BLIND DECONVOLUTION OF MUSIC SIGNALS USING HIGHER ORDER STATISTICS

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ABSTRACT

A method for the blind deconvolution of music recordings using Higher Order Statistics (HOS) is presented. Music signals can be modelled as sinusoids with noise. The noise part is assumed to have a nonGaussian statistics with a nonzero skewness. I show that when the 3rd-order statistics of a reverberated music signal is calculated, the effect of the deterministics part is cancelled and only noise convolved with the room impulse response (RIR) is observed. Therefore, using system identification methods based on 3rd-order statistics, RIR can be obtained and used to remove the reverberation. Simulations performed with real RIR and music signals confirm the method and validity of the ideas.

1. INTRODUCTION

A sound emitted from a source in a room passes through many different paths via the reflections, in addition to the direct path, creating the reverberation effect in the sound. This effect is modelled by a very long FIR filter in the most basic case. Estimating the Room Impulse Response (RIR) or its inverse and removing the reveberation have great potentials in many different applications such as dereverberation and restoration of old audio recordings.

In a real scenario, we do not have any explicit information about the RIR (possibly other than the size of the room) or the anechoic sound source. Only a recorded audio signal is available and tackling this problem using only an audio recording is within the realm of Blind Deconvolution (BD). The problem of BD of audio signals has not been considered other than by a few works, [5], [7], [18], where the source is speech.

We present a novel blind estimation method of the magnitude response of a RIR from music recordings with Higher Order Statistics (HOS). The problem setup is as follows. A music signal, $s(t)$, passes through the RIR, $h(t)$, and this is represented by the linear convolution

$$x(t) = \sum_{i=0}^{L} h(i)s(t-i) \tag{1}$$

where $x(t)$ is the recorded sound and the RIR is assumed to be stationary, the letter t is used here to denote discrete time instants, i.e. $t \in \{\frac{n}{f_s} | n \in \mathscr{Z}\}$ with f_s being the sampling frequency and $\mathscr{Z}$ being the set of integers. In the frequency domain, $h(t)$ is represented by $H(k) = \sum_{t=0}^{L} h(t)e^{-j2\pi t\frac{k}{2L+1}}$. The aim is to estimate $|H(k)|$ using only $x(t)$. There are two unknowns, the source signal and the RIR, while there is only one signal to use which is the recorded sound, as in a typical BD problem. Therefore, some other information must be brought in by other means. We will show that the additional information is gained through statistics of the source.

In the BD methods which were originally developed in the communications and the geophysics literature [2], [6], the assumption that the source signal has independent and identically distributed (i.i.d) samples like a noise is the defining element that leads one to the solution. These methods, however, cannot be applied to audio signals directly because audio signals have strong correlations. But, investigation of the models used in the representation of audio signals lead to some approaches making use of the basic ideas in these conventional methods. For the dereveberation of speech signals, for instance, the speech signal is whitened by using the all-pole model and then a BD algorithm like given in [6] is applied to the whitened signal [5]. Music signals have long-term correlations and a very high-order filter would be needed to model a music signal as an all-pole filter excited by noise. Therefore, such an approach is not suitable to music signals.

We will present a method derived from one of the most widely used models in music signal processing which is the sinusoids+noise model [16] written below

$$s(t) = \sum_{r=1}^{R} A_r \cos(w_r t + \phi_r(t)) + n(t). \tag{2}$$

The sinusoidal part is taken as deterministic while the noise part represent the stochastic part of the signal. The amplitudes and the frequencies of the sinusoidal (harmonic) components are considered as short-time deterministic constants. The method employed for the magnitude response estimation of a RIR is based on the 3rd-order statistics of the noise part in this model with some assumptions.

In the next section, this music model is explained and its (cumulants) statistics will be derived. In section 3, a magnitude response estimation algorithm will be given based on the observations from section 2. The detailed explanation of the methods for the realization of the proposed algorithm is presented in section 4, simulation results will be provided in section 5.

