MARGINALIZATION OF STATIC OBSERVATION PARAMETERS IN A RAO-BLACKWELLIZED PARTICLE FILTER WITH APPLICATION TO SEQUENTIAL BLIND SPEECH DEREVERBERATION

Christine Evers and James R. Hopgood

School of Engineering and Electronics, Institute for Digital Communications, The University of Edinburgh
Mayfield Road, Edinburgh, EH9 3JL, United Kingdom
phone: +44 (131 650) 5655, email: c.evers@ieee.org
web: www.see.ed.ac.uk, http://www.erp.ac.uk/

ABSTRACT

Enhancement of an unknown signal from distorted observations is an extremely important Engineering problem. In addition to noise, the observation space often contains a degrading filter component. A typical example is blind speech enhancement, where a reverberant channel between a stationary source and the receiver can be modeled as a static infinite impulse response component. Particle filters have become popular and versatile estimators for estimating the clean source signal and unknown model parameters by sequentially drawing a large number of samples from a hypothesis distribution. However, direct sampling of static components leads to particle impoverishment as a dynamic is implicitly enforced on the parameters. To circumvent this issue, this paper proposes a novel approach by exploiting analytically tractable substructures of the state space to marginalize static components, facilitating separate estimation of the static parameters using their optimal estimator. The approach is tested for blind dereverberation of speech. Results show that the proposed algorithm effectively removes the effects of the static reverberant channel.

1. INTRODUCTION

State space models in various signal processing applications are often of the conditionally Gaussian state-space (CGSS) form

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{D}_t \mathbf{v}_t, \qquad \mathbf{v}_t \sim \mathcal{N}\left(\mathbf{0}_{Q\times 1}, \mathbf{I}_Q\right), \quad (1a)$$

$$y_t = \mathbf{b}^T \mathbf{y}_{t-1} + \mathbf{c}^T \mathbf{x}_t + \sigma_{w_t} w_t, \qquad w_t \sim \mathcal{N}\left(0, 1\right), \quad (1b)$$

where $\mathbf{x}_t = \{x_k\}_{k=t-Q+1}^{t}$ are the past Q source signal samples, $\mathbf{A}_t$ the source transition matrix, $\mathbf{D}_t \mathbf{D}_t^T$ the innovation covariance, $\mathbf{y}_t = \{y_k\}_{k=t-P+1}^{t}$ the past P observations, $\sigma_{w_t}^2$ the observation noise variance, $\mathbf{c}$ a weighting vector, $[\mathbf{b}]_p = b_p, p \in \mathcal{P}$ are the infinite impulse response (IIR) coefficients, and Q and P are the source and channel model order.

In practice, only the distorted observations are available and thus an important Engineering problem is to estimate the trajectory of clean source signal samples and underlying model parameters. As this problem is underdetermined, prior knowledge must be incorporated. However, exact prior knowledge is often not available. Therefore, *belief* about the system is modeled by considering all unknown variables as stochastic quantities. In order to estimate the most probable restored values from this belief, particle filters (PFs) approximate the desired unknowns by representing their target distributions by a large number of samples drawn from hypothesis distributions. New data is used to correct and evolve estimates in time.

As particle filters track and update the evolution of the state trajectories with time, a dynamic is implicitly enforced on the estimated variables. Therefore, the inclusion of a *static* IIR filter, $\mathbf{b}$, in the particle set leads to particle impoverishment. To circumvent this issue, approaches in the literature propose to introduce an artificial dynamic on the static parameters (see [1] and references therein). However, this approach alters rather than solves the problem at hand.

In previous work [2], the issue of static parameter estimation in PFs is circumvented by utilizing the Rao-Blackwellized PF (RBPF) [3] to separately estimate the source signal using the Kalman filter (KF) and the IIR component by a Bayesian update procedure. As the source signal is dependent on knowledge of the IIR component, the estimate of $\mathbf{b}$ is substituted into the source signal KF. However, as the actual IIR filter parameters rather than an estimate are assumed, implicit dependencies of the source signal in the prediction step, as well as uncertainty introduced through channel estimation are disregarded.

This paper proposes to analytically marginalize the IIR component from the source signal in order to obtain a sound framework that considers any implicit dependencies. Interestingly, in this approach the maximum *a posteriori* (MAP) estimate of the IIR component is itself estimated using a KF. Therefore, by exploiting analytically tractable substructures of the system model, the source signal as well as the static IIR component are obtained using their optimal estimators.

An example for an application utilizing the system in eqn. (1) is blind speech dereverberation: Acoustic signals radiated in confined spaces exhibit reverberation due to reflections off surrounding obstacles. Enhancement of the signal is extremely important for, e.g., speech recognition systems, gunshot detection devices, or hearing aids. Most dereverberation approaches in the literature are based on inversion of an estimated channel [4], spectral subtraction [5], or linear predictive coding (LPC) analysis [6]. However, these approaches often either rely on prior information about the room impulse response (RIR), or difficulties arise due to channel inversion of non-minimum phase RIRs. Furthermore, online enhancement desirable for, e.g., security surveillance applications is not supported due to batch processing. The approach proposed in this paper circumvents these issues by i) *direct source signal estimation* avoiding channel inversion; ii) *Blind channel estimation* avoiding the necessity of prior knowledge of the RIR; and iii) *Sequential processing* facilitating real-time enhancement.

