PERFORMANCE OF MULTI-USER ASYNCHRONOUS CHAOS-BASED COMMUNICATION SYSTEMS THROUGH M-DISTRIBUTED FADING CHANNEL

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ABSTRACT

In this paper, the performance of chaos-based DS-CDMA communication system over an m-distributed fading channel is evaluated. The chaos synchronization is assumed and a coherent reception in multi-user case is considered. Based on the Rician approximation of the energy distribution, an exact analytical expression of the bit error rate is derived. Many chaotic maps are used in order to demonstrate the exactitude of our analytical approach. Analytical and simulated results are presented and compared.

1. INTRODUCTION

Communication systems involving chaotic sequences have received much attention in the past ten years [15]. This may be due to the advantages provided by chaotic signals, such as robustness in multipath environments and resistance to jamming [24]. Chaotic signals are non periodic, broadband, and difficult to predict and to reconstruct. These properties coincide with requirements for signal used in communication systems, in particular for spread-spectrum and secure communications [24, 15]. The results of Pecora and Carrolls [19] on synchronizing two chaotic processes inspired many chaotic communication systems. However, it is practically difficult to achieve, and it is then not used in all chaos-based systems. Others robust synchronization techniques are proposed for chaos-based communication systems [8] [12]. These systems can achieve synchronization for low signal to noise ratio. The basic principle to apply chaotic sequences in spread spectrum systems consists of replacing the conventional binary spreading sequences, such as m-sequences or Gold sequences, by a chaotic sequence. Nowadays, PN sequences are the most popular sequences in direct sequence spread spectrum. These sequences have good correlation properties but have a limited security: they can be reconstructed by linear regression attack because of their short linear complexity [20]. On the other hand, the use of chaotic sequences instead of conventional PN sequences increases transmission security because chaotic signals can be seen as non periodic signals with an infinite number of states [26].

A large literature exists also on chaotic spreading sequences design [3] and optimization [22, 23]. Among the various digital chaos-based communication schemes, coherent chaos-shift-keying (CSK) [14, 25, 26], chaos-based DS-CDMA [22, 23], and noncoherent differential chaos-shift-keying (DCSK)[13] schemes have been most thoroughly analyzed. In our paper, we are only interested by computing the performance of chaos-based DS-CDMA. In our modulation spread spectrum schemes we use the chaotic wideband waveforms directly to represent the binary symbols as in [15]. Coherent systems like chaos-based DS-CDMA system require coherent correlators with the assumption that the receiver is able to generate a locally synchronous chaotic signal.system as that describded in [25], i.e. the bit sequence is spread by multiplying it by the chaotic sequence. Note that other CSK systems can be found in the literature [15].

In order to compute the bit-error rate (BER) performance of chaos-based communication systems, many various assumptions have been presented. The simplest approximation used in [7] for example, is to consider the transmitted chaotic bit energy being constant. This approximation yields to very imprecise BER performance when the considered spreading factors are very small. In fact, because of the non periodic nature of chaotic signals, the transmitted bit energy after spreading by chaotic sequence varies from one bit to another. Another widely used assumption is to consider the Gaussian approximation for the decision parameter at the correlator output [18, 25, 7]. This approximation considers the sum of dependent variables as a Gaussian variable. Since the chaotic signals are generated from a deterministic generator, the Gaussian approximation can be valid for high spreading factors but suffers from precision for small ones [26].

An exact computation of the BER for single and multi-user chaos communication system was recently presented by Lawrance *et al* in [26, 16]. In their approach, they did not use the constant bit energy approximation, neither the Gaussian assumption. Only additive channel noise and multiple access interference noise follow, in their study, a Gaussian distribution. Their approach enables the dynamics properties of the chaotic sequence by integrating the BER expression for a given chaotic map over all possible chaotic sequences for a given spreading factor. This latter method is compared to the BER computation under Gaussian assumption in [8, 26] and seems more realistic to match the exact BER. But, as mentioned in [16], the method has a high computational cost.

