

DISTRIBUTED DETECTION AND ESTIMATION IN DECENTRALIZED SENSOR NETWORKS: AN OVERVIEW

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ABSTRACT

In this work we review some of the most recent in-network computation capabilities that can be used in sensor networks to alleviate the information traffic from the sensors towards the sink nodes. More specifically, after briefly reviewing distributed average consensus techniques, we will concentrate on consensus mechanisms based on self-synchronization of coupled dynamical systems, initialized with local measurements. We will show how to achieve globally optimal distributed detection and estimation through minimum exchange of information between nearby nodes in the case where the whole network observes one common event.

1. INTRODUCTION

In sensor networks, there is typically an evident contrast between very demanding performance requirements on the whole network and, at the same time, very limited capabilities available at the single sensor [1]. This contrast can be overcome through the employment of large scale networks, composed of a large number of tiny, cheap sensors: Collecting and appropriately processing a high number of measurements, gathered by many sensors, in a fusion center can indeed provide the performance improvement necessary to satisfy the user requirements, even if the individual performance of each sensor is limited. The performance improvement can be in terms of detection capabilities, estimation accuracy and fault tolerance with respect to breakdown or stand-by of a significant number of nodes.

To contrast the performance limitation due to scarcity of resources of each node, namely energy, bandwidth and computational complexity, many recent works have addressed the problem of finding optimal uses of the available resources for sending the information from the sensors to a fusion center, according to alternative optimization and constraint criteria [2, 3, 4, 5, 6]. This is a very interesting research field, as it merges distributed detection/estimation theory with medium

access control schemes and optimal power/rate allocation on each link, as a function of the final accuracy or detection probability. However, in many relevant applications, e.g., the detection of hazardous events, it is likely that many nodes would send an alarm towards the control node at the same time, i.e., when the event occurs. Hence, a congestion event around the sink node is more likely to happen just when the network is required to react in the most reliable manner. This boils down to the so called *scalability* problem and it motivates the search for distributed strategies capable of increasing the network reliability, under critical situations, especially for large scale networks. Some recent works have derived the fundamental limits in the transport capacity of a wireless network, under simple interference models. It was proven in [7] that in a wireless network with several one-to-one links, the transport capacity for each user scales with the number n of sources as $1/\sqrt{n \log n}$, for large n . More recently, this result was extended to more general communication models in [8]. The result of [7] pertains primarily to communication networks with many sources and as many destinations. In a sensor network, where many sensors send data to a sink node, it was proven in [9] that the transport capacity scales as $1/n$. This capacity is achievable through proper scheduling of the transmissions from each sensor, in order to limit interference while having, at the same time, the highest number of simultaneous transmissions. This entails some kind of coordination among the sensors that might compromise the overall simplicity. A further fundamental step was derived in [10], where it was shown that if the function to be computed by the network is a *symmetric* function of the measurements, i.e., it is invariant to any permutation of the observed variables, the transport capacity scales as $1/\log n$. This is an important result that reflects the *data-centric* nature of sensor networks, where what is important is the measurement per se and not the knowledge of which node has taken which measurement.

In [9], it was also shown that a performance improvement is achievable by endowing the network with a hierarchical structure where the nodes organize themselves in clusters electing a cluster head. All sensors in a cluster send their data

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to their cluster head, who takes care of forwarding the data to a fusion center, possibly through a multihop path. In [9] it was also shown that some kind of *in-network* processing, as a form of distributed source-channel coding, is useful to better exploit the available resources. A procedure for in-network processing was proposed in [12], based on successive refinement of initial estimates obtained through local exchange of information among nearby nodes. In [12], it was also shown that, as the number of nodes grows, in-network processing always uses less energy than a centralized algorithm, for a desired level of accuracy. Another interesting approach to asynchronous algorithms for distributed computation of functions of the sensor measurement without a fusion node is provided by the so called *gossip* algorithms [13].

In this paper, after reviewing some recent distributed processing techniques, we will concentrate on techniques that allow the totally distributed computation of some important (symmetric) functions of the measurements, using a simple coupling mechanism between nearby nodes.