Sect. §2 discusses the methodology. The background for blind speech dereverberation and results are presented in sect. §3. Conclusions are drawn in sect. §4.

2. METHODOLOGY

Given the stochastic model in eqn. (1), an optimal estimator is sought of the source signal, $\mathbf{x}_{0:t}$. If all system variables are considered as stochastic entities, an estimate of the source signal can be obtained by maximizing its posterior probability density function (pdf), $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})$, where the time-varying model parameters are defined as $\boldsymbol{\theta}_{0:t} \triangleq \{\mathbf{a}_{0:t}, \phi_{v_{0:t}}, \phi_{w_{0:t}}\}$ and are assumed known in this section. According to eqn. (1), the posterior pdf of $\mathbf{x}_{0:t}$ is dependent upon the parameters of the RIR, $\mathbf{b}$, which are unknown in practice. Thus, in order to estimate $\mathbf{x}_{0:t}$, an estimate of $\mathbf{b}$ is required, i.e.,

$$p(\mathbf{z}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}) = p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})\, p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})$$

where $\mathbf{z}_{0:t} \triangleq \{\mathbf{x}_{0:t}, \mathbf{b}\}$. Differentiating the mean squared error between $\mathbf{z}_{0:t}$ and its estimate, $\widehat{\mathbf{z}}_{0:t}$, with respect to $\widehat{\mathbf{z}}_{0:t}$ and setting to zero, the minimum mean-square error (MMSE) estimate is

$$\begin{aligned}
\widehat{\mathbf{z}}_{0:t} &= \mathbb{E}_{p(\mathbf{z}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})}[\mathbf{z}_{0:t}] = \int \mathbf{z}_{0:t}\, p(\mathbf{z}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})\, d\mathbf{z}_{0:t} \\
&= \iint \begin{bmatrix} \mathbf{x}_{0:t} \\ \mathbf{b} \end{bmatrix} p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})\, p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})\, d\mathbf{b}\, d\mathbf{x}_{0:t} \\
&= \begin{bmatrix} \int \mathbf{x}_{0:t}\, p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})\, d\mathbf{x}_{0:t} \\ \int \mathbf{b}\, p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})\, d\mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbb{E}_{p(\mathbf{x}_{0:t} \mid \cdot)}[\mathbf{x}_{0:t}] \\ \mathbb{E}_{p(\mathbf{b} \mid \cdot)}[\mathbf{b}] \end{bmatrix} = \begin{bmatrix} \widehat{\mathbf{x}}_{0:t} \\ \widehat{\mathbf{b}} \end{bmatrix}
\end{aligned} \tag{2}$$

where $\mathbb{E}_{p(\mathbf{u} \mid \mathbf{y}_{1:t}, \cdot)}[\mathbf{u}_{0:t}] \triangleq \int \mathbf{u}_{0:t}\, p(\mathbf{u} \mid \mathbf{y}_{1:t}, \cdot)\, d\mathbf{u}$ is the expected value for any function $\mathbf{u}_{0:t}$. Thus, an MMSE estimate of $\mathbf{z}_{0:t}$ is obtained by separately deriving the MMSE estimates of $\mathbf{x}_{0:t}$ and $\mathbf{b}$. The MMSE estimator of $\mathbf{x}_{0:t}$ maximizes the *marginalized* posterior pdf, $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})$, independent of $\mathbf{b}$. The posterior pdf, $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$, is derived in sect. §2.1. $p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})$ is obtained in sect. §2.2. $\mathbf{b}$ is marginalized from $p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ in sect. §2.3.

2.1 State estimation using Kalman filtering

The Kalman filter is the optimal estimator of the source signal for known model parameters, $\boldsymbol{\theta}_{0:t}$, in CGSS systems such as eqn. (1). KFs sequentially predict $\mathbf{x}_{0:t}$ based on the model parameters and correct the prediction using the most recent measurement, i.e.,

$$p(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b}) \propto \prod_{k=1}^{t} p(\mathbf{x}_k \mid \mathbf{y}_{1:k}, \mathbf{x}_{0:k-1}, \boldsymbol{\theta}_{0:k}, \mathbf{b})$$

is to be estimated. $\mathbf{x}_{0:t}$ is thus recursively propagated in time for each particle. The corresponding KF equations estimating $p(\mathbf{x}_t \mid \mathbf{y}_{1:t}, \mathbf{x}_{0:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ can be found respectively by 1) predicting the states by marginalization of the trajectory of past states, $\mathbf{x}_{0:t-1}$, and 2) updating the estimate using y_t by applying Bayes's theorem (similar to [7]), such that (s.th.)

