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# ON THE ERROR EXPONENT OF THE WIDEBAND RELAY CHANNEL

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## ABSTRACT

*We investigated the error exponent of the wideband relay channel. By computing the random coding error exponent of three different relay strategies, i.e., amplify-and-forward (AF), decode-and-forward (DF) and block Markov code (BMC), we found that relayed transmission can enhance the wireless link reliability significantly in the wideband regime compared to direct transmission. We also studied optimal power allocation and relay placement by maximizing the reliability function. Analytical and numerical results show, for DF and BMC relays, placing the relay node in the middle of source and destination provides the best link reliability. But for the AF relay scheme, the optimal relay placement depends on the path-loss exponent; for large path-loss exponents, half-way relay placement is also optimal.*

## 1. INTRODUCTION

Relayed transmission has received increasing attention as it can provide distributed space diversity to combat the fading impairment in the wireless network. The classical relay channel was introduced by van der Mulen [1], and then further explored by Cover and El Gammal [2]. Laneman et al., [3] analyzed the outage behavior and diversity order for several relay protocols. Their results characterized the diversity multiplexing trade-off at the high signal-to-noise ratio (SNR). Recently, Liang and Veeravalli [4] studied the optimal resource allocation problem for the Gaussian orthogonal relay channel. However, most previous work has primarily focused on narrow-band relay transmission, where the received SNR per degree of freedom is high. In this paper, we study the performance of the relay channel in the wideband extreme, i.e., the available bandwidth is large and the resulting SNR per degree of freedom is low. Relevant examples are wireless ad-hoc and sensor networks.

We use Gallager's random code error exponent [5] (also known as the channel reliability function) as a tool to analyze different relay strategies. Error exponent provides a measure of how fast the decoding error probability decays exponentially as the code block length increases for rates below channel capacity. We show that, for orthogonal relaying, both AF and DF provide higher reliability than the direct transmission, and the DF scheme has better performance than AF for similar settings. If we relax the orthogonal constraint, i.e., the relay node can receive and transmit message at the same time (full duplex), block Markov coding scheme can be used to boost the link reliability even more. The error exponent can serve as a performance measure to optimize the power allocation and relay node placement. We found that placing relay node in the middle between source and destination can provide the best link reliability for DF and BMC schemes. But for the AF scheme, the optimal position depends on the path-loss exponent of the physical wireless propagation model.

The remainder of this paper is organized as follows. Section II introduces the system model for the problem under consideration. Section III defines the the random coding error exponent. Section IV and Section V give out the error exponent results for various relay strategies. Some numerical results are provided in Section VI. Section VII summarizes the main results of the paper.

## 2. SYSTEM MODEL

In this work, communication occurs over a relay network, with one relay node and one source-destination pair, as is shown in Fig. 1. The source $S$ broadcasts the message to both relay $R$ and destination $D$. The relay processes the message and then sends it to the destination to assist the destination decoding the data. Based on the limitation of relay node, we focus on two kinds of relay: 1) orthogonal relay (half-duplex), i.e., transmitting and receiving in the different time or frequency subchannels. The AF and DF schemes fall into this category. 2) Full duplex operation, including block Markov coding transmission. We model the wideband

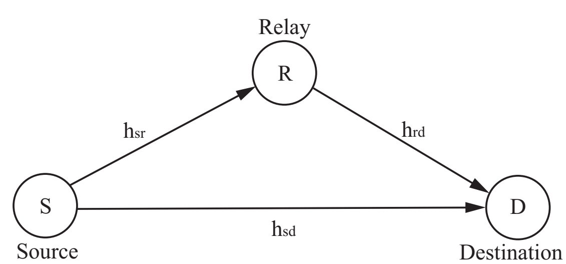


Figure 1: Layout of Relay Network.