2. AVERAGE CONSENSUS

An example of distributed computation for achieving an average consensus in ad hoc networks was proposed, for example, in [11], [14]. A lot of related work has also been done in the decentralized coordination of mobile agents [15], [16]. In its simplest form, average consensus can be achieved through the following strategy. Let us consider a network composed of N nodes and denote with x_i , $i = 1, \dots, n$, the measurement taken from node i . The network connectivity is given through the coefficients a_{ij} , with $a_{ij} = 1$ if nodes i and j are directly connected, otherwise $a_{ij} = 0$. Let us indicate with $\mathbf{x}[n]$ the N size column vector formed with all the sensor measurements, at time index n . Let us assume, with no loss of generality, that each sensor takes a measurement at time $n = 0$, so that $\mathbf{x}[0]$ indicates the set of initial measurements. After the initial measurements, each sensor exchanges its estimate with its neighbors and modifies its own estimate by taking a linear combination of the estimates received by its neighbors, according to the following equation

$$\mathbf{x}[n] = \mathbf{W}\mathbf{x}[n-1] + \mathbf{v}[n], n = 1, 2, \dots, \quad (1)$$

where $\mathbf{v}[n]$ is the noise vector at step n and W_{ij} is the weight associated by node i to the signal received from node j ($W_{ij} \neq 0$ if $a_{ij} \neq 0$, i.e., if nodes i and j are connected). It was shown in [11] that, when the noise is absent, i.e. $\mathbf{v}[n] = \mathbf{0}$, if the matrix \mathbf{W} is symmetric and satisfies the following properties:

$$\begin{aligned} \mathbf{W}\mathbf{1} &= \mathbf{1}; \\ \lambda_1 &= 1; \\ |\lambda_i| &< 1, \text{ for } 2 \leq i \leq N, \end{aligned} \quad (2)$$

where $\{\lambda_i\}_{i=1}^N$ denote the eigenvalues of \mathbf{W} in nonincreasing order and $\mathbf{1}$ is the vector of all ones, then all sequences $x_i[n]$,

$i = 1, \dots, n$, converge to the average value $\bar{x} \triangleq (1/N) \sum_{i=1}^N x_i$, i.e.,

$$\lim_{n \rightarrow \infty} \|x_i[n] - \bar{x}\| = 0, \quad \forall i, \quad (3)$$

where $\|\cdot\|$ denotes the ordinary Euclidean norm. Conditions in (2) mean that \mathbf{W} must be a symmetric matrix whose largest eigenvalue is 1 and the eigenvector associated to this eigenvalue is the vector $\mathbf{1}$. A possible matrix structure satisfying (2) is the following [14]:

$$W_{ij} = \begin{cases} 1/(d+1), & \text{if nodes } i \text{ and } j \text{ are connected;} \\ 1 - d_i/(d+1), & i = j; \\ 0, & \text{if } i \text{ and } j \text{ are not connected.} \end{cases}, \quad (4)$$

where d_i is the degree of node i and the network degree $d = \max_i \{d_i\}$. This choice of \mathbf{W} would require each node to know its own degree and also the network degree, but alternative choices could relax this constraint. In any case, model (1) requires that each receiver is capable to discriminate all the received signals, to be able to assign to each of them, the proper weight W_{ij} .

Furthermore, the algorithm (1) suffers from high sensitivity to noise, as already noticed in [14]. In fact, if we pre-multiply (1) by $\mathbf{1}^T$ and divide by N , we get [14]:

$$\bar{x}[n] = \bar{x}[n-1] + \frac{1}{N} \sum_{i=1}^N v_i[n], \quad (5)$$

with $\bar{x}[n] \triangleq (1/N) \sum_{i=1}^N x_i[n]$. This shows that the running average $\bar{x}[n]$ undergoes a random walk, thus implying that its variance increases linearly with the time index. Hence, as already observed in [14], the average consensus achieved through (1) does not converge in any statistical sense (except in the mean). It was shown in [14] that, with average consensus, what converges to a constant value is the variance of the deviations $z_i[n] := x_i[n] - \bar{x}[n]$. As a numerical example, in Fig. 1 we report the values $x_i[n]$, $i = 1, \dots, N$ vs. n , for a network composed of $N = 21$ nodes, fully connected. Each node is initialized with the measurement $x_i[0] = i$ and the matrix \mathbf{W} is built as in (4), with $d_i = 20$. The additive noise is Gaussian with zero mean and unit variance. We can clearly see that, even though all nodes move together (thus implying that the variance of the deviations tends to remain constant), the estimates follow a random walk that does not converge in any statistical sense. To alleviate this problem, in [14] it was shown how to optimize the choice of the matrix \mathbf{W} in order to minimize the mean square deviation $\delta[n] = E\{\|\mathbf{z}[n]\|^2\}$. But still, this only mitigates the problem, but it does not remove it.