Since previously presented approaches are not valid for small spreading factors or have a high complexity of computation, another accurate approach was recently developed in [9, 11, 10] to compute the exact BER performance for single and multi-user chaos-based DS-CDMA over an Additive White Gaussian Noise (AWGN) channel. The idea is to compute the Probability Density Function (PDF) of the chaotic bit energy and to integrate the BER over all possible values of the PDF. The shape of the PDF bit energy is a qualitative indication concerning expected BER performances.

The system studied in this paper is quiet similar to the coherent CSK system. The earliest study of the performance of chaos-based DS-CDMA system over a Rayleigh channel was performed in [7]. The Gaussian approximation is used in [7] in order to give the empirical BER curves, and the BER performances of the studied systems are computed by numerical simulations. Our motivation in this paper is to give an exact analytical BER expression without neglecting the dynamical properties of chaotic sequences.

The channel used in this paper is an m-distributed fading channel (e.g. Rice and Rayleigh channels). The analog chaotic wideband waveforms are used directly to spread the binary symbols. Based on previous works on the robust synchronization of chaos-based communication systems [8] [12], we assume that the perfect synchronization is achieved. Further, the approach adopted here may be valid for many others chaotic communication systems.

Section 2 presents the transmitter structure and the channel model. In section 3, the demodulation process is defined, and the theoretical BER is provided, and section 4 is dedicated to simulation results.

2. CHAOS BASED DS-CDMA SYSTEM

The multi-user chaos-based DS-CDMA system is represented in figure 1.

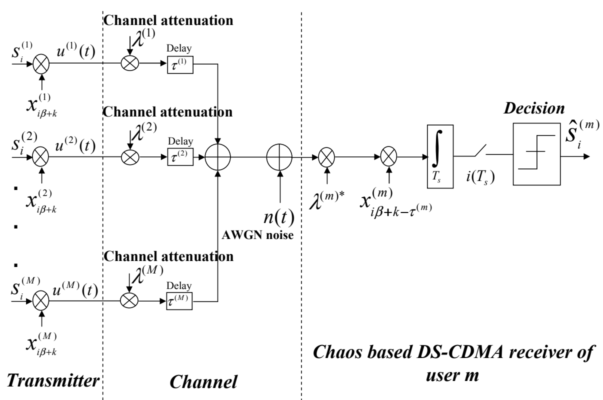

Figure 1: Simplified baseband equivalent of chaos-based DS-CDMA system with a m-distributed fading channel.

2.1 Transmitter structure

The studied system is a chaos-based DS-CDMA system with M asynchronous users. The data information symbols of user m ($s_i^{(m)} = \pm 1$) with period T_s are generated by independent sources. Symbols of user m are spread by a chaotic sequence $x^{(m)}(t)$. The chaotic sequences of all users are generated using the same chaotic generator $f(.)$ with different initial conditions. A new chaotic sample (or chip) is generated every time interval equal to T_c ($x_k^{(m)} = x^{(m)}(kT_c)$) from

$$x_{k+1}^{(m)} = f(x_k^{(m)}) \tag{1}$$

The chaotic sequences generated from $f(.)$ have a common mean μ and a common variance. It is always possible to move the map to achieve $\mu = 0$ without changing the dynamical properties of the map. The mean of all sequences is then assumed egal to zero ($\mu = 0$) in this paper. The transmitted signal of user m at the output of the transmitter is:

$$u^{(m)}(t) = \sum_{i=0}^{\infty} \sum_{k=0}^{\beta-1} s_i^{(m)} x_{i\beta+k}^{(m)} g(t-(i\beta+k)T_c) \tag{2}$$

where $g(t)$ is the pulse shaping filter (in this paper we have chosen a rectangular pulse of unit amplitude on $[0, T_c]$), β is the spreading factor, which is equal to the number of chaotic samples in a symbol duration ($\beta = T_s/T_c$).