channel as a set of $N$ parallel narrowband channels. We assume that the Doppler spread is negligible, which makes the narrowband channels have independently and identically distributed (i.i.d.) statistics. Moreover, we assume that the coherence bandwidth is much larger than the bandwidth of the narrowband channel, such that each channel is flat faded. Using the sampling theorem, the received signal at the relay and the destination for the $n^{th}$ channel and $k^{th}$ symbol time can be written, respectively, as

$$y_r[k,n] = \sqrt{\frac{P_s}{N}} h_{s,r}[k,n] x_s[k,n] + z_r[k,n] \quad (1)$$

$$y_d[k,n] = \sqrt{\frac{P_s}{N}} h_{s,d}[k,n] x_s[k,n] + \sqrt{\frac{P_r}{N}} h_{r,d}[k,n] x_r[k,n] + z_d[k,n] , \quad (2)$$

where $x_i[k,n]$ is the source/relay transmitted signal with $i \in \{s,r\}$. We assume $E[|x_i[k,n]|^2] = 1$, and let the transmit power at the source and relay be $P_s$ and $P_r$ respectively. In $(1)-(2)$, $h_{i,j}[k,n]$ is the fading coefficient, where $i \in \{s,r\}$ and $j \in \{r,d\}$; $z_j[k,n]$ represents the additive white noise for $j \in \{r,d\}$. The pair $(k,n)$ can be considered as indices for the time-frequency slot, or degrees of freedom, to communicate. Statistically, we model $h_{i,j}[k,n]$ as zero-mean, circularly-symmetric complex Gaussian random variables, which are independent across different narrowband channels and links. Additionally, we model $z_j[k,n]$ as zero-mean, independent, circularly-symmetric complex Gaussian random variables with variances $N_0$.

In this work, we simplify the model in Fig. 1. We assume that the distance between the source and destination is normalized to one, and the relay is located on a line between the source and destination. The parameter $d$ represents the distance from source to relay, and $(1-d)$ is the distance from the relay to the destination. Using physical path-loss propagation model for wireless communication [8], we assume $E[|h_{i,j}|^2] = \frac{1}{d_{i,j}^{\alpha}}$, where $d_{i,j}$ is the distance from transmitter $i$ to receiver $j$ and $\alpha$ is path-loss exponent.

Furthermore, we assume there is no decoding delay and coding is across the different narrowband channels. Our goal is to compute the error exponent of this wideband relay transmission and to study the optimal power allocation and relay placement. Since we assume i.i.d. statistics across the narrowband channels, we can aim at one narrowband channel with source and relay power constraint $(\frac{P_s}{N}, \frac{P_r}{N})$. As $N \to \infty$, the power allocated to each narrowband channel goes to 0. Equivalently, we can focus on analyzing a narrowband flat fading channel in the low SNR regime. For convenience, we omit the narrowband index $n$. With a little abuse of notation, let $(P_s, P_r)$ represent the transmit power at source/relay for each narrowband channel, which can take a very small value.

## 3. THE RANDOM CODING ERROR EXPONENT

Gallager [5] established random coding techniques to upper-bound the achievable average error probability over a random code ensemble with maximum-likelihood decoding. Specifically, given a code $\mathbb{C}$ of length $N$ over an alphabet $\chi$ with $2^{nR}$ codewords, we have

$$\bar{P_e} \leq \exp\left(-N(E_0(\rho,Q) - \rho R)\right) , \quad (3)$$

with $E_0(\rho,Q)$ defined as

$$E_0(\rho,Q) = -\ln\left(\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} Q(\mathbf{X}) f(\mathbf{Y}/\mathbf{X})^{1/(1+\rho)} d\mathbf{X}\right]^{1+\rho} d\mathbf{Y}\right) , \quad (4)$$

for $0 \leq \rho \leq 1$. $Q(\mathbf{X})$ is the code symbol (or input) distribution and $f(\mathbf{Y}/\mathbf{X})$ is the channel output distribution conditioned on the input. The random coding exponent is defined to be the one that yields the tightest bound:

$$E_r(R) = \max_{\rho}\max_{Q}\{E_0(\rho,Q) - \rho R\} , \quad (5)$$

where the maximization is over $Q$ and subject to the input power constraint. For linear Gaussian Channel model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} , \quad (6)$$

if we assume input symbol $\mathbf{x}$ has Gaussian distribution $\mathbf{x} \sim CN(0,\mathbf{P})^1$, and noise $\mathbf{z}$ has circular symmetric gaussian distribution $\mathbf{z} \sim CN(0,\mathbf{W})$. Substituting $Q(\mathbf{x})$ and $f(\mathbf{y}/\mathbf{x})$ into (5), we can get the following theorem.