3. IN-NETWORK PROCESSING THROUGH A SELF-SYNCHRONIZATION MECHANISM

An alternative way to reach consensus through a decentralized strategy was proposed in [17], based on the self-synchronization

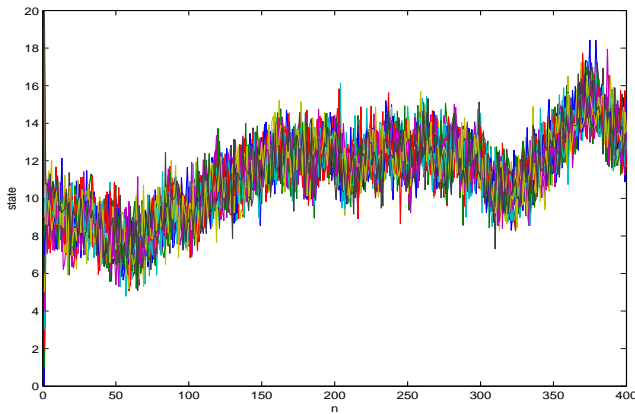


Fig. 1. Running consensus for each sensor vs. time index.

capabilities of a population of mutually coupled pulsed oscillators, borrowed from physiological models describing the heart beating [23]. The system proposed in [17] required full connectivity and it is strongly affected by propagation delays, as a delay is intrinsically indistinguishable from the time shift associated to the sensor measurement. An alternative approach was proposed in [18], where it was shown that a population of nonlinearly coupled dynamical systems can be designed to reach a globally optimal maximum likelihood estimate, also in the case of local coupling, provided that the network is connected, i.e., there is a path, possibly composed of several hops between any pair of nodes, and the global coupling strength exceeds a threshold that depends on the network topology, the observation and the coupling function.

In the sensor network proposed in [18], [19], each node has a dynamical system that evolves in time according to the following equation

$$\dot{\theta}_i(t) = g(x_i) + \frac{K}{c_i} \sum_{j=1}^N a_{ij} f[\theta_j(t) - \theta_i(t)] + v_i(t), \quad (6)$$

for $i = 1, \dots, N$, where $\theta_i(t)$ is the state function of the i -th sensor ($\theta_i(0)$ may be initialized to zero or to any random number, $g(x_i)$ is a function of the local measurement x_i taken by node i , the summation takes into account the coupling with all other nodes, $f(\cdot)$ is a monotonically increasing, odd function of its argument and it describes the mutual coupling among the sensors¹; K is a global control loop gain; c_i is a local coefficient that depends on the SNR at node i (its effect will be clarified later on); the amplitudes a_{ij} account for the local coupling between oscillators and are assumed to be symmetric ($a_{ij} = a_{ji}$) and non negative. We assume that two oscillators are coupled (i.e., $a_{ij} \neq 0$) if their distance is smaller than the coverage radius of each sensor². The run-

¹Without loss of generality, $f(x)$ is normalized so that $df(0)/dx = 1$.

²The coverage radius is assumed to be the same for all sensors, even

ning decision, or estimate, of each sensor is encoded in its pulsation $\dot{\theta}_i(t)$.

We define the synchronization of the population of dynamical systems as the situation where all the derivatives $\dot{\theta}_i(t)$ converge to the same function $\dot{\theta}^*(t)$, for $i = 1, \dots, N$.

Multiplying each equation (6) by c_i and summing over i , for $i = 1, \dots, N$, it is easy to check that, thanks to the symmetry of the coefficients a_{ij} and of the anti-symmetry of the function $f(x)$, if the system synchronizes, in the sense defined above, all the derivatives $\dot{\theta}_i(t)$ tend to

$$\begin{aligned} \dot{\theta}^*(t) &= \frac{\sum_{i=1}^N c_i g(x_i)}{\sum_{i=1}^N c_i} + \frac{\sum_{i=1}^N c_i v_i(t)}{\sum_{i=1}^N c_i} \\ &\triangleq \omega^* + v(t). \end{aligned} \quad (7)$$