2.2 Chaotic maps

In order to generalize our approach, four chaotic maps have been chosen. These maps are widely used in chaos-based communication systems [7, 15, 9]. They have different statistical properties, which allows us to consider a large family of chaotic maps. The first map is the Chebyshev polynomial function of order 2 (CPF); the second one is the Cubic map; the third one is the Hénon map, and the last one is a piecewise linear map (PWL). They are given by:

1- CPF map: $x_{k+1} = 1 - 2x_k^2$

2- Cubic map: $x_{k+1} = 4x_k^3 - 3x_k$

3- Hénon map: $\begin{cases} x_{k+1} = 1 + y_k - 1.4x_k^2 \\ y_{k+1} = 0.3x_k \end{cases}$

4- PWL map: $\begin{cases} z_k = L|x_k| + \phi\,[\bmod 1] \\ x_{k+1} = sign(x_k)(2z_k - 1) \end{cases}$

The PWL map depends on parameters L and ϕ. L is a positive integer and ϕ ($0 < \phi < 1$) is a real number. Throughout the paper, the PWL parameters are fixed as follows: $L = 3$, $\phi = 0.1$ [2].

2.3 Channel model

Figure 1 shows the bloc diagram of the channel model. $\tau^{(m)}$ is the time delay of user m (these delays are different, so that the users are asynchronous), and $\lambda^{(1)}$... $\lambda^{(M)}$ are independant random variables of the form:

$$\lambda^{(m)} = \sqrt{2K^{(m)}} + a^{(m)} + jb^{(m)} \tag{3}$$

where $K^{(m)}$ is the channel gain and $a^{(m)}$, $b^{(m)}$ are two independent Gaussian random variables with zero mean and variances equals to 1. The channel distribution depends on the value of $K^{(m)}$. For low gain, the channel can be seen as a Rayleigh channel, but when $K^{(m)}$ is high the channel follows the Rice distribution. The channels of all users are independent. An additive complex circular white Gaussian noise $w(t)$ is added at the output of the channels, with a two-side power spectral density equal to N_0. The multi-user received signal is finally given by :

$$r(t) = \sum_{n=1}^{M} \sum_{i=0}^{\infty} \sum_{k=0}^{\beta-1} \lambda^{(n)} s_i^{(n)} x_{i\beta+k}^{(n)} g(t-(i\beta+k)T_c - \tau^{(n)}) + w(t). \tag{4}$$

3. BER EXPRESSION

3.1 Demodulation Process

At the receiver, the first step of the detection for the bit i of user m consists of passing the received signal $r(t)$ through the filter matched to $g(t)$ in the interval $[(i\beta+k)T_c + \tau^{(m)}; (i\beta+k+1)T_c + \tau^{(m)}]$, assuming perfect synchronization (i.e. knowledge of the delay $\tau^{(m)}$ for user m only), for all $k = 0, \dots, \beta - 1$, yielding to variables

$$\rho_{(i,m)}^k \triangleq \int_{(i\beta+k)T_c+\tau^{(m)}}^{(i\beta+k+1)T_c+\tau^{(m)}} r(t) g\left(t-(i\beta+k)T_c - \tau^{(m)}\right) dt \tag{5}$$

After channel compensation (which assumes that the channel gain for user m is known), the decision variable is defined as

$$D_i^{(m)} \triangleq \text{real}\left(\left(\lambda^{(m)}\right)^* \sum_{k=0}^{\beta-1} \rho_{(i,m)}^k x_{i\beta+k}^{(m)}\right) \quad (6)$$

and the symbol estimation is defined as

$$\widehat{s}_i^{(m)} = \text{sign}\left(D_i^{(m)}\right) \quad (7)$$

Note that other multi-user detectors have been proposed in [5] for DCSK systems, where the corresponding performances are also provided. It can then be shown that $D_i^{(m)}$ can be expressed as

$$D_i^{(m)} = |\lambda^{(m)}|^2 s_i^{(m)} E_{bc}^{(i,m)} + \sum_{n \neq m} \text{MUI}_m(n) + n_{m,i} \quad (8)$$

where $E_{bc}^{(i,m)} \triangleq T_c \sum_{k=0}^{\beta-1} \left(x_{i\beta+k}^{(m)}\right)^2$ is the chaotic energy corresponding to the i^{th} symbol interval, $\text{MUI}_m(n)$ denotes the interference of user n on user m, and $n_{m,i}$ is a zero-mean Gaussian variable representing the noise term.