2. MUSIC SIGNAL MODEL AND THE HOS OF MUSIC SIGNALS

A BD problem requires more information in addition to the observed signal and, most of the time, it is obtained via some statistics of the source signal. In this case, the source is music signal which has a general model given by (2) and I will try to make use of the HOS hidden in the signal with cumulants employed as the tools of statistics .

In the music signal model (2), the amplitudes and the frequencies of the sinusoidal (harmonic) components are considered as short-time deterministic constants. Because it is assumed that the frequencies are almost constant, the initial phase values $\phi_r(t)$ can be taken as constants even though small fluctuations can occur, i.e. $\phi_r(t) \approx \phi_r(0)$, $\forall t \in \mathscr{Z}$. On the other hand, $\phi_r(t)$ is not the same for different values of $r = 1, \cdots, R$, i.e. the individual cosines are not in phase [11] Hence, the phases of the different harmonic components are assumed to be i.i.d random variables along different frequencies and constant in time. Furthermore, the noise signal $n(t)$ is assumed to be white with a nonGaussian distribution, which will play an important role in the development of our ideas as will be become evident in the next section.

The conventional signal processing is based on second-order statistics. It is a straightforward task to estimate a linear system

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given the input and output signals under certain conditions. In a blind scenario, however, using statistics of order higher than 2 is more suitable to extract some information or to do estimation. To represent the HOS of a random process, mostly cumulants are used. They lead to easy mathematical formulation and cumulants of order ≥ 3 preserve the phase. The multidimensional Fourier transform of cumulants are called the polyspectra and the first three polyspectra are the power spectrum, the bispectrum and the trispectrum. The reader is referred to [12] for a tutorial on this subject. One can do the processing either using the cumulant sequence or polyspectra depending on the nature of the problem. The polyspectra of two convolved signals turn out to be the multiplication of the polyspectra of the signals and this leads to easy observations and mathematical derivations. A method developed using the cumulants means that a parametric model will be employed. Parametric estimation methods require a priori knownledge on the model type and order which creates the major difficulty for a RIR. It is not possible to model a RIR by a MA or ARMA model with a precisely known order(s). Because of this, parametric methods are not suitable in this problem. Frequency domain methods, on the other hand, can tolerate order mismatches and allow us to focus on the magnitude and the phase estimation problems separately. Because of these, I will employ polyspectra in the algorithm that will be developed.

Given the model for the music, its cumulants will be derived and the algorithm development will be carried out through the polyspectra. The crucial points are the existence of the noise part and the assumption that it is white and has a nonGaussian probability density function (pdf).

2.1 Cumulants of Music Signal

The polyspectra of two convolved signals is equal to the multiplication of their polyspectra [12]. I start to develop our ideas from this point. Mathematically, I have $P_{k,x}(w_1,\cdots,w_{k-1}) = \mathrm{DFT}_{k-1}\{c_{k,x}(\tau_1,\cdots,\tau_{k-1})\}$ for the kth-order polyspectra where $c_{k,x}(\tau_1,\cdots,\tau_{k-1})$ is the corresponding cumulant sequence (DFT_{k-1} denotes the discrete Fourier transform of dimension $k-1$).

I only have access to $x(t)$ and its polyspectra equals to the multiplication of those of $s(t)$ and $h(t)$ The noise $n(t)$ and the deterministic parts $d(t)$ are independent, therefore, from $x(t) = h(t)*d(t)+h(t)*n(t)$ I obtain

$$\begin{aligned} P_{k,x}(w_1,\cdots,w_{k-1}) = &P_{k,d}(w_1,\cdots,w_{k-1})P_{k,h}(w_1,\cdots,w_{k-1}) \\ &+P_{k,n}(w_1,\cdots,w_{k-1})P_{k,h}(w_1,\cdots,w_{k-1}) \end{aligned} \quad (3)$$