In [20], it was proved that, in the noiseless case where $v_i(t) = 0$, if the network is connected and K is greater than a critical value K_c , then the equilibrium $\dot{\theta}^*(t) = \omega^*$ is globally asymptotically stable. This means that all derivatives $\dot{\theta}_i(t)$ converge, asymptotically, to the unique (constant) value ω^* , irrespective of the initial conditions. The critical value K_c is upper bounded by the following inequality [20]:

$$K_U = \frac{2 \|\mathbf{D}_c \Delta \omega\|_2}{f_{\max} \lambda_2(\mathbf{L}_A)}, \quad (8)$$

where $\mathbf{D}_c = \text{diag}(c_1, \dots, c_N)$, $\Delta \omega \triangleq \omega - \omega^* \mathbf{1}_N$, with $\omega \triangleq g(x)$ and ω^* defined in (7), $f_{\max} \triangleq \max_{x \in \mathbb{R}} f(x)$; $\lambda_2(\mathbf{L}_A)$ is the so called *algebraic connectivity* of the graph, i.e., the second smallest eigenvalue of the weighted Laplacian $\mathbf{L} \triangleq \mathbf{B} \mathbf{D}_A \mathbf{B}^T$, where \mathbf{B} is the incidence matrix of the graph associated to the network. The algebraic connectivity provides important information about the network connectivity. For example, if the network is disconnected, $\lambda_2(\mathbf{L}_A) = 0$. Conversely, the higher is the degree of connectivity, the higher is the value of $\lambda_2(\mathbf{L}_A)$. The rate of convergence to the unique synchronization state is given by the product $K \lambda_2(\mathbf{L}_A)$. This means that a higher network connectivity (degree) increases the convergence speed, but at the same time it requires more energy to guarantee a higher node degree. But a higher convergence speed implies also a reduced energy to reach the final value. Hence, in general, considering both the energy spent to establish a link and the energy spent to reach the global equilibrium, we may expect an optimal degree of connectivity.

If the coupling function $f(x)$ is linear (or nonlinear unbounded), $f_{\max} = \infty$, so that a linear coupling system can be always made to converge to the common consensus, as, from (8), the critical threshold is zero. Conversely, a nonlinear bounded coupling system has a nonnull critical value K_c

though this could be changed to accommodate for different network topological models, like small worlds or scale-free networks.

that makes possible a variety of behaviors, impossible with linear coupling systems, that could be used when one does not want the whole network to converge to a common value, but rather to form clusters.

The final decision, or estimate, achievable by each sensor is then a function $h(\omega^*)$. Choosing the functions $h(\cdot)$ and $g(\cdot)$, as well as the coefficients c_i , appropriately, we can design the network to converge to a large class of functions of the initial measurements $f(x_1, \dots, x_N)$, like the mean, the maximum, the geometric mean, and so on. The mean is achieved by simply setting $c_i = 1$, for all i and $g(x) = h(x) = x$. The maximum can be achieved, approximately, by setting $c_i = 1$ and choosing $g(x_i) = x_i^p$ and $h(\omega^*) = (N\omega^*)^{(1/p)}$, with p a large integer value. The geometric mean corresponds to $c_i = 1$ $g(x_i) = \log(x_i)$ and $h(\omega^*) = \exp(N\omega^*)$.

There is a basic difference that makes system (6) more robust than (1), against additive noise. In fact, different from (1), the additive noise $v(t)$ in (7) has a constant variance. This is a consequence of having encoded the global estimate on the derivative $\dot{\theta}_i(t)$ of the state, rather than on the state $\theta_i(t)$. Furthermore, if each sensor knows its own noise variance σ_i , it can set $c_i = 1/\sigma_i$. From (7), this implies that the sensor with the smallest noise is the one that mostly influences the final equilibrium and thus the other sensors estimates. Ideally, if one sensor has a vanishing noise, it forces all other sensors to converge to its own estimate. Interestingly, this happens without requiring any sensor to know which other sensor has the smallest noise. An example of application is reported in Fig. 2, showing the behavior of the state derivatives $\dot{\theta}_i(t)$ for each sensor, for the same network topology and noise properties as in Fig. 1. We can clearly see that in this case, each oscilla-

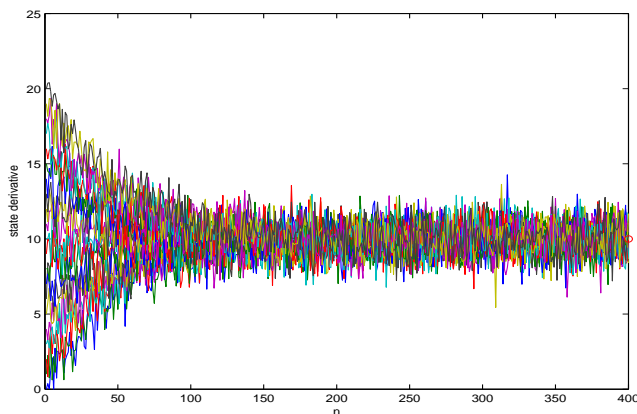


Fig. 2. State derivative of each sensor, as a function of iteration index.

tor converges to a noisy estimate, with a constant estimation variance.

