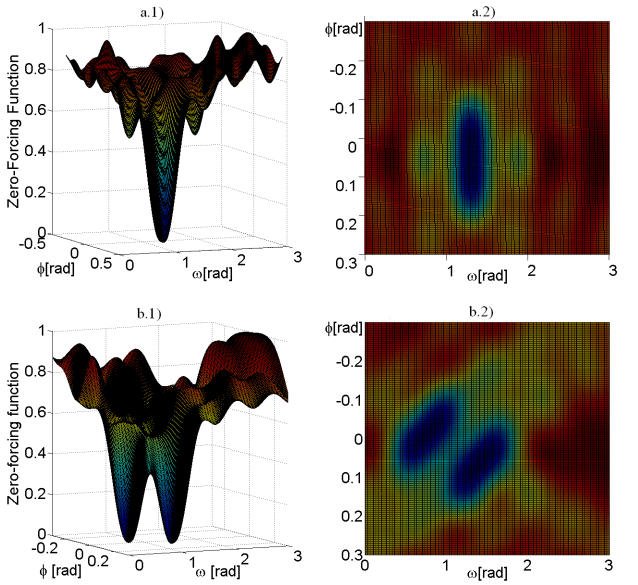

Figure 1: Zero-forcing function for $N = 12$ and ; (a) one source, (b) two sources.

To show the usefulness of the zero-forcing function we consider two cases. The first case is when the two sources are largely spaced. For $(\omega_1, \phi_1) = (1.4, 0.06)$ rad and $(\omega_2, \phi_2) = (0.5, 0.015)$ rad, Fig. 2.(a), Fig. 2.(b), Fig. 2.(c) and Fig. 2.(d) represent the longitudinal and cross section of the normalised NFL MUSIC spectrum and the normalised NFL ZF MUSIC spectrum at $\omega = \omega_1$, $\omega = \omega_2$, $\phi = \phi_1$ and $\phi = \phi_2$, respectively. We notice that, cf. Fig. 2.(a) and Fig. 2.(c), after one iteration, the zero-forcing function canceled the estimated first electric angle at (ω_1, ϕ_1), without affecting too much the lobe of the normalised NFL ZF MUSIC spectrum at (ω_2, ϕ_2). Thus, avoiding the delicate search of multiple maxima, (ω_2, ϕ_2) can be detected by a simple global maximization. The second case is more delicate since we consider two closely spaced sources. For $(\omega_1, \phi_1) = (1.4, 0.05)$ rad and $(\omega_2, \phi_2) = (1.1, 0.015)$ rad, Fig. 3.(a), Fig. 3.(b), Fig. 3.(c) and Fig. 3.(d) represent the longitudinal and cross section of the normalised NFL MUSIC spectrum and the normalised NFL ZF MUSIC spectrum at $\omega = \omega_1$ rad, $\omega = \omega_2$, rad $\phi = \phi_1$ rad and $\phi = \phi_2$ rad, respectively. Even if the sources are closely spaced, we notice that, cf. Fig. 3.(a) and Fig. 3.(c), after one iteration, the zero-forcing function canceled the estimated first electric angle (ω_1, ϕ_1), however the lobe of the normalised NFL ZF MUSIC spectrum is slightly attenuated at (ω_2,ϕ_2), but it remains the global maximum of the normalised NFL ZF MUSIC spectrum (cf. Fig. 3.(b), Fig. 3.(d)). Thus, once again, avoiding the delicate search of multiple maxima, (ω_2, ϕ_2) can be detected by a simple global maximisation.

Finally, Fig. 4 shows that increasing the number of sensors leads to decreasing the main lobe width. Furthermore, we notice that the peak is more selective for the second pulsation.