3.2 BER expression of chaos based DS-CDMA system

Due to the non periodic nature of chaotic sequence, the energy $E_{bc}^{(i,m)}$ is not constant for all transmitted bits. From the central limit theorem, one can assume that the sum of the multi-user interference (MUI) terms in (8) is normally distributed. Moreover, under mild conditions, the variance of the MUI-plus-noise term can be expressed as $\frac{1}{2}|\lambda^{(m)}|^2 E_{bc}^{(i,m)} \psi^{(i,m)}$, where $\psi^{(i,m)}$ depends on the (low) correlations between user m and the other users, on the gains $\lambda^{(n)}$, on the delays $\tau^{(n)}$, and on N_0. The actual expression of $\psi^{(i,m)}$ is not given here for brevity. According to the fact that the symbols are equally distributed on the set $\{-1,+1\}$, it comes from (7) and (8) that the detection probability of symbol i (with energy $E_{bc}^{(i,m)}$) for a given gain $\lambda^{(m)}$ and for given energie $E_{bc}^{(i,m)}$ is given by:

$$P_{er}^{(i,m)} = Q\left(\frac{\left|\lambda^{(m)}\right|^2 E_{bc}^{(i,m)}}{\sqrt{\frac{1}{2}\left|\lambda^{(m)}\right|^2 E_{bc}^{(i,m)} \psi^{(i,m)}}}\right) \quad (9)$$

The mean BER of the system is obtained by integrating (9) over all possible values of the bit energy and channel gain (using the notation $\widetilde{\lambda}$=$|\lambda|$):

$$\text{BER}^{(i,m)} = \int_0^\infty \int_0^\infty Q\left(\sqrt{\frac{2\widetilde{\lambda}^2 E_{bc}}{\psi^{(i,m)}}}\right) p(E_{bc}) p(\widetilde{\lambda}) dE_{bc} d\widetilde{\lambda}$$

One can also expressed the mean BER as

$$\text{BER}^{(i,m)} = \int_0^\infty Q\left(\sqrt{\frac{2V^2}{\psi^{(i,m)}}}\right) p(V)\, dV, \text{ with } V = \widetilde{\lambda}\sqrt{E_{bc}} \quad (10)$$

The computation of (10) requires the knowledge of the pdf of V. Now, eq. (10) is similar to the BER obtained in the framework of mobile radio channels. Indeed, the BER expression for a BPSK transmission over a radio channel with gain ρ and with bit energy-to-noise ratio E_b/N_0 is:

$$\text{BER}_{\text{Radio channel}} = \int_0^\infty Q\left(\sqrt{\frac{2\rho^2 E_b}{N_0}}\right) p(\rho)\, d\rho \quad (11)$$

Closed form expressions are available for (11) in the case of channels following Rayleigh [4], Nakagami [6] or Rice distributions [17] . Expression (10) has the same form as (11) and all previous results on (11) can be used for getting an analytical form of integral (10).

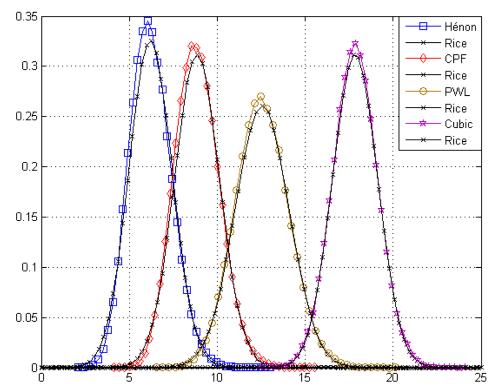

Figure 2: Examples of PDFs of variable V for different chaotic maps (Hénon, CPF, PWL, Cubic), and corresponding approximated Rice PDFs.

4. BER COMPUTATION

In order to compute analytically the integral in (10), one needs an expression of the pdf of $V = \widetilde{\lambda}\sqrt{E_{bc}}$. Now, the actual pdf is not standard, since it depends on the particular distribution of the energy $E_{bc}^{(i,m)}$. One proposes then to approximate this distribution by one of the three distributions (Rayleigh, Rice, and Nakagami) which allows us to obtain a closed-form expression from (10) and (11). For many different examples of system parameters (in particular, for different chaotic sequences), these three theoretical densities have been considered to approximate the density of V. The parameters of these distributions have been chosen in order to best fit the actual distribution. From these different examples, it always appeared that the Rice distribution gives the best approximation. Therefore, one focused on this distribution. Figure 2 gives some examples of the distribution of V with gain $K = 0$dB of the Hénon (with $\beta = 10$), CPF ($\beta = 20$), PWL ($\beta = 30$), Cubic ($\beta = 40$) chaotic sequences, along with the corresponding approximated Rice densities. The histogram of each sequence is obtained from 10^6 samples. The procedure to derive the Rice parameters is describded below.