I assumed that the noise part is white which implies $P_{k,n}(w_1,\cdots,w_{k-1}) = \gamma_k$ resulting in

$$\begin{aligned} P_{k,x}(w_1,\cdots,w_{k-1}) = &P_{k,d}(w_1,\cdots,w_{k-1})P_{k,h}(w_1,\cdots,w_{k-1}) \\ &+\gamma_k P_{k,h}(w_1,\cdots,w_{k-1}) \end{aligned} \quad (4)$$

where γ_k is the scaling constant. Since we do not have direct access to $d(t)$, we cannot obtain $P_{k,d}(w_1,\cdots,w_{k-1})$ from $x(t)$. But, we can find the order k such that $P_{k,d}(w_1,\cdots,w_{k-1})$ is zero and this will result in $P_{k,x}(w_1,\cdots,w_{k-1}) = \gamma_k P_{k,h}(w_1,\cdots,w_{k-1})$. This way, we will be able to exploit the noise part and eliminate the effect of the sinusoidal part in music signals; the effective signal will be equal to the noise convolved with the RIR and the problem will be in a similar form studied in the conventional BD methods. Hence, $h(t)$ can be estimated from $P_{k,x}(w_1,\cdots,w_{k-1})$ since it will indeed correspond to the polyspectra of the $h(t)$.

After this preliminary discussion, I now concentrate on the cumulants of a music signal and explain that 3 as the cumulant order should be chosen so that I can get rid of the sinusoidal part.

The deterministic part is a sum of sinusoids with different phases which are assumed to be uniformly distributed in the interval $[0,2\pi)$. In the 2nd- and 4th-order statistics, the deterministic part appears as sinusoids [17], [1]. For the order $k=3$, the cumulants of $d(t)$ becomes zero. In order to see this, let us calculate the 3rd-order cumulant of $s(t)$

The 3rd-order cumulant of $s(t) = \sum_{r=1}^{R} A_r cos(w_r t + \phi_r(t)) + n(t)$, $c_{3,s}(\tau_1,\tau_2) = E_{\phi_r,n}\{s(t)s(t+\tau_1)s(t+\tau_2)\}^1$, by the assumption that $\phi_r(t)$ is a random variable through r and constant along t, becomes

$$\begin{aligned} c_{3,s}(\tau_1,\tau_2) \;=\; &\frac{A_r^3}{4} E_{\phi_r}\Bigg\{ \Big(\sum_{r=1}^{R} \cos(w_r(t-\tau_1+\tau_2)-\phi_r(t)) \\ &+ \cos(w_r(t+\tau_1+\tau_2)-\phi_r(t)) \\ &+ \cos(w_r(t+\tau_1-\tau_2)+\phi_r(t)) \\ &+ \cos(w_r(3k+\tau_1+\tau_2)+3\phi_r(t)) \Big) \Bigg\} \\ &+ \delta(\tau_1,\tau_2)\gamma_{3,n} \end{aligned} \quad (5)$$

where the phases are zero mean random variables in the cosine terms. The expectation operation cancels them out because each phase ϕ_r has zero mean, hence,

$$c_{3,s}(\tau_1,\tau_2) = \delta(\tau_1,\tau_2)\gamma_{3,n}. \quad (6)$$

From this result, we have $P_{3,s}(w_1,w_2) = \mathrm{DFT}_2\left\{\gamma_{3,n}\delta(\tau_1,\tau_2)\right\} = \gamma_{3,n}$ and using this fact, (4) gets the following form

$$P_{3,x}(w_1,w_2) = \gamma_{3,n} H(w_1)H(w_2)H^*(w_1+w_2) \quad (7)$$

where $P_{3,h}(w_1,w_2)$ is replaced by $H(w_1)H(w_2)H^*(w_1+w_2)$. The 3rd-order polyspectra is called the bispectrum and the letter B is used to denote it [13]. I adopt this convention from this point on and $B_x(w_1,w_2)$ is used instead of $P_{3,x}(w_1,w_2)$ and (7) is rewritten as

$$B_x(w_1,w_2) = \gamma_{3,n} H(w_1)H(w_2)H^*(w_1+w_2). \quad (8)$$

The expression (8) implies that if the assumption on the phase of the different harmonics in a music signal hold then one can use the 3rd-order cumulants to obtain the information regarding the RIR from the cumulants of the remaining noise part in the signal. Given the estimated polyspectra of a FIR filter, there are different algorithms to estimate the filter [9], [10], [14], [15]. I only present a method for the magnitude estimation $|H(k)|$. The phase algorithms given for nonparametric methods result in very inaccurate estimates for the phase becasue of ambiguities. My investigation of the phase estimation methods in [9], [15] showed that the level of inaccuracy becomes bigger for long impulse responses rendering the estimates useless.