$$p(\mathbf{x}_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b}) = \mathcal{N}\left(\mathbf{x}_t \mid \boldsymbol{\mu}_{t|t-1}, \boldsymbol{\Sigma}_{t|t-1}\right) \tag{3a}$$

$$p(\mathbf{x}_t \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b}) = \mathcal{N}\left(\mathbf{x}_t \mid \boldsymbol{\mu}_{t|t}, \boldsymbol{\Sigma}_{t|t}\right), \tag{3b}$$

where, for each particle trajectory,

$$\boldsymbol{\mu}_{t|t-1} = \mathbf{A}_t \boldsymbol{\mu}_{t-1|t-1}, \tag{4a}$$

$$\boldsymbol{\Sigma}_{t|t-1} = \mathbf{D}_t \mathbf{D}_t^T + \mathbf{A}_t \boldsymbol{\Sigma}_{t-1|t-1} \mathbf{A}_t^T \tag{4b}$$

$$\boldsymbol{\mu}_{t|t} = \boldsymbol{\mu}_{t|t-1} + \left(y_t - \mathbf{b}^T \mathbf{y}_{t-1} - \mathbf{c}^T \boldsymbol{\mu}_{t|t-1}\right) \mathbf{k}_t \tag{4c}$$

$$\boldsymbol{\Sigma}_{t|t} = \left(\mathbf{I}_Q - \mathbf{k}_t \mathbf{c}^T\right) \boldsymbol{\Sigma}_{t|t-1}., \tag{4d}$$

with residual variance, $\sigma^2_{z_t}$, and Kalman gain, $\mathbf{k}_t$,

$$\sigma^2_{z_t} = \mathbf{c}^T \boldsymbol{\Sigma}_{t|t-1} \mathbf{c} + \sigma^2_{w_t} \quad \text{and} \quad \mathbf{k}_t = \frac{1}{\sigma^2_{z_t}} \boldsymbol{\Sigma}_{t|t-1} \mathbf{c}. \tag{4e}$$

This marginalization of the states from the joint pdf $p(\boldsymbol{\theta}_{0:t}, \mathbf{x}_{0:t}, \mathbf{b} \mid \mathbf{y}_{1:t})$ causes the marginalization of $\mathbf{x}_{0:t}$ from $p(y_t \mid \mathbf{y}_{1:t-1}, \mathbf{x}_{0:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ which, by probability transformation of eqn. (1), is given as

$$p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{x}_{0:t}, \mathbf{b}) = \mathcal{N}\left(y_t \mid \mathbf{b}^T \mathbf{y}_{t-1} + \mathbf{c}^T \mathbf{x}_{0:t}, \sigma^2_{w_t}\right).$$

Marginalizing $\mathbf{x}_{0:t}$ from $p(y_t \mid \mathbf{y}_{1:t-1}, \mathbf{x}_{0:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ thus yields

$$p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b}) = \mathcal{N}\left(y_t \mid \mathbf{y}^T_{t-1} \mathbf{b} + \boldsymbol{\mu}^T_{t|t-1} \mathbf{c}, \sigma^2_{z_t}\right). \tag{5}$$

By marginalizing $\mathbf{x}_{0:t}$ from the joint posterior the source signal can be estimated using its optimal estimator. However, both $p(\mathbf{x}_t \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ and $p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ are still dependent on $\mathbf{b}$, which is unknown in practice.

2.2 Static IIR component estimation using the KF

The static IIR component, $\mathbf{b}$, does not exhibit a dynamic over time. Predicting future values would thus be futile. Nonetheless, *belief* in the static parameters can be updated as new data becomes available. Using Bayes's theorem, this belief can be sequentially updated via

$$p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}) = \frac{p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b})\, p(\mathbf{b} \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t-1})}{p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t})},$$

where the posterior pdf at time $t-1$, $p(\mathbf{b} \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t-1})$, acts as the prior pdf at t to recursively update $p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})$. Assuming that the posterior at $t-1$ is Gaussian with mean $\boldsymbol{\mu}_{\mathbf{b},t-1}$ and covariance $\boldsymbol{\Sigma}_{\mathbf{b},t-1}$,

$$p(\mathbf{b} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}) = \mathcal{N}\left(\mathbf{b} \mid \boldsymbol{\mu}_{\mathbf{b},t}, \boldsymbol{\Sigma}_{\mathbf{b},t}\right), \tag{6}$$

where the covariance, $\boldsymbol{\Sigma}_{\mathbf{b},t}$, and mean, $\boldsymbol{\mu}_{\mathbf{b},t}$, are given by

$$\boldsymbol{\Sigma}_{\mathbf{b},t} = \left(\mathbf{I}_P - \mathbf{k}_{\mathbf{b},t}\,\tilde{\mathbf{y}}_{t-1}^T\right)\boldsymbol{\Sigma}_{\mathbf{b},t-1} \tag{7a}$$

$$\boldsymbol{\mu}_{\mathbf{b},t} = \boldsymbol{\mu}_{\mathbf{b},t-1} + \mathbf{k}_{\mathbf{b},t}\left(\tilde{y}_t - \tilde{\mathbf{y}}_{t-1}^T\boldsymbol{\mu}_{\mathbf{b},t-1}\right) \tag{7b}$$

with gain, $\mathbf{k}_{\mathbf{b},t}$, and residual covariance, $\sigma^2_{z_{t,\mathbf{b}}}$,

$$\mathbf{k}_{\mathbf{b},t} = \frac{1}{\sigma^2_{z_{t,\mathbf{b}}}}\boldsymbol{\Sigma}_{\mathbf{b},t-1}\tilde{\mathbf{y}}_{t-1} \text{ and } \sigma^2_{z_{t,\mathbf{b}}} = \sigma^2_{z_t} + \tilde{\mathbf{y}}_{t-1}\boldsymbol{\Sigma}_{\mathbf{b},t-1}\tilde{\mathbf{y}}_{t-1}^T$$

and where $\tilde{y}_t = y_t - \mathbf{c}^T\boldsymbol{\alpha}_{t|t-1}$ and $\tilde{\mathbf{y}}_{t-1}^T = \mathbf{y}_{t-1}^T + \mathbf{c}^T\boldsymbol{\Gamma}_{t|t-1}$.