4. SIMULATIONS

In this section, the NFL ZF-MUSIC algorithm is compared with the standard NFL MUSIC algorithm and with the SOS-

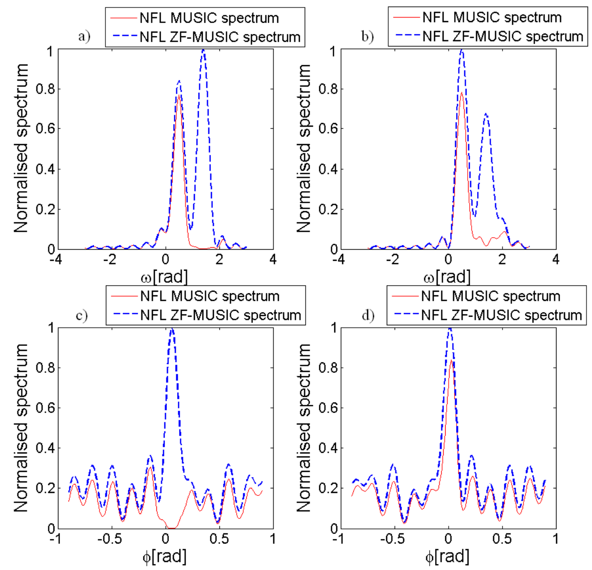

Figure 2: The longitudinal and cross section of the normalised NFL MUSIC spectrum and the normalised NFL ZF MUSIC spectrum at : (a)$\omega_1 = 1.4$ rad, (b)$\omega_2 = 0.5$ rad, (c)$\phi_1 = 0.015$ rad, (d)$\phi_2 = 0.06$ rad.

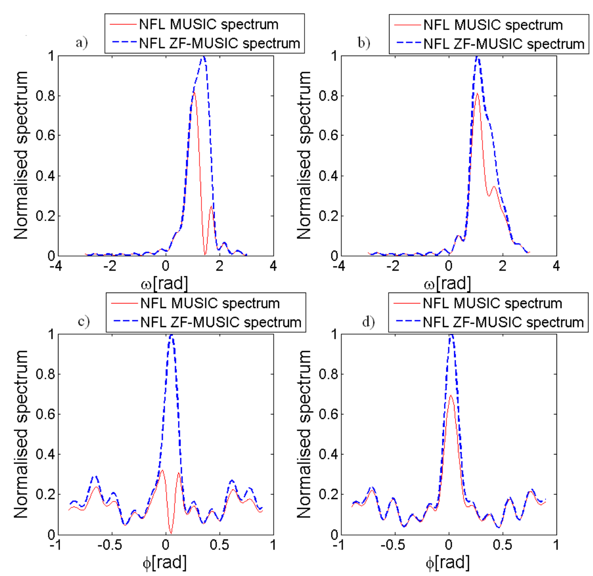

Figure 3: The longitudinal and cross section of the normalised NFL MUSIC spectrum and the normalised NFL ZF MUSIC spectrum at : (a)$\omega_1 = 1.4$ rad, (b)$\omega_2 = 1.1$ rad, (c)$\phi_1 = 0.015$ rad, (d)$\phi_2 = 0.05$ rad.

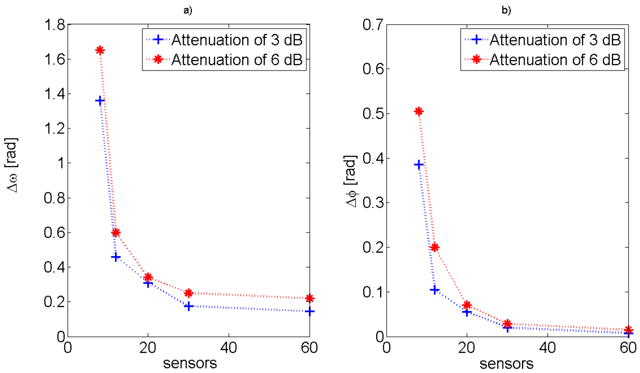

Figure 4: The width of the zero-forcing function corresponding to an attenuation of 3 and 6 dB for $(\omega,\phi) = (1.3, 0.05)$ rad and different values of N ; (a) the width of the main lobe of the longitudinal section at $\phi = 0.05$ rad, (b) the width of the main lobe of the cross section at $\omega = 1.3$ rad.