Starting from (8), I next present the algorithm for the magnitude estimation.

3. BLIND MAGNITUDE RESPONSE ESTIMATION OF A RIR USING 3RD-ORDER STATISTICS FROM MUSIC SIGNALS

What we conclude from the result given in (8) is that applying the blind system identification techniques available for the 3rd-order statistics on the observed reverberated music signal will provide an estimate for the RIR. In real signals, however, the whiteness assumption of the noise part will not hold perfectly. The noise would rather have some structure in it. After calculating the 3rd-order cumulant of the observed signal, one can do a whitening on this 3rd-order statistics as in the 2nd-order statistics. The structure causing the coloring is modeled as an AR filter as in conventional whitening

[1] I put ϕ_r and n as a subscript to the expectation to imply that the expectation is with respect to both ϕ_r and n since they are the random variables involved in this equation, but because they are independent the result will simply be the expectation applied to each of them individually without any regard to if they are added or multiplied

applied on speech dereverberation [5], [8]. In doing this, it is assumed that a short-order AR filter captures only the coloring structure in the noise without distorting the RIR. By this whitening procedure we expect that, when we apply the system identification algorithm, the signal will be closer to the ideal form noise-convolved-with-the-RIR.

The method for the AR model estimation using 3rd-order spectra provided in the paper [4] will be used since it guarantees a unique and stable solution in principle.

Involving the whitening, the procedure defined for the blind magnitude response estimation of a RIR from the reverberated music signal on a finite set of samples $\{x(t)\}_{t=1}^{S}$ is given as below.

3.1 Magnitude Estimation Procedure

1. Estimate the AR model of order p from estimated the cumulant sequence and apply 3rd-order whitening, $x'(t)$.
2. Estimate the truncated cumulant sequence using the whitened sequence, $\hat{c}_{3,x'}(\tau_1,\tau_2)$.
3. Calculate $\hat{B}_x(w_1,w_2) = \mathrm{DFT}_2\{\hat{c}_{3,x'}(\tau_1,\tau_2), 2N+1\}$.
4. Estimate $|\hat{H}(w)|$ from $\hat{B}_x(w_1,w_2)$

The whitened signal obtained from the original signal $x(t)$ is denoted by $x'(t)$ and the estimated quantities are given with a hat. The detailed description of the algorithms employed to realize this approach along with some implementation remarks are given next.

4. ALGORITHMS AND IMPLEMENTATION ISSUES

The steps given in magnitude estmation procedure are given in detail in this section. I also discuss some practical issues.

4.1 Estimation of the Cumulant Sequence

Divide data into K portions with equal samples such that each portion has M samples of data, i.e. $S = KM$. The mean of each segment is calculated and subtracted from each sample. I, then, obtain the biased estimates[2] of the cumulants for all segments using

$$\hat{c}^{i}_{3,x}(\tau_1,\tau_2) = \frac{1}{M}\sum_{k=l_1}^{l_2} x(t)x(t+\tau_1)x(t+\tau_2) \tag{9}$$

where $i = 1,\cdots,K$, $l_1 = \max(0,-\tau_1,-\tau_2)$, $l_2 = \min(M-1, M-1-\tau_1, M-1-\tau_2)$, $\tau_1 = -L,\cdots,L$, $\tau_2 = -L,\cdots,L$, and average the cumulant estimates obtained from all segments of the signal

$$\hat{c}_{3,x}(\tau_1,\tau_2) = \frac{1}{K}\sum_{i=1}^{K} \hat{c}^{i}_{3,x}(\tau_1,\tau_2). \tag{10}$$

The value of L will be chosen according to where this cumulant sequence estimation is used. Steps 1 and 2 require the estimation of the cumulant sequence. For step 1, which is the whitening, L will be around 30-50 and for step 2 it will be chosen as the truncated RIR length which would be 2000 or 4000 depending on the room considered.