4.1 Derivation of the Rice distribution parameters

Let R be a random variable distributed according to a Rice distribution. The general Rice pdf is a function defined for positive real numbers by:

$$p_R(r) = \frac{2(K_r+1)r}{\Omega} e^{-K_r - \frac{(K_r+1)r^2}{\Omega}} I_0\left(2\sqrt{\frac{K_r(K_r+1)}{\Omega}}r\right) \tag{12}$$

where $r \geq 0$, $K_r \geq 0$, $\Omega \geq 0$, and I_0 is the modified Bessel function of the first kind with order zero. The shape parameter K_r can be expressed in terms of γ explicitly as [1]:

$$K_r = \frac{\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}} \tag{13}$$

Parameters Ω and γ can be obtained from the second-order moments of variable R by [17]:

$$\Omega = E[R^2], \qquad \gamma = \mathrm{var}(R^2)/\Omega^2 \tag{14}$$

Therefore, we propose to define parameters Ω and γ of the Rice distribution which approximates the distribution of V so that they match the same equations (14) with V instead of R. Therefore, we set:

$$\begin{aligned}\Omega &= E[V^2] = T_c E\left[(\sqrt{2K^{(m)}}+a)^2+b^2\right] E\left[\textstyle\sum_{k=0}^{\beta-1}\left(x_{i\beta+k}\right)^2\right] \\ &= 2T_c(1+K^{(m)})E\left[\textstyle\sum_{k=0}^{\beta-1}\left(x_{i\beta+k}\right)^2\right]\end{aligned} \tag{15}$$

and

$$\begin{aligned}\gamma &= \mathrm{var}(V^2)/\Omega^2 = \frac{E[V^4]-\Omega}{\Omega^2} \\ &= \frac{1}{\Omega^2}\left(4T_c^2((K^{(m)}+1)^2+1)E\left[\left(\textstyle\sum_{k=0}^{\beta-1}\left(x_{i\beta+k}\right)^2\right)^2\right]-\Omega\right)\end{aligned} \tag{16}$$

In (15) and (16), the mathematical expectations are estimated from the chaotic sequences. K_r is then obtained from (13).

4.2 Analytical BER expression

Once the pdf of V has been approximated by the Rice distribution as above, the BER (10) is obtained from (11), yielding to [17] :

$$\mathrm{BER}^{(i,m)} = Q(u,v) - \frac{I_0(uv)}{2}\left[1+\sqrt{\frac{d}{1+d}}\right]\exp\left(-\frac{u^2+v^2}{2}\right) \tag{17}$$

where $u = \sqrt{\frac{K_r[1+2d-2\sqrt{d(d+1)}]}{2(1+d)}}$, $v = \sqrt{\frac{K_r[1+2d+2\sqrt{d(d+1)}]}{2(1+d)}}$, and $d = \frac{\Omega}{K_r+1}\frac{E_b}{\psi^{(i,m)}}$. Moreover E_b is the constant bit energy before spreading, and $Q(.,.)$ is the Marcum Q-function [17]. Note that $\mathrm{BER}^{(i,m)}$ actually depends on i (and m), through the term $\psi^{(i,m)}$. However, due to the low correlations between chaotic sequences, this dependency is weak, and $\mathrm{BER}^{(i,m)}$ remains almost constant for all i, as shown in the simulations.