WP (Second-Order Statistics Weighted Prediction) introduced by Grosicki *et al.* [7]. The context of these simulations is an Uniform and Linear Array (ULA) of 12 sensors spaced by a quarter wavelength. Thus, according to (4) and (5), the Fresnel region is delimited by: $|\omega| < 1.5708$ rad and $0 < \phi < 0.0694$ rad. Two sources are assumed to be near-field narrowband complex circular Gaussian sequences with zero mean and variance, σ_s^2, equal to one. Consequently, the SNR and the source covariance matrix are defined, respectively, as $\text{SNR[dB]} = 10\log_{10}(\frac{1}{\sigma^2})$, and

$$\mathbf{R_S} = \begin{bmatrix} 1 & \gamma \\ \gamma^* & 1 \end{bmatrix} \tag{12}$$

where $|\gamma|$ determines the degree of correlation between these two sources. where $|\gamma|$ determines the degree of correlation between these two sources. The simulations were run with $T = 15$ snapshots and $N_{trial} = 100$ Monte Carlo trials. For each scheme, the Mean Squared Error (MSE) for the first electric angle is evaluated by $MSE(\omega) = \frac{1}{2N_{trial}}[\sum_{i=1}^{N_{trial}}(\hat{\omega}_1(i) - \omega_1)^2 + \sum_{i=1}^{N_{trial}}(\hat{\omega}_2(i) - \omega_2)^2]$, where $\hat{\omega}_1(i)$ and $\hat{\omega}_2(i)$ represent the estimates of ω_1 and ω_2 repectively. The MSE for the second pulsation is evaluated in the same way. As a comparison, we have also plotted the corresponding Cramér-Rao Bounds (CRB) [7].

In the first case, we consider two largely spaced sources which belong to the Fresnel zone, i.e., $(\omega_1, \phi_1) = (0.5, 0.015)$ rad and $(\omega_2, \phi_2) = (1.2, 0.06)$ rad. According to Fig. 5 and Fig. 6, where we have considered different degrees of correlation between the two largely spaced sources, the SOS-WP algorithm cannot estimate the electric angles with a good accuracy even with high SNRs. The NFL ZF-MUSIC and the standard NFL MUSIC algorithms are equivalent for SNRs higher than 20 dB. However, we note that the standard NFL MUSIC algorithm is in good agreement with the CRB from about 15 dB, unlike the sequential NFL ZF-MUSIC algorithm which is more accurate at low SNRs and continues to be in very good agreement with the CRB from 0 dB and 5 dB when the sources are moderately and highly correlated, respectively.

In the second case, we consider two closely spaced sources which belong to the Fresnel zone, ie., $(\omega_1, \phi_1) =$

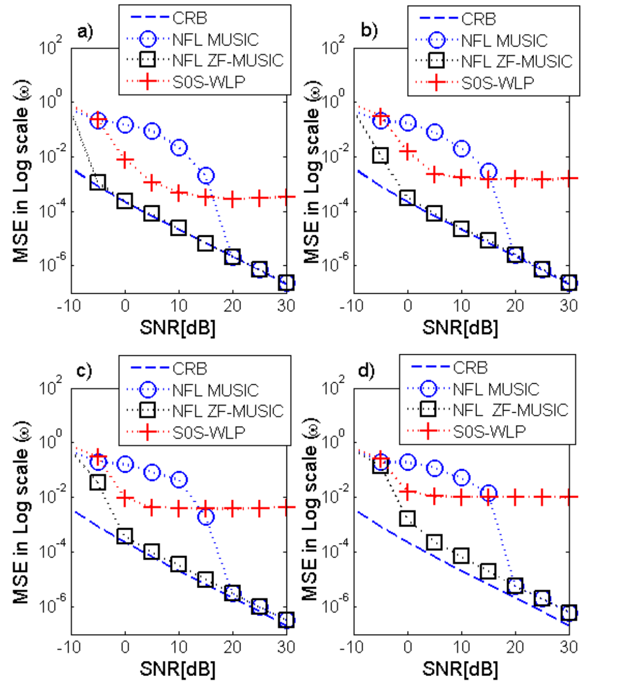

Figure 5: MSE(ω rad) vs. SNR[dB], Largely spaced sources ($\omega_1 = 0.5$, $\phi_1 = 0.015$) rad and ($\omega_2 = 1.2$, $\phi_2 = 0.06$) rad, with 12 sensors and 15 snapshots ; (a) $\gamma = 0$, (b) $\gamma = 0.3$, (c) $\gamma = 0.5$, (d) $\gamma = 0.8$.