**Theorem 1** *(*Gaussian Error Exponent*) of (6):*

$$E_0(\rho,\mathbf{P}) = \rho \ln \mathbf{E}_H\left|\mathbf{I} + \frac{1}{1+\rho}\mathbf{W}^{-1}\mathbf{H}\mathbf{P}\mathbf{H}^{\dagger}\right| , \quad (7)$$

*where $\mathbf{E}$ denotes expectation, and $|\cdot|$ represents determinant of matrix.*

We omit the proof of this theorem because the result can be found in other literature [6]. If the channel model (6) reduces to the scalar one, Eq. (7) can be written as $E_0(\rho,P) = \rho \ln \mathbf{E}(1 + \frac{P|h|^2}{N_0(1+\rho)})$, which is the well known error exponent for the scalar fading channel.

## 4. ERROR EXPONENT FOR ORTHOGONAL RELAY CHANNEL

For orthogonal relay operation, the relay node can not transmit and receive at the same time. We partition the transmission as two steps. First, source $S$ broadcasts message and relay $R$ keeps silent, i.e.,let $x_r[k,n] = 0$ in Eq. $(1)-(2)$. In the next step, relay $R$ transmits the processed message to destination and source $S$ stops transmission. Mathematically, for the first step, the received signal of each equivalent narrowband channel can be written as

$$y_r = \sqrt{P_s} h_{s,r} x_s + z_r \quad (8)$$

$$y_d[1] = \sqrt{P_s} h_{s,d} x_s + z_d[1] , \quad (9)$$

For the next step, we obtain

$$y_d[2] = \sqrt{P_r} h_{r,d} x_r + z_d[2] . \quad (10)$$

### 4.1 Amplify-and-forward (AF) Relay

Using the amplify-and-forward relay scheme, the relay node amplifies the message it received in the first phase and forwards it to the destination in the second phase, i.e.,

$$\sqrt{P_r} x_r = \beta y_r , \quad (11)$$

here we define the amplifier gain as $\beta = \sqrt{\frac{P_r}{P_s h_{sr}^2 + N_0}}$. Substitute (11) into (10) and write the received signal during the two phases in vector form

$$\underbrace{\begin{pmatrix} y_d[1] \\ y_d[2] \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} h_{s,d} \\ h_{r,d}\beta h_{s,r} \end{pmatrix}}_{\mathbf{H}} \sqrt{P_s} x_s + \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ h_{r,d}\beta & 0 & 1 \end{pmatrix}\begin{pmatrix} z_r \\ z_d[1] \\ z_d[2] \end{pmatrix}}_{\mathbf{z}} ,$$

[1]To choose $Q(x)$ as Gaussian is not optimal and a distribution concentrated on a "thin spherical shell" will give better results [5], nonetheless Gaussian error exponent is a convenient lower bound for the optimal error exponent.

Note that

$$E(\mathbf{z}\mathbf{z}^{\dagger}) = \begin{pmatrix} N_0 & 0 \\ 0 & |h_{r,d}\beta|^2 N_0 + N_0 \end{pmatrix} .$$

We observed that the AF relay is equivalent to a single-input-multiple-output (SIMO) channel. Using Theorem 1, we have following result.

**Theorem 2** *(*Error Exponent of AF relay*) :*

$$E_r^{AF}(R) = \max_{0\le\rho\le 1}\left\{\frac{1}{2}\rho \ln \mathbf{E}\left(1+\frac{P_s}{(1+\rho)N_0}\left(h_{s,r}^2 + h_{s,d}^2 - \frac{h_{s,r}^2}{|h_{r,d}\beta|^2+1}\right)\right) - \rho R\right\} . \quad (12)$$

For fair comparison with direct transmission, we have halved the degree of freedom and doubled the rate as $2R$ to account for the half-duplex transmission.

If we fix the total power budget as $P$, our goal is to optimize the power allocation $(P_s, P_r)$ between source and relay transmission to maximize the error exponent of (12). Also, we try to find the optimal position in the line between source and destination to place the relay node. For wideband AF relay system, we assume the amplifier coefficient $\beta$ takes the same value for all the parallel narrowband channel. Practically, it is a reasonable assumption and need not use passband filters for each narrow band channel.