4.2 3rd-Order Whitening

The whitening will be carried out by assuming that the structure in the noise can be modelled by an all-pole (AR) filter. The reason why the AR model is taken is that if we choose an ARMA model it might remove the effect of the RIR as well. In addition, it is not easy to solve for the MA part in an ARMA model. The noise part $n(t)$ modelled by an all-pole filter of order p is given by

$$n(t) = -\sum_{i=1}^{p} a_i n(t-i) + e(t) \tag{11}$$

[2] Biased estimates are generally preferred over the unbiased estimates since they have smaller variance.

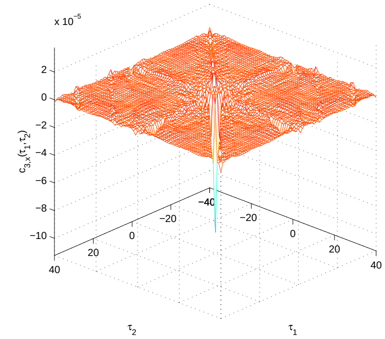

Figure 1: The 3rd-order cumulant sequence after whitening.

where $e(t)$ is the assumed excitation signal. This noise signal is in $x(t)$ and we do not have access to it directly. Hence, the familiar AR recursion formula for the 3rd-order cumulants of $x(t)$ is written as [4]

$$c_{3,x}(\tau_1,\tau_2) + \sum_{i=1}^{p} a_i c_{3,x}(\tau_1 - i,\tau_2) = 0,\ \tau_1 > 0. \tag{12}$$

It has been proved in [4] that the set of equations obtained by fixing τ_1 at each one of the values $\tau_1 = 1,\cdots,p$ and varying τ_2 as $\tau_2 = -p,\cdots,0$ yield the vector $\mathbf{a} = [a_1 \cdots a_p]^T$ such that the corresponding all-pole filter is stable.

The solution $\mathbf{a} = -\mathbf{R}^{-1}\mathbf{c}$ will be employed to represent a short 3rd-order modelling of the noise structure in the music signal for the purpose of whitening, where $\mathbf{R}$ is a matrix obtained from the cumulants of x as given in [4]. The cumulant values in $\mathbf{R}$ and $\mathbf{c}$ are replaced by their estimated versions, $\hat{c}_{3,x}(\tau_1,\tau_2)$, calculated using (9) and (10) in the implementation. The AR order, p, is determined experimentally according to the music signal in consideration. In the simulations I have undertaken, it was chosen around 30 to 50. The inverse FIR filter $\mathbf{a}' = [1\ -\mathbf{a}^T]^T$ is applied on $x(t)$ to obtain the whitened signal, $x'(t) = \sum_{i=0}^{p-1} a'_i x(t-i)$.

A 3rd-order whitening was applied on the same signal obtained from brass. The resulting cumulant sequence is plotted from two different views in Figure 1. The order of the AR filter was 10. The effect of the whitening is making the cumulant sequence closer to Kronecker delta function. It removes the structure to some degree.