Comparing eqns. (7a) and (7b) to eqns. (4c) and (4d), the channel estimation is of the form of the update Kalman equations. As more knowledge about the observations becomes available, the *belief* in the static IIR component is updated (as opposed to predicting a dynamic into the future and correcting using measurements as in sect. §2.1).

2.3 Marginalization of channel parameters

The Kalman equations for $\mathbf{x}_{0:t}$ are dependent on the channel parameters through $\boldsymbol{\mu}_{t|t}$ (eqn. (4c)). In fact, as can be shown by induction, $\boldsymbol{\mu}_{t|t}$ is *linearly dependent* in $\mathbf{b}$, s.th. eqn. (4c) at $t-1$ is equivalent to,

$$\boldsymbol{\mu}_{t-1|t-1} = \underbrace{\boldsymbol{\mu}_{t-1|t-2} + \left(y_t\mathbf{k}_t - \mathbf{c}^T\boldsymbol{\mu}_{t|t-1}\right)\mathbf{k}_t}_{\boldsymbol{\alpha}_{t-1|t-1}} - \underbrace{\mathbf{k}_t^T\mathbf{y}_{t-1}^T}_{\boldsymbol{\Gamma}_{t-1|t-1}}\mathbf{b}.$$

Inserting into the prediction in eqn. (4a) at t,

$$\boldsymbol{\mu}_{t|t-1} = \mathbf{A}_t\boldsymbol{\mu}_{t-1|t-1} = \underbrace{\mathbf{A}_t\boldsymbol{\alpha}_{t-1|t-1}}_{\boldsymbol{\alpha}_{t|t-1}} + \underbrace{\mathbf{A}_t\boldsymbol{\Gamma}_{t-1|t-1}}_{\boldsymbol{\Gamma}_{t|t-1}}\mathbf{b}. \tag{8}$$

Thus, $\boldsymbol{\mu}_{t|t-1}$ is *implicitly linear* in $\mathbf{b}$ via $\boldsymbol{\mu}_{t-1|t-1}$. Inserting eqn. (8) in eqn. (4c) and defining $\mathbf{B}_t \triangleq \mathbf{I}_Q - \mathbf{k}_t\mathbf{c}^T$, the update equation is thus linear in $\mathbf{b}$ through the relation

$$\boldsymbol{\mu}_{t|t} = \underbrace{\mathbf{B}_t\,\boldsymbol{\alpha}_{t|t-1} + \mathbf{k}_t y_t}_{\boldsymbol{\alpha}_{t|t}} + \underbrace{\left[\mathbf{B}_t\,\boldsymbol{\Gamma}_{t|t-1} - \mathbf{k}_t\mathbf{y}_{t-1}^T\right]}_{\boldsymbol{\Gamma}_{t|t}}\mathbf{b}. \tag{9}$$

This linear dependency of $\boldsymbol{\mu}_{t|t}$ in $\mathbf{b}$ facilitates marginalization of $\mathbf{b}$ from $p(\mathbf{x}_t \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ as shown in eqn. (2).

2.3.1 Marginalization of channel from state posterior

Inserting eqns. (3b) and (6) into the marginalization of $\mathbf{b}$ in eqn. (2) and solving the integral using the Gaussian identity, the marginalized posterior becomes

$$p(\mathbf{x}_t \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t}) = \mathcal{N}\left(\mathbf{x}_t \mid \hat{\boldsymbol{\mu}}_{t|t}, \hat{\boldsymbol{\Sigma}}_{t|t}\right) \tag{10}$$

with marginal covariance, $\hat{\boldsymbol{\Sigma}}_{t|t}$, and marginal mean, $\hat{\boldsymbol{\mu}}_{t|t}$,

$$\hat{\boldsymbol{\mu}}_{t|t} = \boldsymbol{\alpha}_{t|t} + \boldsymbol{\Gamma}_{t|t}\boldsymbol{\mu}_{\mathbf{b},t} \tag{11a}$$

$$\hat{\boldsymbol{\Sigma}}_{t|t} = \left(\mathbf{I}_Q - \mathbf{k}_t\mathbf{c}^T\right)\boldsymbol{\Sigma}_{t|t-1} + \boldsymbol{\Gamma}_{t|t}\boldsymbol{\Sigma}_{\mathbf{b},t}\boldsymbol{\Gamma}_{t|t}^T. \tag{11b}$$

Recalling eqn. (6), the marginalized mean is thus equivalent to inserting the MAP estimate of the channel in the KF update in eqn. (9). As opposed to [2,8], the error covariance and hence the uncertainty introduced by channel estimation is taken into account through inclusion of $\boldsymbol{\Sigma}_{\mathbf{b},t}$ in eqn. (11b).