Let us define $SNR = \frac{P^{dir}}{N_0 W}$, where $P^{dir}$ is the direct transmission power in each channel use, and $W$ is bandwidth of each narrowband channel. Then we have $P_s = 2SNR\gamma, P_r = 2SNR(1-\gamma)$, where $0 \le \gamma \le 1$, denotes the fraction of power allocated to the source transmission. Let $\mathbf{E}[|h_{i,j}|^2] = \lambda_{i,j} = \frac{1}{d_{i,j}^{\alpha}}$. Hence, we can express $\beta$ as $\beta = \sqrt{\frac{2SNR(1-\gamma)}{2SNR\gamma\lambda_{sr}+1}}$.

Substituting all the terms into Eq. (12), and computing expectation value with respect to the channel gain, we have the following lemma.

**Proposition 1** *For the AF relay channel, the error exponent is given by*

$$E_r^{AF}(R) = \max_{0\le\rho,\gamma,d\le 1}\left\{\frac{\rho}{1+\rho}\gamma SNR\left(1+\frac{1}{d^{\alpha}} + \frac{1}{d^{\alpha}}C\exp(C)E_i(-C)\right) - \rho R\right\} , \quad (13)$$

*where $d$ is the distance between source and relay; $C = \frac{(2SNR\gamma+d^{\alpha})(1-d)^{\alpha}}{2SNR(1-\gamma)d^{\alpha}}$; $E_i(\cdot)$ is the exponential integral function [7, pp. 925].*

**Remark 1** *The optimal values $(d^*, \gamma^*)$ to maximize the error exponent (13) depend on the path-loss exponent $\alpha$. For $\alpha \ge 4$, $d^* \approx 1/2$, hence placing the relay node in the middle point of source S and destination D is optimal for large path-loss exponent. The optimal value $(d^*, \gamma^*)$ monotonically decreases from 1 to 0.5 as path-loss exponent $\alpha$ increases.*

*Proof:* We omit the proof here due to space limitations.

Table 1: Optimal Relay Position $d^*$ and Power Allocation $\gamma^*$

| $\alpha$ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $d^*$ | 0.99 | 0.79 | 0.52 | 0.50 | 0.50 |
| $\gamma^*$ | 0.98 | 0.87 | 0.59 | 0.54 | 0.53 |

Maximizing the AF error exponent (13) over $d$ and $\gamma$ can be easily decoupled from maximization with respect to $\rho$. Hence, we can numerically search the two-dimensional space of $d$ and $\gamma$. Although we were not able to show analytically that $E_r^{AF}(R)$ is concave in $(d,\gamma)$, our simulation results indicate it. Also, note that the relay placement and power allocation are independent of the SNR values. We summarize optimal value $(d^*, \gamma^*)$ for typical $\alpha$ value in Table I.

### 4.2 Decode-and Forward (DF) Relay

For DF relay, the relay node decodes the source message it received from the source as $\hat{\mathbf{x}}_s$ for $N$ narrowband carriers, re-encodes the information and sends it to the destination in the second step. In this work, we assume the simple repetition-coded scheme. The relay retransmits the signal as

$$x_r[n] = \hat{x}_s[n] ,$$

where $n$ is the narrowband channel index; $\hat{x}_s$ is the decoded data at the relay node that was sent from the source. The error probability of DF relay transmission is:

$$\begin{aligned} P_e^{DF} &= \exp(-NE_r^{SR}) + \left(1-\exp(-NE_r^{SR})\right)\cdot\exp(-NE_r^{MAC}) \\ &\approx \exp(-NE_r^{SR}) + \exp(-NE_r^{MAC}) , \end{aligned} \quad (14)$$

where $E_r^{SR}$ is the error exponent of source-relay transmission; $E_r^{MAC}$ denotes the destination decoding error exponent given repeated transmission from source and relay in two steps. Here we have assumed the number of narrowband carriers $N$ or code block length is large enough that the error probability of source-relay is very small. According to Theorem 1, we have

$$E_r^{SR} = \max_{0\le\rho\le 1}\left\{\frac{1}{2}\rho\ln\left(1+\frac{P_s\lambda_{sr}}{N_0(1+\rho)}\right) - \rho R\right\} \quad (15)$$

$$E_r^{MAC} = \max_{0\le\rho\le 1}\left\{\frac{1}{2}\rho\ln\left(1+\frac{P_s\lambda_{sd}+P_r\lambda_{rd}}{N_0(1+\rho)}\right) - \rho R\right\} . \quad (16)$$

Again, we halved degree of freedom and doubled the rate to $2R$ for the half-duplex communication.