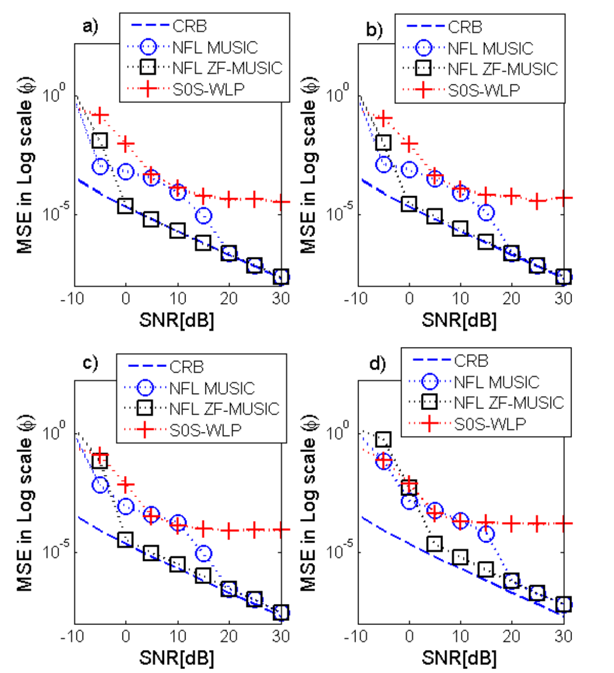

Figure 6: MSE(ϕ rad) vs. SNR[dB], Largely spaced sources ($\omega_1 = 0.5$, $\phi_1 = 0.015$) rad and ($\omega_2 = 1.2$, $\phi_2 = 0.06$) rad, with 12 sensors and 15 snapshots ; (a) $\gamma = 0$, (b) $\gamma = 0.3$, (c) $\gamma = 0.5$, (d) $\gamma = 0.8$.

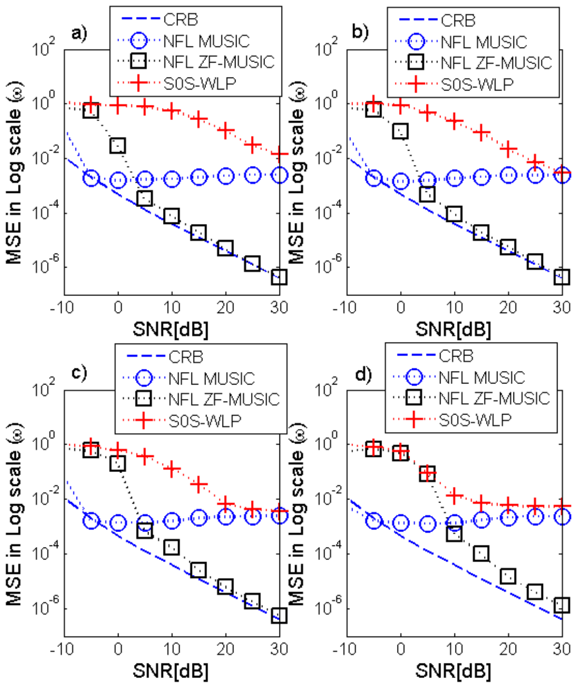

Figure 7: MSE(ω rad) vs. SNR[dB], Closely spaced sources $(\omega_1, \phi_1) = (0.78, 0.019)$ rad and $(\omega_2, \phi_2) = (0.85, 0.063)$ rad, with 12 sensors and 15 snapshots ; (a) $\gamma = 0$, (b) $\gamma = 0.3$, (c) $\gamma = 0.5$, (d) $\gamma = 0.8$.

$(0.78, 0.019)$ rad and $(\omega_2, \phi_2) = (0.85, 0.063)$ rad. According to Fig. 7 and Fig. 8, where we have considered different values of γ for the two closely spaced sources, the standard NFL MUSIC and the SOS-WP algorithms are no longer reliable. However, the sequential NFL ZF-MUSIC algorithm is in good agreement with the CRB of the first electric angle from 5 dB and 10 dB for moderately and highly correlated sources, respectively. It is the same case for the CRB of the second pulsation from 5 dB and 15 dB for moderately and highly correlated sources, respectively.