Similarly to sect. §2.1, the marginalization of $\mathbf{b}$ from the joint posterior causes the marginalization of $\mathbf{b}$ from the likelihood, $p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{b})$ (eqn. (5)) s.th.

$$p(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}) = \mathcal{N}\left(y_t \mid \mu_{y_t}, \sigma^2_{z_{t,\mathbf{b}}}\right). \tag{12}$$

where $\mu_{y_t} \triangleq \mathbf{y}_{t-1}^T\boldsymbol{\mu}_{\mathbf{b},t-1} + \mathbf{c}^T\left(\boldsymbol{\alpha}_{t|t-1} + \boldsymbol{\Gamma}_{t|t-1}\boldsymbol{\mu}_{\mathbf{b},t-1}\right)$.

2.4 Parameter estimation using particle filtering

In sects. §2.1 to §2.3, the model parameters, $\boldsymbol{\theta}_{0:t}$, where assumed known, but are generally unknown in practice. Thus, similarly to eqn. (2), by appending all unknowns in an augmented state space $\mathbf{f}_{0:t} \triangleq \{\mathbf{z}_{0:t}, \boldsymbol{\theta}_{0:t}\}$, the posterior pdf $\mathbf{f}_{0:t}$ is given as

$$p(\mathbf{f}_{0:t} \mid \mathbf{y}_{1:t}) = p(\mathbf{z}_{0:t} \mid \mathbf{y}_{1:t}, \boldsymbol{\theta}_{0:t})\, p(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t}) \tag{13}$$

such that the MMSE estimate of $\mathbf{f}_{0:t}$ is given as

$$\begin{aligned}\widehat{\mathbf{f}}_{0:t} &= \mathbb{E}_{p(\mathbf{f}_{0:t} \mid \mathbf{y}_{1:t})}[\mathbf{f}_{0:t}] \\ &= \int \mathbf{f}_{0:t}\, p(\mathbf{f}_{0:t} \mid \mathbf{y}_{1:t})\, d\mathbf{f}_{0:t} = \begin{bmatrix}\widehat{\mathbf{z}}_{0:t} \\ \widehat{\boldsymbol{\theta}}_{0:t}\end{bmatrix}\end{aligned} \tag{14}$$

Thus, $\boldsymbol{\theta}_{0:t}$ and $\mathbf{z}_{0:t}$ can be estimated separately, where $\mathbf{z}_{0:t}$ can be obtained using the KFs as proposed in sects. §2.1 to §2.3. If a proposal distribution, $\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})$, is available that approximates and has the same support as $p(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})$, the MMSE estimate of $\boldsymbol{\theta}_{0:t}$ in eqn. (14) can be expanded using Bayes's theorem,

$$\begin{aligned}\widehat{\boldsymbol{\theta}}_{0:t} &= \frac{\int \boldsymbol{\theta}_{0:t}\, p(\mathbf{y}_{1:t} \mid \boldsymbol{\theta}_{0:t})\, p(\boldsymbol{\theta}_{0:t})\, d\boldsymbol{\theta}_{0:t}}{\int p(\mathbf{y}_{1:t} \mid \boldsymbol{\theta}_{0:t})\, p(\boldsymbol{\theta}_{0:t})\, d\boldsymbol{\theta}_{0:t}} \\ &= \frac{\int \boldsymbol{\theta}_{0:t}\, \frac{p(\mathbf{y}_{1:t} \mid \boldsymbol{\theta}_{0:t}) p(\boldsymbol{\theta}_{0:t})}{\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})}\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})\, d\boldsymbol{\theta}_{0:t}}{\int \frac{p(\mathbf{y}_{1:t} \mid \boldsymbol{\theta}_{0:t}) p(\boldsymbol{\theta}_{0:t})}{\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})}\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})\, d\boldsymbol{\theta}_{0:t}} \\ &= \frac{\int \boldsymbol{\theta}_{0:t}\, w_t\, \pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})\, d\boldsymbol{\theta}_{0:t}}{\int w_t\, \pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})\, d\boldsymbol{\theta}_{0:t}} = \frac{\mathbb{E}_{\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})}[\boldsymbol{\theta}_{0:t} w_t]}{\mathbb{E}_{\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})}[w_t]}.\end{aligned}$$

where the unnormalized weights, w_t, are defined as

$$w_t = \frac{p(\mathbf{y}_{1:t} \mid \boldsymbol{\theta}_{0:t})\, p(\boldsymbol{\theta}_{0:t})}{\pi(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t})} \tag{15}$$

As an exercise in *stochastic* integration, Monte Carlo sampling can be used to approximate $\widehat{\boldsymbol{\theta}}_{0:t}$ by drawing N independent and identically distributed (i. i. d.) samples, $\boldsymbol{\theta}_{0:t}^{(i)}$, $i \in \mathcal{N}$ from the proposal pdf, leading to the MMSE estimate

$$\tilde{\boldsymbol{\theta}}_{0:t} = \frac{\frac{1}{N}\sum_{i\in\mathcal{N}} \boldsymbol{\theta}_{0:t}^{(i)} w_t^{(i)}}{\frac{1}{N}\sum_{j\in\mathcal{N}} w_t^{(j)}},$$

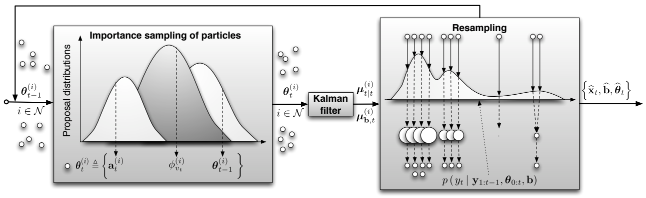


Figure 1: *Rao-Blackwellized SIR PF*

where the normalized weights, $\tilde{w}_t^{(i)}$, are defined as

$$\tilde{w}_t^{(i)} \triangleq w_t^{(i)} \Big/ \sum_{j \in \mathcal{N}} w_t^{(j)}. \tag{16}$$

Thus, even though the KFs for estimation of the source signal, $\mathbf{x}_{0:t}$, and the channel parameters, $\mathbf{b}$, cannot be applied directly due to unknown model parameters, $\boldsymbol{\theta}_{0:t}$, their estimates can be obtained by evaluating an ensemble of KFs for *stochastically* selected parameters using the PF (see Fig. 1).