**Proposition 2** (Error Exponent of fixed DF relay) *:* $E_r^{DF} = \min\{E_r^{SR}, E_r^{MAC}\}$.

We want to maximize the error exponent by power allocation and relay placement. We are using physical path-loss model of wireless propagation, and let $P_s = 2SNR\gamma$, $P_r = 2SNR(1-\gamma)$. Mathematically, we have following the optimizing problem,

$$\max_{0\le d,\gamma\le 1}\min\left\{2SNR\gamma\frac{1}{d^{\alpha}}, (2SNR\gamma + 2SNR(1-\gamma)\frac{1}{(1-d)^{\alpha}})\right\} . \quad (17)$$

Since the first term monotonically increases as $\gamma$ and $d$ increase from 0 to 1, but the second term is a monotonically decreasing function of $\gamma$ and $d$, the minimum in (17) can be achieved when the first term equals to the second one. Hence, we can reduce the problem to

$$\max_{0\le d,\gamma\le 1} \gamma\frac{1}{d^{\alpha}} \qquad \text{subject to} \quad \gamma\frac{1}{d^{\alpha}} = \gamma + (1-\gamma)\frac{1}{(1-d)^{\alpha}} \ . \tag{18}$$

It is a one dimensional maximization, we can readily get the solution. We summarize the above analysis of the optimal $(d^*, \gamma^*)$ in the following remark.

**Remark 2** *The optimal value to maximize DF error exponent is $(d^*, \gamma^*) = (\frac{1}{2}, \frac{1}{2-2^{-\alpha}})$. Hence placing the relay node in middle point of source-relay line is optimal to boost the link reliability, and the power allocation $\gamma$ is close to one half as path-loss exponent $\alpha$ increase.*

It is well known that adaptive type DF, i.e., switch back to direction transmission in the event of relay decoding error, can achieve full diversity in the high SNR regime. However, in our wideband relay case, adaptive DF amounts to choose the better error exponent between direct transmission and DF transmission. Since DF relay has much higher error exponent value than direct transmission, adaptive type DF can not improve the performance anymore in our case.

## 5. ERROR EXPONENT FOR BLOCK MARKOV CODING (BMC)

In this section, we focus on the full-duplex relay operation, i.e., when relay node can receive and transmit at the same time. Block Markov Coding (BMC) was first proposed by Cover and El Gammal [2] to derive the lower bound for the relay channel capacity. For convenience, we briefly restate the BMC process in the wideband multi-carrier background. The information bearing bits stream (message) at the source is parsed into blocks, each with $N$ symbols; each block of $N$ symbols can be transmitted in $N$ narrowband carrier for one channel use. Let $w_i \in [1, 2^{NR}]$ be the message sent by the source during $i^{th}$ block. The set of message $\mathscr{W} = \{1, 2, \cdots, 2^{NR}\}$ is randomly partitioned into bins $\mathscr{S} = \{S_1, S_2, \cdots, S_{2^{NR_0}}\}$ with $R_0 < R$. A random codebook $\mathscr{X} = \{\mathbf{x}_1(w|s), \mathbf{x}_2(s)\}$ is generated based on the joint probability distribution $p(x_1, x_2)$, where $w \in [1, 2^{NR}]$ and $s \in [1, 2^{NR_0}]$. After the relay successfully decodes the message from the source during the $(i-1)^{st}$ block, it transmits a codeword $\mathbf{x}_2(s_i)$ in the $i^{th}$ block to help destination decode the previously received message. For detailed description, please refer to [2]. If we assume the entries of codewords $\mathbf{x}_1(w|s)$ and $\mathbf{x}_2(s)$ are independent, identical Gaussian distribution with zero mean and unit variance. The simultaneously transmitted signal vectors by source and relay in $i^{th}$ block are given, respectively, by