4.3 Estimation of the Truncated Cumulant Sequence For Step-2

A RIR is very long. When the cumulant sequence is estimated, it has to be assumed to be of certain length. If I consider a RIR sampled at 16 kHz, one would need to take 5000 to 10000 samples depending on the room. Figure 2 shows a RIR measured at a concert hall at Queens University of Belfast. The sampling rate was 16 kHz. As seen, only 3000 to 5000 samples have significantly high magnitude values. Therefore, the cumulant sequence can be truncated to a maximum range around 3000 to 5000. This truncation is necessary and the truncation length can only be chosen roughly by assuming a prior knowledge on the type of the room, which is quite reasonable since most of the case it is known where the recording took place. The truncation length will be the assumed RIR length and denoted by N. The number of music samples to be used in (9) must be chosen in accordance with the truncation length. The sample number used in each block must be as high as possible to alleviate the effects of the transients in the convolution of music samples with the RIR. In practice this is not very easy to choose because it is related to the stationarity of the music signal. But, loosely speaking, the number of music samples must be at least twice the truncation length. In the simulations, I chose 4 times the truncation length music samples. With these values chosen, the formulae given in (9) and (10) are, then, used to estimate the cumulant sequence $\hat{c}_{3,x'}(\tau_1,\tau_2)$ for $\tau_1,\tau_2 \in \{-N,\cdots,0,\cdots,N\}$.

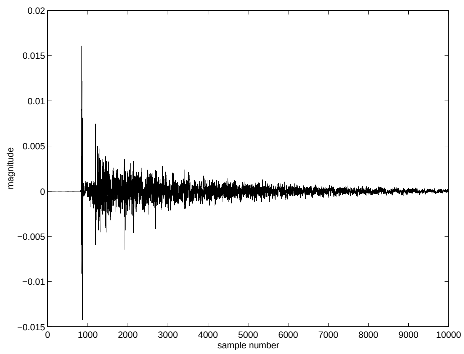

Figure 2: The impulse response of the Whittla Concert Hall sampled at 16 kHz.

4.4 Calculation of the Bispectra

The bispectra $\hat{B}_{x'}(w_1,w_2)$ are obtained simply by taking 2D-DFT of the estimated cumulant sequence. If the truncation length is denoted by N then the FFT length must be $2N+1$ due to the fact that cumulant sequence indices will run from $-N$ to N. Therefore, the bispectra estimation with w_1, w_2 replaced by the discrete frequency values k_1 and k_2, respectively, using the Matlab notation, follows from

$$\begin{aligned}\hat{B}_{x'}(k_1,k_2) &= \text{FFT2}\left\{\hat{c}_{3,x'}(\tau_1,\tau_2),2N+1\right\}\\ &= \gamma_{s,3}H(k_1)H(k_2)H^*(k_1+k_2)\end{aligned} \quad (13)$$

where $k_1,k_2 \in \{-N,\cdots,0,\cdots,N\}$, $H(k)=\sum_{t=0}^{2N} h(t)e^{-j\frac{2\pi kt}{2N+1}}$.

4.4.1 Magnitude Response Estimation

I will use the spectrum magnitude estimation given in [3]. The spectrum magnitude estimation from bispectra formula is

$$\ln\left(\left|\gamma_{s,3}\right|^{1/3}|H(k)|\right) = \frac{-2\sum_{i=0}^{k-1}\ln\left(|H(i)|\left|\gamma_{s,3}\right|^{1/3}\right)+M(k)}{(k+3)}, \quad (14)$$
$$k=1,\cdots,L$$

where $M(k)=\sum_{i=0}^{k}\ln|B_x(i,k-i)|$, L corresponds to the angular frequency π. The value of the zero frequency magnitude is calculated from $\ln\left(\left|\gamma_{s,3}\right|^{1/3}|H(0)|\right)=\frac{1}{3}\ln|B(0,0)|$.

The magnitude spectrum is estimated up to a scale indeterminacy, as in all blind approaches. Practical implementations will be carried out using a finite number of data samples $\{x(t)\}_{t=1}^{N}$ and $B_x(k_1,k_2)$ are estimated from $\{x(t)\}_{t=1}^{N}$.

5. SIMULATIONS

A 10 second long music signal from brass was taken and windowed into 10000 samples (no overlapping) each of which was used in the estimation of the 3rd-order cumulant sequence. The averaged cumulant sequence is plotted in Figure 3.