The performance of particle filters is highly dependent on the choice of $\pi\left(\boldsymbol{\theta}_{0:t} \,|\, \mathbf{y}_{1:t}\right)$. The optimal importance function minimizes the variance upon $\boldsymbol{\theta}_{0:t}^{(i)}$ and the observations [9]. However, generally $\boldsymbol{\theta}_{0:t}^{(i)}$ are non-linear in the likelihood and $w_t^{(i)}$ cannot be evaluated. Sampling from the prior, $p\left(\boldsymbol{\theta}_t \mid \boldsymbol{\theta}_{t-1}\right)$, is often used instead, s.th. eqn. (15) reduces to

$$w_t^{(i)} = w_{t-1}^{(i)} \times p\left(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}^{(i)} \right). \tag{17}$$

As discussed in sects. §2.1 to §2.3, $p\left(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}^{(i)} \right)$ is obtained by marginalizing the source signal and channel from $p\left(y_t \mid \mathbf{y}_{1:t-1}, \boldsymbol{\theta}_{0:t}, \mathbf{x}_{0:t}, \mathbf{b}\right)$, as given in eqn. (12).

As $\pi\left(\boldsymbol{\theta}_{0:t} \,|\, \mathbf{y}_{1:t}\right)$ only approximate $p\left(\boldsymbol{\theta}_{0:t} \mid \mathbf{y}_{1:t}\right)$, the discrepancy between the proposal and the posterior pdf increases stochastically with time. After few iterations all but one importance weight are close to zero and computational effort is dissipated to tracking particle trajectories not contributing to the final estimate. Resampling ensures that only statistically relevant samples are retained [9]. The complete algorithm is summarized in Algorithm 1.

3. EXPERIMENTS

3.1 Application: Blind speech dereverberation

In this section, blind speech dereverberation is considered as an example for a dynamic state space model in order to evaluate the performance of the proposed algorithm.

Algorithm 1 PF with marginalized channel parameters

for $t = 1, \ldots,$ number of samples **do**
  for $i = 1, \ldots,$ number of particles **do**
    Importance sampling of $\boldsymbol{\theta}_{0:t}$ (eqs. (18), (19)).
    KF prediction of $\boldsymbol{\mu}_{t|t-1}$, $\boldsymbol{\Sigma}_{t|t-1}$ (eqs.(4b), (4a)).
    Evaluate $\boldsymbol{\alpha}_{t|t-1}$, $\boldsymbol{\Gamma}_{t|t-1}$, $\boldsymbol{\alpha}_{t|t}$, $\boldsymbol{\Gamma}_{t|t}$ (eqs. (8), (9)).
    KF estimation of $\boldsymbol{\mu}_{\mathbf{b},t}$ and $\boldsymbol{\Sigma}_{\mathbf{b},t}$ (eqs. (7a), (7b)).
    KF correction of $\boldsymbol{\mu}_{t|t}$, $\boldsymbol{\Sigma}_{t|t}$ (eqns. (11a), (11b)).
    Evaluation of weights w_t (eqs. (12), (17)).
  end for
  Normalization of importance weights (eqn. (16)).
  Resampling.
end for

3.1.1 *Source model*

Autoregressive (AR) processes are a popular approach for modeling the vocal tract of a speaker due to their accurate modeling of the short-term spectrum of speech. However, stationary AR processes result in poor modeling of speech signals due to the continually changing nature of the vocal tract. To reconcile this time-variance, time-varying AR (TVAR) processes can be used. The source transition matrix, $\mathbf{A}_t$, in eqn. (1a) can thus be expressed as

$$\mathbf{A}_t \triangleq \begin{bmatrix} \multicolumn{2}{c}{\mathbf{a}_t^T} \\ \mathbf{I}_{Q-1} & \mathbf{0}_{Q-1\times 1} \end{bmatrix}.$$

where $\left[\mathbf{a}_t\right]_q = \{a_q\}, q \in \mathcal{Q}$ are the TVAR source parameters. The square root of the innovation covariance is given as $\mathbf{D}_t \triangleq \begin{bmatrix}\sigma_{v_t} & \mathbf{0}_{1\times Q-1}\end{bmatrix}^T$, and $\mathbf{c}^T \triangleq \begin{bmatrix}1 & 0 & \ldots & 0\end{bmatrix}$.