5. CONCLUSION

A sequential algorithm for the estimation of the range and bearing of near-field sources has been proposed. The algorithm is an extension of the far-field zero-forcing MUSIC algorithm presented recently. Without any deflation of the signal subspace, the MUSIC criterion is scaled by an appropriate function which removes the previously estimated ranges and bearings. The proposed algorithm has been tested in some critical scenarios, e.g., for highly correlated sources, and/or for closely spaced sources, the results have shown that the variance of the NFL ZF-MUSIC algorithm achieves the CRB for sufficient SNR and outperforms the standard NFL MUSIC and the SOS-WP algorithms.

REFERENCES

[1] R. O. Schmidt, "Multiple emitter location and signal parameter estimation", *IEEE Trans. Antennas Propagat.*, vol. 34, pp.276-280, 1986.

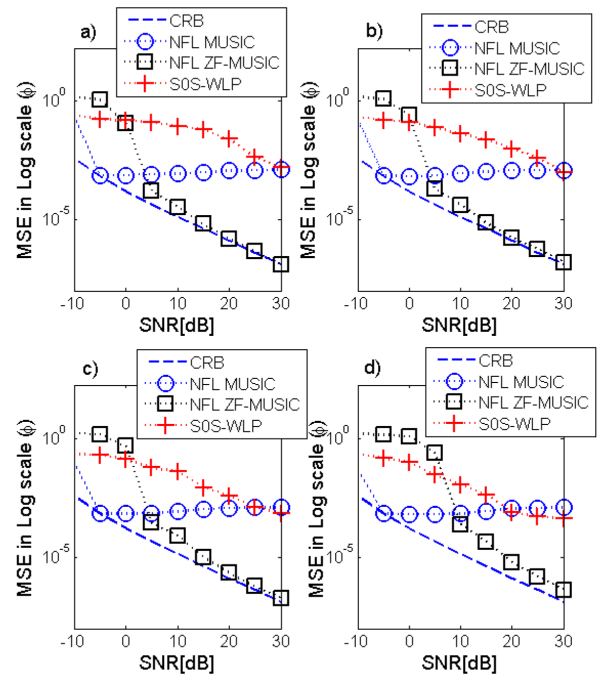

Figure 8: MSE(ϕ rad) vs. SNR[dB], Closely spaced sources $(\omega_1, \phi_1) = (0.78, 0.019)$ rad and $(\omega_2, \phi_2) = (0.85, 0.063)$ rad, with 12 sensors and 15 snapshots ; (a) $\gamma = 0$, (b) $\gamma = 0.3$, (c) $\gamma = 0.5$, (d) $\gamma = 0.8$.

[2] A. J. Van der Veen, E. F. Deprettere and A. L. Swindlehurst, "Subspace based signal analysis using Singular Value Decomposition", *Proceedings of the IEEE*, Vol. 81, No. 9, pp.1277-1308, Sept. 1993.

[3] J. Li, D. Zheng, P. Stoica, "Angle and waveform estimation via RELAX", *IEEE Trans. on Aerospace and Electronic Systems*, vol. 33, Issue 3, pp.1077 - 1087, July 1997.

[4] S. K. Oh, and C. K. Un, "A sequential estimation approach for performance improvement of eigenstructure-based methods in array processing", *IEEE Trans. on Signal Processing*, Vol. 41, Jan. 1993, pp.457-463.

[5] G. Bouleux and R. Boyer, "Zero-forcing based sequential MUSIC algorithm", *IEEE International Conference on Acoustics, Speech and Signal Processing, 2007. ICASSP 2007*, vol. 3, 2007.

[6] Y. D. Huang and M. Barkat, "Near-field multiple source localization by passive sensor array", *IEEE Trans. Antennas Propag.*, vol. 39, no. 7, pp. 968-975, Jul. 1991.

[7] E. Grosicki, K. Abed-Meraim, and Y. Hua, "A weighted linear prediction method for near-field source localization", *IEEE Trans. Signal Process.*, vol. 53, pp. 3651-3660, Oct. 2005.

[8] Lagarias, J.C., J. A. Reeds, M. H. Wright, and P. E. Wright, "Convergence properties of the nelder-mead simplex method in low dimensions", *SIAM Journal of Optimization*, Vol. 9 Number 1, pp. 112-147, 1998.