4. GLOBALLY OPTIMAL ESTIMATION THROUGH LOCAL INTERACTIONS

In the case of linear observation models, the self-synchronization algorithm described before can be used to achieve globally optimum maximum likelihood estimates, using only local exchange of information, provided that the network is fully connected, as proposed in [18]. Let us consider the observation model

$$\mathbf{y}_i = \mathbf{A}_i \mathbf{x} + \mathbf{w}_i, \quad (9)$$

where \mathbf{x} is the unknown parameter vector, assumed to be the same for all sensors; \mathbf{A}_i is the mixing matrix of sensor i , and \mathbf{w}_i is the observation noise vector, modeled as a circularly symmetric complex Gaussian vector with zero mean and covariance matrix \mathbf{C}_i . We assume that the noise vectors affecting different sensors are statistically independent of each other (however, the noise vector present in each sensor may be colored). Let us denote with L the number of unknowns, so that \mathbf{x} is a column vector of size L . The observation vector \mathbf{y}_i has dimension M . We consider the case where the single sensor must be able, in principle, to recover the parameter vector from its own observation. This requires that $M \geq L$ and that \mathbf{A}_i is full column rank. Generalizing the strategy described in the previous section to the vector case, in this case the dynamical system in each node evolves according to the following vector state equation

$$\dot{\boldsymbol{\theta}}_i(t) = \hat{\mathbf{x}}_{ML}^{(i)} + K(\mathbf{A}_i^H \mathbf{C}_i^{-1} \mathbf{A}_i)^{-1} \sum_{j=1}^N a_{ij} f(\boldsymbol{\theta}_j(t) - \boldsymbol{\theta}_i(t)), \quad (10)$$

with $i = 1, \dots, N$, where $\hat{\mathbf{x}}_{ML}^{(i)} = (\mathbf{A}_i^H \mathbf{C}_i^{-1} \mathbf{A}_i)^{-1} \mathbf{A}_i^H \mathbf{C}_i^{-1} \mathbf{y}_i$ is the local ML estimate. Hence, if the system has the capability to reach a synchronization state, where $\dot{\boldsymbol{\theta}}_i(t) = \dot{\boldsymbol{\theta}}^*(t)$, for all i , that state must necessarily be

$$\dot{\boldsymbol{\theta}}^*(t) = \left(\sum_{i=1}^n \mathbf{A}_i^H \mathbf{C}_i^{-1} \mathbf{A}_i \right)^{-1} \left(\sum_{i=1}^n \mathbf{A}_i^H \mathbf{C}_i^{-1} \mathbf{y}_i \right). \quad (11)$$

The conditions insuring the global asymptotic stability of the synchronized state (11) were established in [20] and are of the same form as (8). Hence, if K is sufficiently large, every sensor reaches the globally optimum ML estimate. The same approach was also extended to achieve globally optimum decentralized detection in [21].

In the presence of propagation delays, the system (6) becomes

$$\dot{\boldsymbol{\theta}}_i(t) = \omega_i + \frac{K}{c_i} \sum_{j=1}^N a_{ij} f(\boldsymbol{\theta}_j(t - \tau_{ij}) - \boldsymbol{\theta}_i(t)), \quad (12)$$

where τ_{ij} denotes the propagation delay of the signal going from node j to node i . In [22], we proved that, if $f(x)$ is linear and the maximum delay is bounded, also this system may

be guaranteed to converge to a unique stable equilibrium. If the system is capable to detect the sign of the delay (i.e., to distinguish between delay and anticipation), the final equilibrium converges to the equilibrium achievable with no delays, otherwise the final equilibrium is biased. In the case of a non-linear $f(x)$, the analysis is more complicated. Nevertheless, in [22] a small perturbation analysis was used to validate simulation results showing the convergence of the network.

In summary, in-network processing based on self synchronization mechanisms appears to be a promising strategy. Further research developments include the possibility of implementing a totally distributed spatial smoothing, instead of simple averaging. An interesting aspect to be studied is energy consumption, in relationship with both network connectivity and convergence rate. The analysis of time-varying and inhomogeneous phenomena through a network with, possibly, time-varying topology, is another topic of interest. Finally, it can be shown that (6) can be rewritten, under specific assumptions on the weights a_{ij} and on the topology, as a diffusion equation. This remark suggests that information can propagate through the network simply as a diffusion process. This would pave the way to totally innovative mechanisms to route the information from where the event occurred to the control centers.

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