The time-varying behavior of the process is determined by the dynamic of $\mathbf{a}_t$. A model on $\mathbf{a}_t$ that facilitates the use of PFs is a random walk on $\mathbf{a}_t$ and ϕ_{v_t} [3],

$$p\left(\mathbf{a}_t \mid \mathbf{a}_{t-1}\right) \propto \mathcal{N}\left(\mathbf{a}_t \,\middle|\, \mathbf{a}_{t-1}, \boldsymbol{\Delta}_{\mathbf{a}}\right) \mathbb{I}_{\mathcal{A}_Q}(\mathbf{a}_t) \tag{18a}$$

$$p\left(\phi_{v_t} \mid \phi_{v_{t-1}}\right) = \mathcal{N}\left(\phi_{v_t} \,\middle|\, \phi_{v_{t-1}}, \delta_v^2\right) \tag{18b}$$

where[1] $\phi_{v_t} = \ln \sigma_{v_t}^2$, $\mathbb{I}_{\mathcal{A}_Q}(\mathbf{a}_t)$ denotes the indicator function for the region of support, $\mathcal{A}_Q$, of the source parameters. The initial states are given by $p\left(\mathbf{a}_0\right) \propto \mathcal{N}\left(\mathbf{a}_0 \,\middle|\, \mathbf{0}_{Q\times 1}, \boldsymbol{\Delta}_{\mathbf{a}_0}\right) \mathbb{I}_{\mathcal{A}_Q}(\mathbf{a}_0)$ and $p\left(\phi_{v_0}\right) \triangleq \mathcal{N}\left(\phi_{v_0} \,\middle|\, 0, \delta_{e_0}^2\right)$. The set of Markov parameters $\left\{\boldsymbol{\Delta}_{\mathbf{a}}, \boldsymbol{\Delta}_{\mathbf{a}_0}, \delta_v^2, \delta_{v_0}^2\right\}$ is assumed known and constant.

3.1.2 *Channel model*

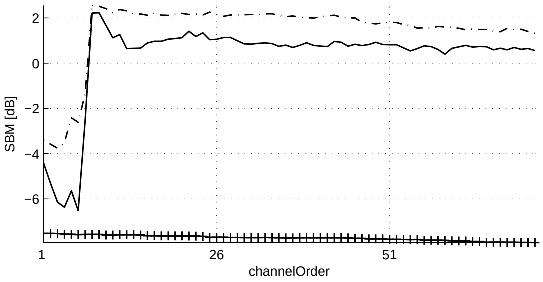
The solution of the acoustic wave equation indicates that a room transfer function can be modeled by a conventional pole-zero model [10]. Due to identifiability issues of poles and zeros, all-zero models are often used instead. However, all-zero models require large numbers of coefficients and may be effective only for a very limited spatial combination of source and receiver positions, as all-zero models lead to large variations in the room transfer function (RTF) for even small changes in source–observer positions [10]. All-pole models are widely used instead, requiring significantly fewer parameters for approximating RTFs and having lower sensitivity to changes in source-observer positions. Distortion by a P^{th} order all-pole channel and white Gaussian noise (WGN) can be expressed by eqn. (1b). Similar to $\sigma_{v_t}^2$, the logarithm of the variance on the measurement noise, $\sigma_{w_t}^2 = \exp \phi_{w_t}$, can be modeled as a random walk with initial pdf $p\left(\phi_{w_0}\right) \triangleq \mathcal{N}\left(\phi_{w_0} \,\middle|\, 0, \delta_{w_0}^2\right)$, i.e.,

$$p\left(\phi_{w_t} \mid \phi_{w_{t-1}}\right) \triangleq \mathcal{N}\left(\phi_{w_t} \,\middle|\, \phi_{w_{t-1}}, \delta_w^2\right). \tag{19}$$

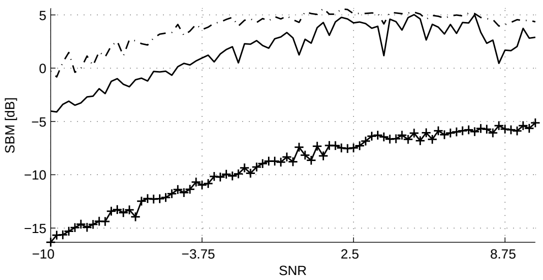
3.2 Results

Algorithm 1 is tested on a synthetic source signal generated according to the speech model in sect. §3.1.1 and compared against the KF assuming known $\boldsymbol{\theta}_{0:t}$ for 1000 samples, $Q = 4$ parameters, $\left\{\Delta_{\mathbf{a}_0}, \delta_{v_0}^2\right\} = 0.5 \times \mathbf{I}_Q$, and

[1] Positive variances are enforced by sampling from $\ln \sigma_{v_t}^2$.



(a) *Channel order*



(b) *SNR*

Figure 2: *SBM degradation with increasing channel order mismatch and SNR for the KF (- - -) for known $\boldsymbol{\theta}_{0:t}$, the proposed PF (——) for unknown $\boldsymbol{\theta}_{0:t}$, and the observed signal (—+—) for a synthetic source signal of order $Q = 4$.*

$\left\{\Delta_{\mathbf{a}}, \delta_v^2\right\} = 5 \times 10^{-4}$. The signal is filtered through an 8^{th} order horn channel [11] and distorted by WGN with $\delta_{w_0}^2 = 5 \times 10^{-3}$ and $\delta_w^2 = 5 \times 10^{-4}$. In Fig. 2b, the signal-based measure (SBM)[2] performance of the PF is tested for increasing SNR of the measurement noise for 200 particles and 30 Monte Carlo iterations using synthetic data. The PF provides an SBM improvement of approximately 12dB. At high SNRs, the PF approaches the performance of the KF, however, at the expense of increased estimate variance. This is due to a narrow likelihood at high SNRs: If the variance on the prior is too broad or the prior lies in the tail of the likelihood, sampling of the particles leads to poor results.