$$\begin{aligned}\mathbf{x}_s &= \sqrt{P_1}\mathbf{x}_1(w_i|s_i) + \theta\sqrt{(1-\gamma_2)P_2}\mathbf{x}_2(s_i)\\ \mathbf{x}_r &= \sqrt{\gamma_2 P_2}\mathbf{x}_2(s_i)\ ,\end{aligned} \tag{19}$$

where $P_1$ and $P_2$ are transmitted power of $\mathbf{x}_1(w|s)$ and $\mathbf{x}_2(s)$; $\gamma_2 \in (0,1)$ denotes the fraction of power $P_2$ allocated to relay. $\theta$ is the phase tuning factor to assist source-relay combining, which satisfies $|\theta|^2 = 1$. The received vector at the relay and destination can be expressed, respectively, as

$$\begin{aligned}\mathbf{y}_r &= \mathbf{h}_{sr}\cdot\mathbf{x}_s + \mathbf{z}_r\\ \mathbf{y}_d &= \mathbf{h}_{sd}\cdot\mathbf{x}_s + \mathbf{h}_{rd}\cdot\mathbf{x}_r + \mathbf{z}_d,\end{aligned} \tag{20}$$

where $\mathbf{h}_{ij}$ represents channel coefficient vector for i.i.d. narrowband carriers, for $i \in \{s, r\}$ and $j \in \{r, d\}$; $(\cdot)$ denotes componentwise multiplication. For BMC relay strategies, there are two transmissions for each message, one for source-relay link; the other are the source and relay multiple-access to the destination. By the Theorem 1, we have the following result for BMC relay.

**Proposition 3** (Error Exponent of BMC relay) *: $E_r^{BMC} = \min\{E_r^{B-SR}, E_r^{B-MAC}\}$, where*

$$E_r^{B-SR} = \max_{0\le\rho\le 1}\left\{\rho\ln\left(1 + \frac{P_1\lambda_{sr}}{N_0(1+\rho)}\right) - \rho R\right\} \tag{21}$$

$$\begin{aligned}E_r^{B-MAC} = \max_{0\le\rho,\gamma_2\le 1,\gamma_2}\Bigg\{\rho\ln\Bigg(1\\ + \frac{P_1\lambda_{sd} + (1-\gamma_2)P_2\lambda_{sd} + \gamma_2 P_2\lambda_{rd}}{(1+\rho)N_0} - \rho R\Bigg\}\ .\end{aligned} \tag{22}$$

In our system model, $\lambda_{rd} \ge \lambda_{sd}$, so the $E_r^{B-MAC}$ is maximized when $\gamma_2 = 1$. Hence the transmitted signal in (19) reduces to

$$\mathbf{x}_s = \sqrt{P_1}\mathbf{x}_1(w_i|s_i), \quad \mathbf{x}_r = \sqrt{P_2}\mathbf{x}_2(s_i)\ . \tag{23}$$

Let us assume $\frac{P_1}{N_0} = \gamma_1\,\text{SNR}$ and $\frac{P_2}{N_0} = (1-\gamma_1)\,\text{SNR}$. The error exponent of BMC relay degenerates to a form similar to DF relay, but with half rate and double degree of freedom. The difference here comes from the full-duplex relay, rather than the orthogonal operation in the DF scheme. The results of optimal power allocation and relay placement for DF relay can also be applied here directly.

## 6. NUMERICAL RESULTS

In this section, we present numerical results to illustrate the advantage of the relayed transmission. We assume the relay node is placed in the line connecting the source and destination; the distance between source-destination is normalized to one. For each link, we consider the physical path-loss channel model with $\alpha = 4$. We normalized the SNR value and degree of freedom for a fair comparison among direct transmission, orthogonal relay and BMC relay, i.e. for the orthogonal relay, the degree of freedom is halved, rate and SNR value for each message transmission is doubled.