The 2D-cumulant sequence is very close to a 2D Kronecker delta function which is what we expect. The noise part of the music signal seems fine in this occasion. But, each music instrument has its own characteristic properties. Because of this, the statistical

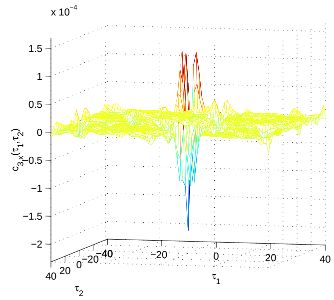

Figure 3: The 3rd-order cumulant sequence of a music signal obtained from brass.

properties of the noise part will differ from instrument to instrument. The whiteness assumption will not hold at all in some instruments. For example, piano has very weak noise component which is very far from being described as noise. In some other instruments, the noise might be very obvious but have very small skewness which will prevent us from using it with the 3rd-order statistics. Our simulations showed that brass is not the only instrument having non-zero skewness. Drum presents a noise having similar 3rd-order statistics.

5.1 Test-1

A recording of brass sampled at 16 kHz was used as the input to the 512-sample long RIR impulse reponse which was taken from the concert hall impulse response measurement. A 3rd-order whitening was applied prior to the magnitude estimation procedure. The order of the all-pole filter used in whitening was chosen to be 50. The estimated magnitude response is shown in Figure 4 with the true one. The estimated magnitude response shown by black color is very close to the true one which is the red colored plot.

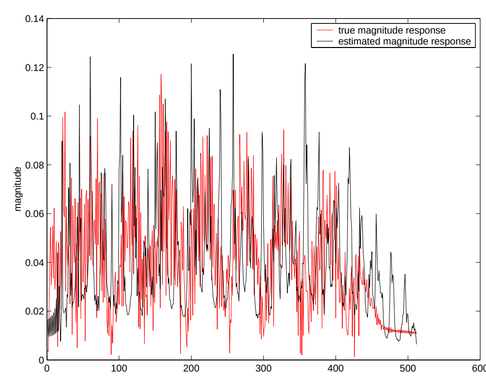

Figure 4: Estimated magnitude response with algorithm-2.

5.2 Test-2

In Figure 5, the magnitude estimate for the 1024-sample portion of the concert hall impulse response is shown with the true one. The input signal was a drum recording and the whitening filter was of order 50. In this simulation, in order to see the affect of truncation, I actually used the first 2048 samples of the impulse response of the concert hall at Queens University of Belfast in creating the data $x(t)$. But, the magnitude estimation was carried out for 1024 samples. Other than a few spiky estimates, in general, the estimation method is able to extract the magnitude response to some degree. This means that the the magnitude response is not affected much by truncation and the magnitude estimation algorithm is quite robust against such imperfections. Again red plot shows the true magnitude response and black colored plot is the estimated one.

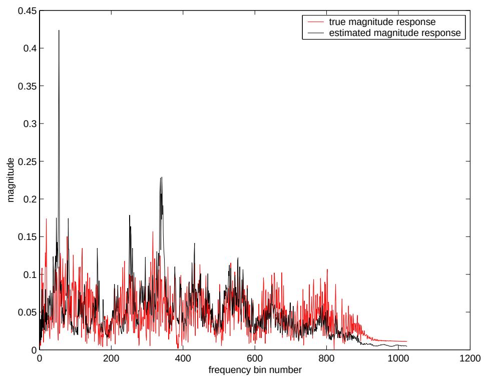

Figure 5: Estimated magnitude response with algorithm-2.

6. CONCLUSIONS

A novel method using 3rd-order statistics of the music signal was outlined for the blind estimation of the magnitude response of a RIR from music recordings. Blind dereverberation of music signals has a lot of potential in film making and restoration of old recordings. The method I provided for the magnitude response estimation seems promising and this is a step further towards obtaining blind dereverberation of audio recordings.

The next step is work on the phase estimation problem. The main difficulty in estimating the phase comes from the fact that phase estimation algorithms using HOS are prone to generate erroneous results due to the ambiguity in the phase values used in the estimation process. However, using some subband processing, improvements can be obtained.

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