5. SIMULATIONS

In the simulations presented below, all user channels have the same gain. In figure 3, the BER curves are obtained from (17), and from Monte Carlo simulations for the chaos-based DS-CDMA system. The simulated BER curve of the conventional DS-CDMA system using Gold codes is also plotted to compare the performances between both systems. It clearly appears that there is an excellent match between simulations and analytical results of chaos-based DS-CDMA system. In multi-user case, the performance of the DS-CDMA system is worse than the chaos-based DS-CDMA system when the spreading factor is low. This is due to the fact that the chaotic sequences in multi-user case have a lower cross-correlation magnitude than short Gold sequences [21]. But when the spreading factor is high, the cross-correlation magnitude of Gold sequences is low, and the Gold sequences outperform the chaotic sequences. The degradation is related in this case to the non constant transmitted bit energy after spreading by the chaotic signal [9, 10]. But for a sufficiently high spreading factor ($\beta = 127$), both performances are very close. The higher is the spreading factor, the less is the difference between the performances [9, 10]. In fact, a low spreading factor for a transmission over a fading channel has a limited benefit. In order to improve the performance of these systems, high spreading factors must be used. The use of chaotic sequences in the spreading spectrum systems increases the security of the transmission for a very small degradation, especially when the spreading factor is high. Figure 4 presents the

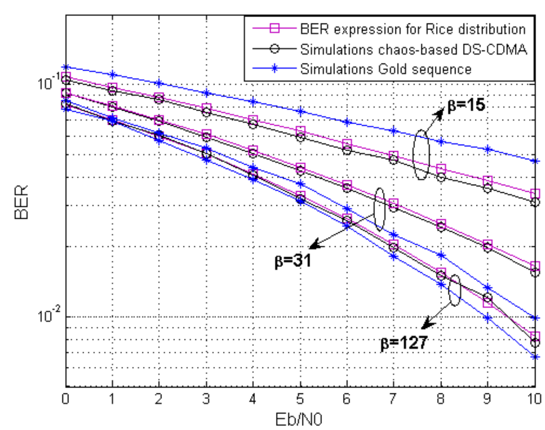

Figure 3: Simulated BER for chaos-based DS-CDMA system, analytical BER expression for Rice distribution, Simulated BER for DS-CDMA system for $M = 3$ users, spreading factors $\beta = 15, 31, 127$ and channel gain $K = 2$dB.

performance results for different chaotic maps and different parameters, defined as follows. Hénon map: ($\beta = 10$, $M = 10$, $K = -20$dB), CPF map: ($\beta = 20$, $M = 8$, $K = -10$,dB) PWL map: ($\beta = 40$, $M = 6$, $K = -5$dB) and Cubic map: ($\beta = 80$, $M = 4$, $K = 5$dB). One can see that for all chaotic maps, spreading factors, number of users and channel gains, the excellent match between analytical and simulated BERs remains.

6. CONCLUSION

This paper presents a new methodology for computing the analytical expression of the BER for asynchronous multi-user chaos-based DS-CDMA system. The use of the Rice distribution in the derivation of the theoretical BER leads to an analytical expression of this latter. For all considered chaotic maps, spreading factors, number of users and channel gains, there is an excellent match between analytical and simulated BERs. The chaos-based DS-CDMA system system is compared with a conventional DS-CDMA system using Gold codes. Since the cross-correlation of the Gold codes is higher than the chaotic sequences for low spreading factor, the chaos-based DS-CDMA system system outperforms the conventional DS-CDMA system in this case. When the spreading factor is high, the performances of the two systems are very close. The motivation to use chaotic sequences instead of Gold sequences over fading channels is justified by the high security of transmission with low performance degradation.

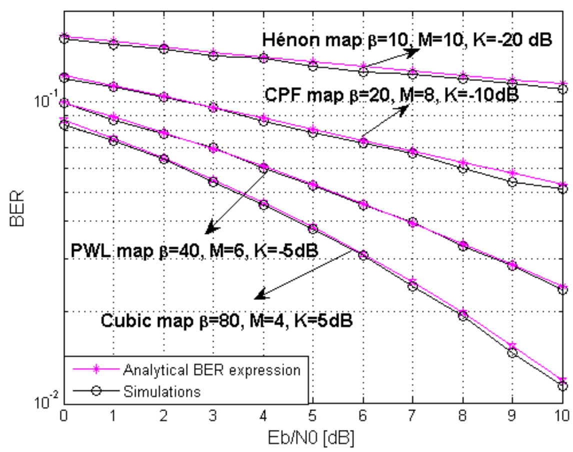

Figure 4: Simulated and analytical BERs for chaos-based DS-CDMA system with different parameters.

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