Fig. 1 compares the error exponent of different transmission strategies with optimal power allocation and relay placement. The SNR value is $-3$ dB, which accounts for our wideband low SNR assumption. We observed that the BMC relay has the highest reliability because we allow the full-duplex operation. Note that the DF scheme has an advantage over AF schemes in the error exponent sense. This observation is in contrast to the existing results in the literature that both AF and adaptive DF achieve full diversity in the high SNR regime. All of the above relay transmissions provide a significant reliability gain over the direct transmission.

To further illustrate the advantage of using relay in the wideband wireless transmission, Fig. 2 plots the minimum number of narrowband carriers to achieve a prescribed decoding error probability. The SNR value is defined as total power per channel use divided by $N_0$. The rate represents the sum rate of all the narrowband carriers. It is required to solve for $N$ in the following equation $P_e = \exp(-NE_r(\frac{SNR}{N}, \frac{R}{N}))$ Fig. 2 shows that the relay strategies require far fewer carriers to achieve the prescribed decoding error probability for the same SNR value and transmission rate, compared with the direct transmission. Hence, it requires less bandwidth or provides higher spectral efficiency.

## 7. CONCLUSION

Random coding error exponents provide more information than the capacity. For any rate below the capacity, they quantify (lower bound) the exponential decay rate of the maximum-likelihood decoding error probability averaged over randomly selected codes. In this paper, we derived the random error exponent of the relay channel wideband relay strategies, analytical and numerical results show that using relay can indeed improve the system reliability significantly for rate below the capacity, which can save power or reduce bandwidth required in the practical wireless system. Furthermore, using physical path-loss wireless propagation model, we investigated the optimal relay placement and power allocation to further boost the system reliability.

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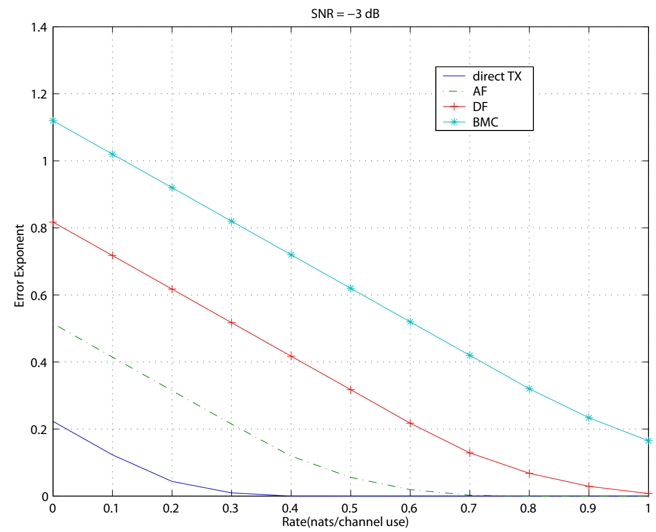


Figure 2: Error exponent vs. rate with optimal power allocation and relay placement.

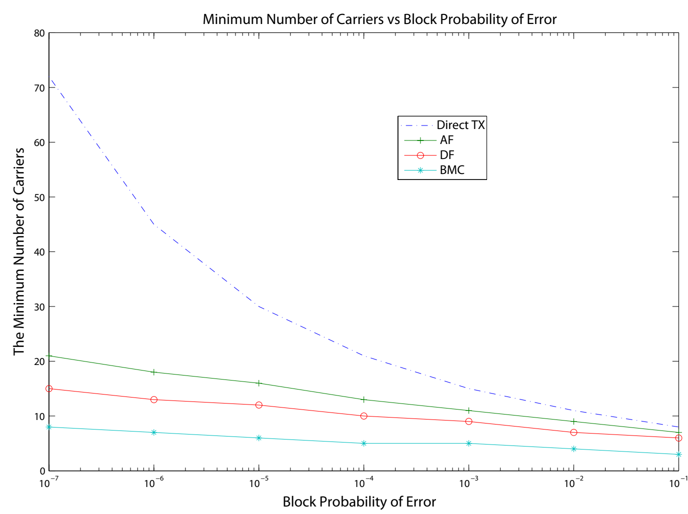


Figure 3: Minimum no. of carriers needed to get prescribed error probability, SNR = 18 dB, R = 10 nats/channel use.