The model order is generally unknown and a channel order estimate, P_{est}, is used. As illustrated in Fig. 2a, undermodeling ($P_{est} < P$) leads to a SBM degradation of up to 8dB as high-energy taps of the channel model are potentially ignored. In contrast, overmodeling ($P_{est} > P$) only causes a SBM degradation of up to 1dB. Using channel order estimates thus does not cause significant deterioration as long as the channel is not undermodeled.

The PF is also applied to a 1.9s speech signal sampled at 8kHz, distorted by a 72^{nd} order horn channel [11] as well as WGN. The resulting speech waves of the estimated, source, and observed signal are shown in Fig. 3 and online[3]. Although musical noise is introduced, the intelligibility of the speech signal is clearly improved over the distorted signal.

[2] $\text{SBM}_{\text{dB}} = 10 \log_{10} \left(\|\mathbf{x}_{0:t-1}\|^2 \Big/ \|\tilde{\mathbf{x}}_{0:t-1} - \mathbf{x}_{0:t-1}\|^2 \right)$, where $\tilde{\mathbf{x}}_{0:t-1}$ is either the estimated or the distorted signal sequence.

[3] http://www.christine-evers.com/EUSIPCO2009/

4. CONCLUSIONS

This paper proposed a generic approach applicable to various state space models and applications that circumvents issues due to static IIR components in PFs. The source signal and static parameters are marginalized from the remaining unknown variables in the model and can thus be separately estimated using the KF rather than by importance sampling. Performance of the proposed method was demonstrated for blind dereverberation of speech. Results for both synthetic and real data verified the effectiveness of the approach.

References

[1] J. Liu and M. West, “Combined parameter and state estimation in simulation-based filtering,” in *Sequential Monte Carlo methods in practice*, A. Doucet, J. F. G. de Freitas, and N. J. Gordon, Eds. New York, NY: Springer, 2000, pp. 197–223.

[2] C. Evers, J. R. Hopgood, and J. Bell, “Acoustic models for online blind source dereverberation using sequential monte carlo methods,” in *Proc. IEEE Conf. ICASSP*, Las Vegas, NV, 24 Mar. - 4 Apr. 2008.

[3] J. Vermaak, C. Andrieu, A. Doucet, and S. J. Godsill, “Particle methods for Bayesian modeling and enhancement of speech signals,” *IEEE Trans. Speech Audio Process.*, vol. 10, no. 3, pp. 173–185, Mar. 2002.

[4] T. Nakatani, M. Miyoshi, and K. Kinoshita, “Single-microphone blind dereverberation,” in *Speech enhancement*, ser. Signals and Commnication Technology, J. Benesty, S. Makino, and J. Chen, Eds. Springer, 2005, pp. 247–270.

[5] E. A. P. Habets, S. Gannot, I. Cohen, and P. C. W. Sommen, “Joint dereverberation and residual echo suppression of speech signals in noisy environments,” *IEEE Trans. Audio, Speech Lang. Process.*, vol. 16, no. 8, pp. 1433–1451, Nov. 2008.

[6] M. Delcroix, T. Hikichi, and M. Miyoshi, “Dereverberation and denoising using multichannel linear prediction,” *IEEE Trans. Audio, Speech Lang. Process.*, vol. 15, no. 6, pp. 1791–1801, Aug. 2007.

[7] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter - Particle Filters for Tracking Applications.* Artech House, 2004.

[8] C. Evers, J. R. Hopgood, and J. Bell, “Blind speech dereverberation using batch and sequential monte carlo methods,” in *Proc. IEEE Conf. ISCAS*, Seattle, WA, 18-21 May 2008.

[9] A. Kong, J. S. Liu, and W. H. Wong, “Sequential imputations and Bayesian missing data problems,” *J. Amer. Stat. Assoc.*, vol. 89, pp. 278–288, 1994.

[10] J. N. Mourjopoulos and M. A. Paraskevas, “Pole and zero modeling of room transfer functions,” *J. Sound Vibr.*, vol. 146, no. 2, pp. 281–302, April 1991.

[11] J. R. Hopgood and P. J. W. Rayner, “Bayesian formulation of subband autoregressive modelling with boundary continuity constraints,” in *Proc. IEEE Conf. ICASSP*, vol. 6, Hong Kong, Apr. 2003, pp. 317–320.

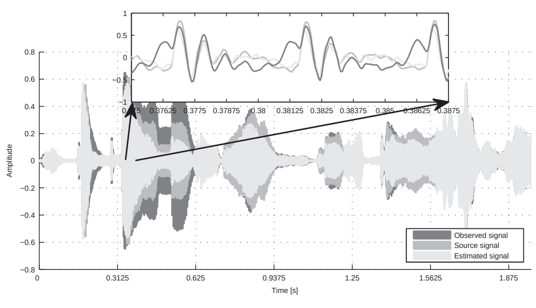


Figure 3: *Results for speech distorted by a horn channel.*